18

20,

BLOQUET
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PL1
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1). $\begin{pmatrix} 1 & 3 & -2 & 0 \\ -1 & 0 & -1 & 0 \\ 2 & 1 & 1 & 0 \end{pmatrix}$ 2

2). $\begin{pmatrix} 1 & 3 & -2 & 0 \\ -1 & 0 & -1 & 0 \\ 2 & 1 & 1 & 0 \end{pmatrix} \leftarrow +L_2 \quad \leftarrow +2L_2$

$\begin{pmatrix} 0 & 3 & -3 & 0 \\ -1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix} \leftarrow -3L_3$

$\begin{pmatrix} 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 \\ 0 & 1 & -1 & \end{pmatrix} \leftarrow L_1 \leftrightarrow L_2$

$\begin{pmatrix} -1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \times -1$

$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ 3

le 3) est a la dernière page.

4). $X = \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}$ $\quad {}^tX = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$.

${}^tX X = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{pmatrix}$ 2

5). $U = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 1 \\ 2 & 4 & 6 \end{pmatrix}$ $\quad {}^tU = \begin{pmatrix} 1 & 4 & 2 \\ 2 & 5 & 4 \\ 3 & 1 & 6 \end{pmatrix}$.

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$\frac{1}{2}(U + {}^tU) = \begin{pmatrix} 1 & 3 & 5/2 \\ 3 & 5 & 5/2 \\ 5/2 & 5/2 & 6 \end{pmatrix}$ $\quad \frac{1}{2}(U - {}^tU) = \begin{pmatrix} 0 & -1 & 1/2 \\ 1 & 0 & -3/2 \\ -1/2 & 3/2 & 0 \end{pmatrix}$

$U = \frac{1}{2}(U + {}^tU) + \frac{1}{2}(U - {}^tU)$.

$$\begin{pmatrix} 1 & 3 & 5/2 \\ 3 & 5 & 5/2 \\ 5/2 & 5/2 & 6 \end{pmatrix} + \begin{pmatrix} 0 & -1 & 1/2 \\ 1 & 0 & -3/2 \\ -1/2 & 3/2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 1 \\ 2 & 4 & 6 \end{pmatrix} = U$$

← Vous avez lu l'énoncé trop vite !

6). $M = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

$M^2 = \begin{pmatrix} 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix}$.

$M^3 = \begin{pmatrix} 16 & 16 & 16 & 16 \\ 16 & 16 & 16 & 16 \\ 16 & 16 & 16 & 16 \\ 16 & 16 & 16 & 16 \end{pmatrix}$.

On remarque $M^2 = 4M$.

7). $M^2 = 4M$.

$M^3 = 4^2 M$ on conjecture $M^k = 4^{k-1} M$.

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8). Initialisation : $M^2 = 4M$.

$H_R \Rightarrow M^k = 4^{k-1} M$

~~$H_{R+1} \Rightarrow$~~ $M^{k+1} = M^k M$

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$= 4^{k-1} M M$

$= 4^{k-1} M^2$

$M^{k+1} = 4^k M$. H_{k+1}

3). $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ $\begin{cases} x + z = 0. \\ y - z = 0. \end{cases} \leftrightarrow \begin{cases} x = -z \\ y = z \end{cases}$

$S = \{(-z, z, z) \mid z \in \mathbb{R}\}$. 3