Intégrales · Corrigé partiel.

$$I = \int \frac{\cos x}{1+\sin x} \qquad \begin{aligned}\sin x &= u\\ \cos x\,dx &= du\end{aligned}$$

$$I = \int \frac{du}{1+u} = \underset{+C}{\ln|1+u|} = \ln|1+\sin x| + C$$

$$\int \frac{\sin x}{\cos x}\,dx : \text{Forme} \int -\frac{u'}{u} \qquad \int \frac{\sin x}{\cos x} = -\ln|\cos x| + C$$

$$\int \frac{dx}{\cos x} = \int \frac{\cos x\,dx}{\cos^2 x} = \int \frac{\cos x\,dx}{1-\sin^2 x} = \int \frac{du}{1-u^2}$$

$$= \int \frac{du}{(1-u)(1+u)} = \frac{1}{2}\int du \left(\frac{1}{1+u} + \frac{1}{1-u} \right) = \ln\left|\frac{1+u}{1-u}\right| + C = \ln\left|\frac{1+\sin x}{1-\sin x}\right| + C$$

$$\int \sin^7 x \cos x\,dx = \int u^7\,du = \frac{u^8}{8} + C = \frac{\sin^8 x}{8} + C$$

$\sin x = u$

$$\int \cos^2 x \sin^7 x\,dx = \int \cos^2 x \sin^6 x \,\sin x\,dx = \int -u^2(1-u^2)^3\,du$$

$\cos x = u$

$-\sin x\,dx = du$ à développer et primitiver.

$$\int \frac{e^{2t}}{1+e^t}\,dt = \int \frac{u^2}{1+u}\frac{du}{u} = \int \frac{u}{1+u} = \int \frac{u+1-1}{u+1} = \int 1 - \frac{1}{1+u}$$

$e^t = u$

$du = e^t dt$

$t = \ln u \quad dt = \frac{du}{u}$

$$\cancel{\frac{u^2}{u^2+u} = \frac{u^2+u-u}{u^2+u} = 1 - \frac{u}{u(u+1)} = 1 - \frac{1}{1+u}}$$

$$\cancel{\frac{u}{u(u+1)}}$$

$$= u - \ln|1+u| + C = e^t - \ln|1+e^t| + C$$

$$\int_1^3 \frac{dx}{\sqrt{x}(1+x)} = \int_1^{\sqrt{3}} \frac{2u\,du}{u(1+u^2)} = \left[2\,\mathrm{At}(u)\right]_1^{\sqrt{3}} = 2(\mathrm{At}\sqrt{3} - \mathrm{At}\,1)$$

$\sqrt{x} = u \quad x = u^2$

$dx = 2u\,du$

$$= 2\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

$$= \frac{2\pi}{12} = \frac{\pi}{6}$$

$$\int \frac{x}{(x-1)(x^2+1)}$$

$$\frac{x}{(x-1)(x^2+1)} = \frac{a}{x-1} + \frac{bx+c}{x^2+1} \qquad a = \frac{1}{2} \quad b = -\frac{1}{2} \quad c = \frac{1}{2}$$

$$= \frac{1}{2}\left(\frac{1}{x-1} + \frac{-x+1}{x^2+1} \right)$$

$$\int \frac{x}{(x-1)(x^2+1)} = \frac{1}{2}\ln|x-1| - \frac{1}{4}\int \frac{2x}{x^2+1} + \frac{1}{2}\int \frac{dx}{1+x^2}$$

$$= \frac{1}{2}\ln|x-1| - \frac{1}{4}\ln(x^2+1) + \frac{1}{2}\,\mathrm{At}\,x + C$$