

1 2 3 4 5 6 7 8
1 1 1 1 1 1 0 0

9
1 1

Exercice 1:

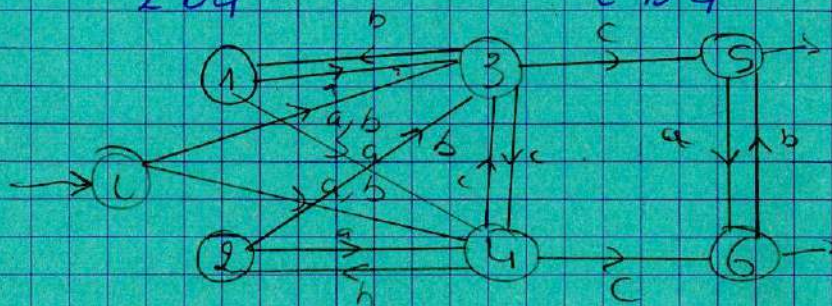
	a	b	c	
12	34	34	P	E
34	2	1	3456	
1	34	P	P	
2	P	34	P	
3456	26	15	3456	S
15	346	P	P	S
26	P	345	P	S
346	2	15	3456	S
345	26	1	3456	S
P	P	P	P	

Exercice 2:

a) On ~~ajoute~~ ^{ajoute} les transitions:

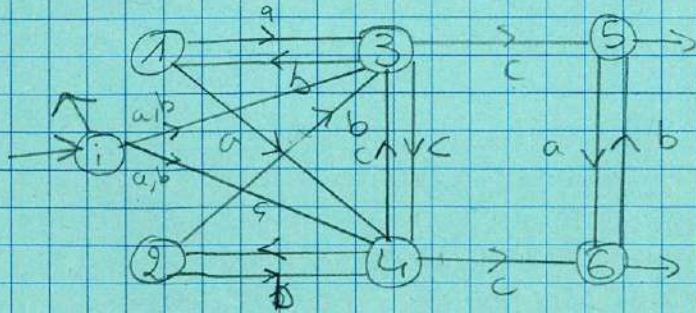
1 a 3
1 a 4
2 b 3
2 b 4

3 a 3
3 a 4
3 b 3
3 b 4



b)

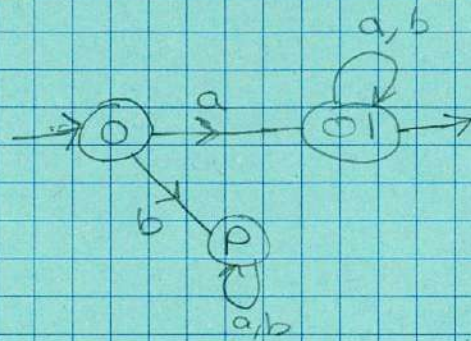
Non le mot vide n'est pas reconnu.



Exercice 3

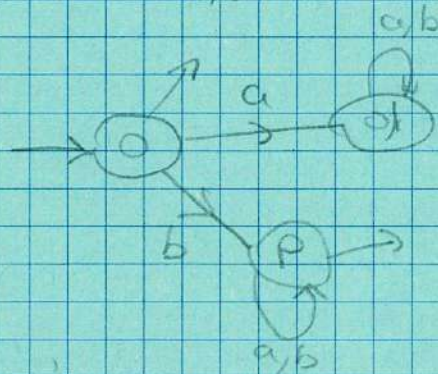
il faut d'abord determiniser l'automate et le completer.

E/S		a	b
0	0	01	P
1	01	01	01
P	P	P	P



Puis le completer;

E/S		a	b
0	0	01	P
1	01	01	01
S	P	P	P



Exercice 4.

Il faut d'abord completer l'automate

		a	b	c
1	3	4	P	P
2	3	4	P	P
S	3	S	1	4
S	4	9	6	3
	5	4	P	P
	6	P	S	P
E	7	3	4	P
	P	P	P	P

- Separer les etats terminaux des etats NT.

$$O_0 = \{ NT, T \} = \{ (1, 2, 5, 6, P), (3, 4) \}$$

		a	b	c
3	S	T	1	NT
4	2	NT	6	NT
				4
				3
				T

ils ne sont pas compatibles

$$O_1 = \{ (1, 2, 5, 6, P), 3, 4 \}$$

- Regrouper les etats NT.

$$O_2 = \{ 1, 2, 5, 6, P, 3, 4 \}$$

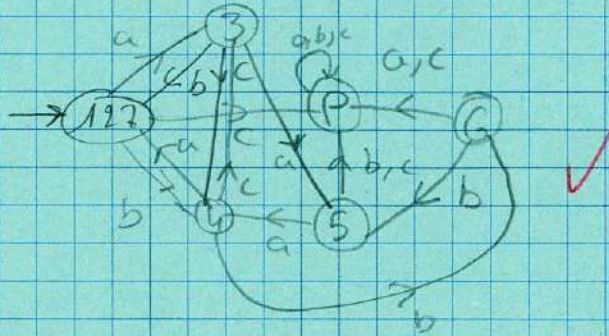
		a	b	c
1	3	T	4	T
2	3	T	4	T
3	4	T	P	NT
4	2	NT	S	NT
5	4	T	P	NT
6	P	NT	S	NT
P	P	NT	P	NT
7	3	T	4	T
				P
				NT

- Verifier pour compatibilite

	a	b	c	
1	3	4	6P	✓
2	3	4	6P	
3	3	4	6P	
6	6P	5	6P	X
P	6P	6P	6P	

$$\Theta_3 = \Theta_{fin} = \{12, 5, 6, P, 3, 4\}$$

	a	b	c
E	123	4	P P P
S	5	P	P P P
P	6	P	P P P
S	3	5	123
S	4	123	6



Exercise 5

a)

$$\begin{cases} 0 = \epsilon + 0a \\ 1 = \epsilon + 1b \\ 2 = 0a + 1b + 2a \\ 3 = 0b + 1b + 3b \end{cases}$$

b)

$$\begin{cases} 0 = \epsilon a^* = a^* \\ 1 = \epsilon b^* = b^* \\ 2 = a^*a + b^*b + 2a \\ 3 = a^*b + b^*b + 3b \end{cases}$$

$$2 = (a^*a + b^*b)a^* = a^*aa^* + b^*aa^*$$

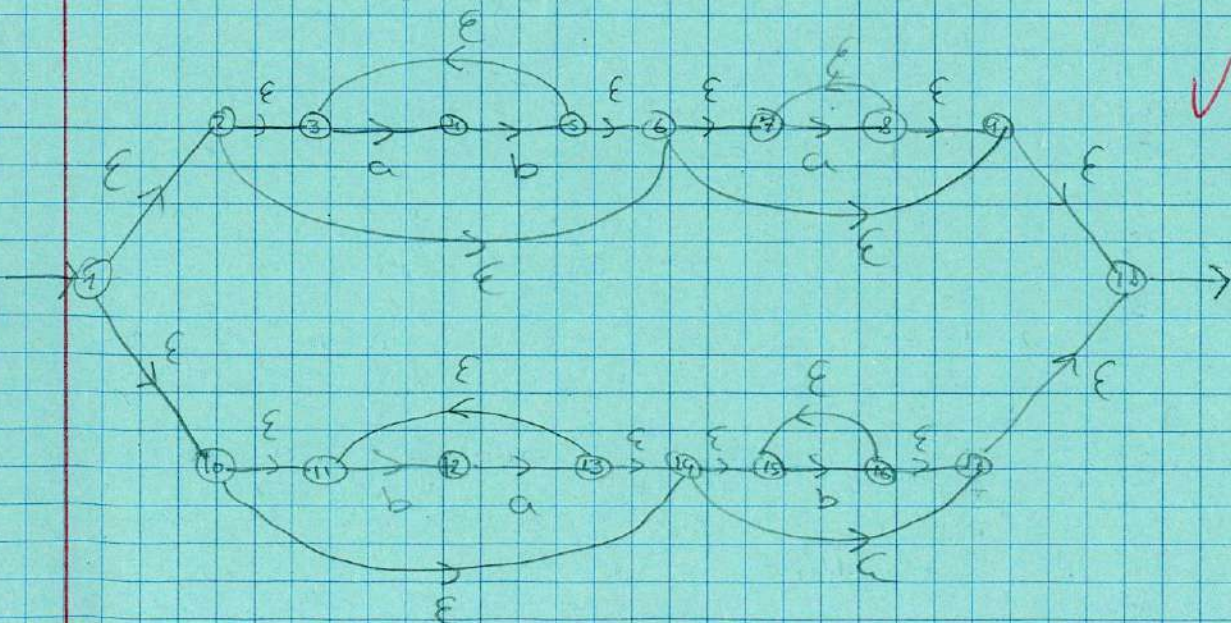
$$3 = (a^*b + b^*b)b^* = (a^*bb^* + b^*bb^*)$$

$$L = a^*aa^* + b^*aa^* + a^*bb^* + b^*bb^*$$

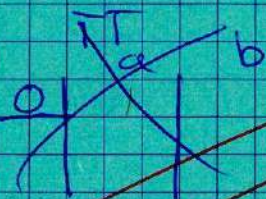
Exercise 6

a)

$$L = (ab)^*a^* + (ba)^*b^*$$



b)



donc c'est incorrect

0249

€/s	0	a	b
S	1249	12349	249
S	5689	689	56789
S	12349	12349	12349
S	249	2349	249
S	689	9	67
S	56789	689	56789
S	2349	2349	2349
S	249	2349	249
S	9	9	-
S	67	6	67
S	6	-	67

Exercice 7

$$8^1 - 1 = 7 \times 1 = 7k \quad \checkmark$$

$$\text{On veut que } 8^{n+1} - 1 = 7k$$

$$\begin{aligned} \text{or } 8^{n+1} - 1 &= 8 \cdot 8^n - 1 \\ &= (7+1)(8^n) - 1 \\ &= 7(8^n) + 8^n - 1 \quad \text{or } 8^n - 1 = 7k \\ &= 7(8^n) + 7k \\ &= 7(8^n + k) = 7k \quad \checkmark \end{aligned}$$

Exercice 9

$$a) \text{ le PGCD est } 2^5 \times 3^5 \times 5^3 \times 11 \times 17 \quad \checkmark$$

Ces tous ce qui y a en commun chez a et b.