Dérivées. Corrigé

- $[\ln(3x-1)]' = \dfrac{3}{3x-1}$ $\qquad x > \frac{1}{3}$
- $\left(e^{5x^2}\right)' = 10x e^{5x^2}$ $\qquad x \in \mathbb{R}$
- $[e^{3x}(4x+7)]' = 3e^{3x}(4x+7) + 4e^{3x} = e^{3x}(12x+25)$ $\qquad x \in \mathbb{R}$
- $\left((3x+1)^4\right)' = 4(3x+1)^3 \times 3 = 12(3x+1)^3$ $\qquad x \in \mathbb{R}$
- $[(4x+7)^3(3x-1)]' = 12(4x+7)^2(3x-1) + 3(4x+7)^3 = 3(4x+7)^2(16x+3)$
- $\left(\dfrac{3x^2+x-1}{5x^2+x+1}\right)' = \dfrac{(6x+1)(3x^2+x-1)-(3x^2+x-1)(10x+1)}{(5x^2+x+1)^2} = \dfrac{-2x^2+16x+2}{(5x^2+x+1)^2}$ $\qquad x \in \mathbb{R}$ $\qquad x \in \mathbb{R}$

  le dénominateur ne s'annule pas : le domaine est $\mathbb{R}$.
- $\left(\dfrac{\cos x}{1+\sin x}\right)' = \dfrac{-\sin x(1+\sin x)-\cos^2 x}{(1+\sin x)^2} = -\dfrac{1+\sin x}{(1+\sin x)^2} = -\dfrac{1}{1+\sin x}$

  $x \neq -\frac{\pi}{2} + 2k\pi$
- $[\sin(4x^2+1)]' = 8x\cos(4x^2+1)$ $\qquad x \in \mathbb{R}$
- $[e^{3x}\ln(x-1)]' = e^{3x}\left(3 + \dfrac{1}{x-1}\right) = \dfrac{e^{3x}}{x-1}(3x-2)$ $\qquad x > 1$

$[(7x+3)(4x-1)(3x+2)]' = 7(4x-1)(3x+2) + 4(7x+3)(3x+2) + 3(7x+3)(4x-1)$

$= 252x^2 + 142x + 1$ $\qquad x \in \mathbb{R}$

$\left(\dfrac{1+\sqrt{x}}{1+2\sqrt{x}}\right)' = \dfrac{\frac{1}{2\sqrt{x}}(1+2\sqrt{x}) - (1+\sqrt{x})\frac{1}{\sqrt{x}}}{(1+2\sqrt{x})^2} = -\dfrac{1}{2\sqrt{x}(1+2\sqrt{x})^2}$ $\qquad x \in \mathbb{R}_+^*$

$\left((4x^3-x+1)^7\right)' = 7(4x^3-x+1)^6 \times (12x^2-1)$ $\qquad x \in \mathbb{R}$

- $\left(\dfrac{1}{3\sqrt{x}} - e^{\sin x}\right)' = \left(\frac{1}{3}x^{-1/2} - e^{+\sin x}\right)' = -\frac{1}{6}x^{-3/2} - e^{\sin x}\cos x$ $\qquad x \in \mathbb{R}_+^*$
- $(\ln(1+x))' = \dfrac{1}{1+x} = (1+x)^{-1}$ $\qquad x \in \mathbb{R}_+^*$

  $\left((1+x)^{-1}\right)' = -1(1+x)^{-2} = (\ln(1+x))''$

  $(\ln(1+x))^{(3)} = (-1)(-2)(1+x)^{-3} = (-1)^2\, 2!\,(1+x)^{-3}$

  Conjecture $(\ln(1+x))^{(n)} = (-1)^{n-1}(n-1)!\,(1+x)^{-n}$

  On la prouve par récurrence.