Corrigé Dérivées et intégrales

$\int \sin 3x = -\frac{\cos 3x}{3} + C$ $\quad \int \cos\left(2x+\frac{\pi}{3}\right)dx = \frac{\sin\left(2x+\frac{\pi}{3}\right)}{2} + C$ $\quad \int (7x^2+9x-1)dx = \frac{7x^3}{3} + \frac{9x^2}{2} - x + C$

$\underbrace{\int (4x^2+1)8x\,dx}_{u \cdot u'} = \frac{(4x^2+1)^2}{2} + C$ $\quad \underbrace{\int \frac{\ln x}{x}dx}_{u \cdot u'} = \frac{1}{2}\ln^2 x + C$ $\quad \int\left(\frac{3}{x} - 2\sqrt{x}+1\right)dx = 3\ln|x| - \frac{4}{3}x^{3/2} + x + C$

2) $\left((7x^2+1)^{35}\right)' = 35(7x^2+1)^{34} \times 14x$ $\quad \left((4\cos x-1)^{1000}\right)' = 1000(4\cos x-1)^{999} \times (-4\sin x)$ $\quad \left(e^{7x^{30}}\right)' = e^{7x^{30}} \times 210x^{29}$

$\left[(4x+1)^9(x+3)\right]' = 4 \times 9(4x+1)^8(x+3) + (4x+1)^9$ $\quad \left((4x-1)^8(3x+17)^{15}\right)' = 8(4x-1)^7 \times 4(3x+17)^{15} + (4x-1)^8 \times 15(3x+17)^{14} \times 3$

3) $\int \frac{dx}{1+x^2} = \mathrm{At}\,x$ $\quad \int \frac{dx}{x^2+25} = \frac{1}{5}\mathrm{At}\frac{x}{5}$ $\quad \int \frac{dx}{x^2+3} = \frac{1}{\sqrt{3}}\mathrm{At}\frac{x}{\sqrt{3}}$ $\quad \int \frac{dx}{4x^2+1} = \frac{1}{4}\int \frac{dx}{x^2+\frac{1}{4}} = \frac{2}{4}\mathrm{At}\,2x$

$\int \frac{dx}{(x-1)^2+3} = \frac{1}{\sqrt{3}}\mathrm{At}\frac{x-1}{\sqrt{3}}$ $\quad \int \frac{dx}{5(x-1)^2+2} = \frac{1}{5}\int \frac{dx}{(x-1)^2+\frac{2}{5}} = \frac{1}{5}\times\sqrt{\frac{5}{2}}\,\mathrm{At}\sqrt{\frac{5}{2}}(x-1)$

4) $\int \frac{dx}{(x-3)^2} = -\int \frac{dx}{x-3}$ $\quad \int \frac{dx}{x-2} = \ln|x-2|$ $\quad \int \frac{x\,dx}{x-7} = \int \frac{x-7+7}{x-7}dx = x + 7\ln|x-7|$

$\int \frac{4x+1}{x^2+1} = \mathrm{At}\,x + 2\ln(x^2+1)$ $\quad \int \frac{2x}{(x+2)^2+4} = \int \frac{2x+4-4}{(x+2)^2+4} = \ln((x+2)^2+4) - 2\,\mathrm{At}\frac{x+2}{2}$

$\int \frac{4x-1}{(x+1)^2+2} = 2\int \frac{2x+2}{(x+1)^2+2} - 5\int \frac{1}{(x+1)^2+2} = 2\ln((x+1)^2+2) - \frac{5}{\sqrt{2}}\mathrm{At}\frac{x+1}{\sqrt{2}}$

5) $\int \frac{dx}{(x-1)(x-3)} = \frac{1}{2}\int\left(\frac{1}{x-3} - \frac{1}{x-1}\right)dx = \frac{1}{2}\ln|x-3| - \frac{1}{2}\ln|x-1|$

$\int \frac{x}{(x+2)(x+5)}dx = \frac{1}{3}\int\left(\frac{5}{x+5} - \frac{2}{x+2}\right)dx = \frac{1}{3}\ln|x+5| - \frac{2}{3}\ln|x+2|$

$\int \frac{x+2}{x(x-1)(x-3)} = 2\ln|x-2| + \ln|x| - 3\ln|x-1|$ $\quad \int \frac{dx}{1-x^2} = \frac{1}{2}\ln|1+x| - \frac{1}{2}\ln|1-x|$

6) $\int (4x^2+3x+2)dx = 4\frac{x^3}{3} + \frac{3x^2}{2} + 2x$ $\quad \int (3\sin x + x^3 - 1)dx = -3\cos x + \frac{x^4}{4} - x$

$\int (2x^7 + \sqrt{x} + x^{3/2})dx = 2\frac{x^8}{8} + \frac{2}{3}x^{3/2} + \frac{2}{5}x^{5/2}$

7) $\int e^{2x}(x^2+x+1) = \frac{e^{2x}}{2}\left(x^2+x+1 - \frac{2x+1}{2} + \frac{2}{4}\right)$ $\quad \int e^{-x}(x^4-1) = \frac{e^{-x}}{-1}\left(x^4 - \frac{4x^3}{-1} + \frac{12x}{1} - \frac{12}{-1}\right)$

$\int \ln x\,dx = x\ln x - \int dx = x\ln x - x$

$u' = 1 \mid u = x$

$v = \ln x \mid v' = \frac{1}{x}$

$\int x\,\mathrm{At}\,x = -\frac{1}{2}\int \frac{x^2}{1+x^2} + \frac{x^2}{2}\mathrm{At}\,x$

$u' = x \mid u = x^2/2$

$v = \mathrm{At}\,x \mid v' = \frac{1}{1+x^2}$

$\int \frac{x^2}{1+x^2} = \int \frac{x^2+1-1}{x^2+1} = x - \mathrm{At}\,x$

8 $\int \frac{\cos x\,dx}{1+\sin^2 x} = \int \frac{du}{1+u^2} = \mathrm{At}\,u = \mathrm{At}(\sin x)$ $\quad \int \frac{1+e^t}{1-e^t} = \int \frac{1+u}{1-u}\frac{du}{u} = \ldots$

$\sin x = u \quad du = \cos x\,dx$