

## Relations de trigonométrie

- Formules d'addition :

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\sin(a + b) = \sin a \cos b + \sin b \cos a$$

$$\tan(a + b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\cos(a - b) = \cos a \cos b + \sin a \sin b$$

$$\sin(a - b) = \sin a \cos b - \sin b \cos a$$

$$\tan(a - b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

- Relations entre les carrés :

$$\cos^2 a + \sin^2 a = 1 \quad 1 + \tan^2 a = \frac{1}{\cos^2 a}$$

- Formules de duplication :

$$\cos(2a) = \cos^2 a - \sin^2 a = 1 - \sin^2 a = 2 \cos^2 a - 1$$

$$\sin(2a) = 2 \sin a \cos a$$

$$\tan(2a) = \frac{2 \tan a}{1 - \tan^2 a}$$

- Définition en fonction de la tangente de l'arc moitié :

sous réserve de définition de la tangente, en notant  $t = \tan \frac{a}{2}$

$$\cos a = \frac{1 - t^2}{1 + t^2} \quad \sin a = \frac{2t}{1 + t^2} \quad \tan a = \frac{2t}{1 - t^2}$$

- Formules de linéarisation :

$$\cos a \cos b = \frac{1}{2}(\cos(a + b) + \cos(a - b)) \quad \cos^2 a = \frac{1 + \cos(2a)}{2}$$

$$\sin a \sin b = -\frac{1}{2}(\cos(a + b) - \cos(a - b)) \quad \sin^2 a = \frac{1 - \cos(2a)}{2}$$

$$\sin a \cos b = \frac{1}{2}(\sin(a + b) + \sin(a - b))$$

- Transformation d'une somme ou différence en produit :

$$\cos a + \cos b = 2 \cos \left( \frac{a + b}{2} \right) \cos \left( \frac{a - b}{2} \right) \quad \cos a - \cos b = -2 \sin \left( \frac{a + b}{2} \right) \sin \left( \frac{a - b}{2} \right)$$

$$\sin a + \sin b = 2 \sin \left( \frac{a + b}{2} \right) \cos \left( \frac{a - b}{2} \right) \quad \sin a - \sin b = 2 \cos \left( \frac{a + b}{2} \right) \sin \left( \frac{a - b}{2} \right)$$

## Dérivée et primitive des fonctions usuelles

function  $f(x)$ , derivative  $f'(x) = \frac{df}{dx}$  and primitive  $\int f(x)dx = F(x) + c$ .

$f'(x)$	$f(x)$	$F(x)$	$f'(x)$	$f(x)$	$F(x)$
0	$c$	$cx$	$e^x$	$e^x$	$e^x$
1	$x$	$\frac{1}{2}x^2$	$a^x \ln(a)$	$a^x$	$\frac{a^x}{\ln(a)}$
$ax^{a-1}$	$\frac{x^a}{a-1}$	$\frac{x^{a+1}}{a+1}$	$\frac{1}{x}$	$\ln(x)$	$x \ln(x) - x$
$-\frac{1}{x^2}$	$\frac{1}{x}$	$\ln x $	$\frac{1}{x \ln(a)}$	$\log_a(x)$	$\frac{x \ln(x) - x}{\ln(a)}$
$\cos(x)$	$\sin(x)$	$-\cos(x)$	$\frac{1}{\sqrt{1-x^2}}$	$\arcsin(x)$	$x \arcsin(x) + \sqrt{1-x^2}$
$-\sin(x)$	$\cos(x)$	$\sin(x)$	$\frac{-1}{\sqrt{1-x^2}}$	$\arccos(x)$	$x \arccos(x) - \sqrt{1-x^2}$
$1 + \tan^2 x = \frac{1}{\cos^2(x)}$	$\tan(x)$	$-\ln \cos(x) $	$\frac{1}{1+x^2}$	$\arctan(x)$	$x \arctan(x) - \frac{1}{2} \ln(1+x^2)$
$\frac{-1}{\sin^2(x)}$	$\cot(x)$	$\ln \sin(x) $	$\frac{-1}{1+x^2}$	$\operatorname{arccot}(x)$	$x \operatorname{arccot}(x) + \frac{1}{2} \ln(1+x^2)$
$\cosh(x)$	$\sinh(x)$	$\cosh(x)$	$\frac{1}{\sqrt{x^2+1}}$	$\operatorname{Arsinh}(x)$	$x \operatorname{Arsinh}(x) - \sqrt{x^2+1}$
$\sinh(x)$	$\cosh(x)$	$\sinh(x)$	$\frac{1}{\sqrt{x^2-1}}$	$\operatorname{Arcosh}(x)$	$x \operatorname{Arcosh}(x) - \sqrt{x^2-1}$
$\frac{1}{\cosh^2(x)}$	$\tanh(x)$	$\ln(\cosh(x))$	$\frac{1}{1-x^2}$	$\operatorname{Artanh}(x)$	$x \operatorname{Artanh}(x) + \frac{1}{2} \ln(1-x^2)$
$\frac{-1}{\sinh^2(x)}$	$\coth(x)$	$\ln \sinh(x) $	$\frac{1}{1-x^2}$	$\operatorname{Arcoth}(x)$	$x \operatorname{Arcoth}(x) + \frac{1}{2} \ln(x^2-1)$

### differentiation and integration rules

short	$f' = \frac{d}{dx} f(x)$	short	$\int_a^b f dt = \int_a^b f(t) dt$
integral	$\left( \int_a^x f dt \right)' = f(x)$	fundamental theorem	$\int_a^x f dt = F _a^x = F(x) - F(a)$
constant	$(c \cdot f)' = c \cdot f'$	constant	$\int_a^b c \cdot f dt = c \cdot \int_a^b f dt$
sum	$(f \pm g)' = f' \pm g'$	sum	$\int_a^b (f \pm g) dt = \int_a^b f dt \pm \int_a^b g dt$
product	$(f \cdot g)' = f' \cdot g + f \cdot g'$	integration by parts	$\int_a^b f' \cdot g dt = f \cdot g _a^b - \int_a^b f \cdot g' dt$
chain rule	$(f(g(x)))' = \frac{df(g)}{dg} \cdot g'(x)$	substitution	$\int_a^b f(g) \cdot g'(t) dt = \int_{g(a)}^{g(b)} f(g) dg$
logarithm. diff. $f, g > 0$	$(f^g)' = f^g \left( g' \ln(f) + g \frac{f'}{f} \right)$	logarithm. integrat.	$\int_a^b \frac{f'}{f} dt = \ln f(t)  _a^b$

### further rules

quotient	$\left( \frac{u}{v} \right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$	intervals	$\int_a^c f dt + \int_c^b f dt = \int_a^b f dt$
inverse funct.	$(f^{-1}(x))' = \frac{1}{\frac{d}{dx} f(f^{-1}(x))}$	limits $b = a$	$\int_a^a f dt = 0$
3-product	$(fgh)' = f'gh + fg'h + fgh'$	commutation	$\int_a^b f dt = - \int_b^a f dt$
3-chain	$(f(g(h(x))))' = f'(g)g'(h)h'(x)$	$\begin{matrix} f \text{ odd} \\ a = -b \end{matrix}$	$f(t) = -f(-t) : \int_{-b}^b f dt = 0$