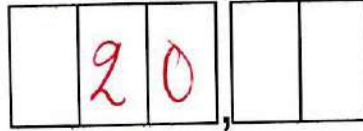


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BLOQUET
RomainPL1
2013

$$1). \begin{pmatrix} 1 & 3 & -2 & 0 \\ -1 & 0 & -1 & 0 \\ 2 & 1 & 1 & 0 \end{pmatrix} \quad 2$$

$$2). \begin{pmatrix} 1 & 3 & -2 & 0 \\ -1 & 0 & -1 & 0 \\ 2 & 1 & 1 & 0 \end{pmatrix} \begin{matrix} \leftarrow +1/2 \\ \\ \leftarrow +2/2 \end{matrix} \quad \begin{pmatrix} 0 & 3 & -3 & 0 \\ -1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix} \begin{matrix} \leftarrow -3/3 \\ \\ \end{matrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ -1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix} \begin{matrix} \\ \leftarrow L_1 \leftrightarrow L_2 \\ \end{matrix} \quad \begin{pmatrix} -1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{matrix} \times -1 \\ \\ \end{matrix}$$

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad 3$$

le 3) est a la dernière page.

$$4). \quad X = \begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix}$$

$${}^t X = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$$

$${}^t X X = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \\ 4 & 8 & 12 & 16 \end{pmatrix} \quad 2$$

5). $u = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 1 \\ 2 & 4 & 6 \end{pmatrix}$

${}^t u = \begin{pmatrix} 1 & 4 & 2 \\ 2 & 5 & 4 \\ 3 & 1 & 6 \end{pmatrix}$

$\frac{1}{2}(u + {}^t u) = \begin{pmatrix} 1 & 3 & 5/2 \\ 3 & 5 & 5/2 \\ 5/2 & 5/2 & 6 \end{pmatrix}$

$\frac{1}{2}(u - {}^t u) = \begin{pmatrix} 0 & -1 & 1/2 \\ 1 & 0 & -3/2 \\ -1/2 & 3/2 & 0 \end{pmatrix}$

$u = \frac{1}{2}(u + {}^t u) + \frac{1}{2}(u - {}^t u)$

$\begin{pmatrix} 1 & 3 & 5/2 \\ 3 & 5 & 5/2 \\ 5/2 & 5/2 & 6 \end{pmatrix} + \begin{pmatrix} 0 & -1 & 1/2 \\ 1 & 0 & -3/2 \\ -1/2 & 3/2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 1 \\ 2 & 4 & 6 \end{pmatrix} = u$

← Vous avez lu l'énoncé trop vite!

6). $M = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

~~$M^2 = \begin{pmatrix} 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix}$~~

~~$M^3 = \begin{pmatrix} 16 & 16 & 16 & 16 \\ 16 & 16 & 16 & 16 \\ 16 & 16 & 16 & 16 \\ 16 & 16 & 16 & 16 \end{pmatrix}$~~

On remarque $M^2 = 4M$.

7). $M^2 = 4M$

$M^3 = 4^2 M$

on conjecture

$M^k = 4^{k-1} M$

2

8). Initialisation: $M^2 = 4M$

$H_k \Rightarrow M^k = 4^{k-1} M$

~~$H_{k+1} \Rightarrow M^{k+1} = M^k M$~~

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$$= 4^{k-1} M M$$

$$= 4^{k-1} M^2$$

$$\underline{M^{k+1}} = 4^k M. \quad H_{k+1}$$

3).

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{cases} x + z = 0. \\ y - z = 0. \end{cases} \Leftrightarrow \begin{cases} x = -z \\ y = z \end{cases}$$

$$\mathcal{L} = \{(-z, z, z) \mid z \in \mathbb{R}\}. \quad \mathcal{L}$$

