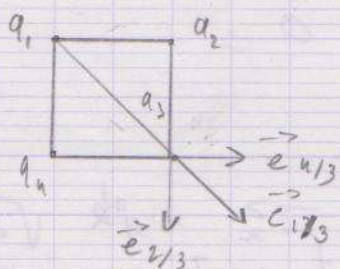


TD 1

Exercice 1

1

1)



2) constante $k = 9 \times 10^9$

$$\vec{F}_{1/3} = k \frac{q_1 q_3}{(r_{1/3})^2} \vec{e}_{1/3} = k \frac{q^2 q}{(a\sqrt{2})^2} = k \frac{q^2}{a^2} \vec{e}_{1/3}$$

$$\vec{F}_{2/3} = k \frac{q_2 q_3}{(r_{2/3})^2} \vec{e}_{2/3} = -k \frac{2q^2}{a^2} \vec{e}_{2/3}$$

$$\vec{F}_{4/3} = k \frac{q_4 q_3}{r_{4/3}^2} \vec{e}_{4/3} = -k \frac{4q^2}{a^2} \vec{e}_{4/3}$$

$$\vec{F}_{1/3} = F_{1/3}^x \vec{e}_x + F_{1/3}^y \vec{e}_y$$

$$\begin{aligned} F_{1/3}^x &= \vec{F}_{1/3} \cdot \vec{e}_x = \|\vec{F}_{1/3}\| \cdot \|\vec{e}_x\| \cdot \cos(\alpha) \\ &= \frac{kq^2}{a^2} \times 1 \times \frac{\sqrt{2}}{2} = \frac{kq^2}{a^2} \frac{\sqrt{2}}{2} \end{aligned}$$

$$\begin{aligned} F_{1/3}^y &= \vec{F}_{1/3} \cdot \vec{e}_y = \|\vec{F}_{1/3}\| \cdot \|\vec{e}_y\| \cdot \cos(-\alpha) \\ &= -\frac{kq^2}{a^2} \frac{\sqrt{2}}{2} \end{aligned}$$

$$\vec{F}_{2/3} = F_{2/3}^x \vec{e}_x + F_{2/3}^y \vec{e}_y$$

$$\begin{aligned} F_{2/3}^y &= \|\vec{F}_{2/3}\| \cdot \|\vec{e}_y\| \cdot \cos(0) \\ &= \frac{2kq^2}{a^2} \end{aligned}$$

$$F_{2/3}^x = 0$$

$$F_{u/3}^2 = -k \frac{4q}{a^2}$$

$$F_{u/3}^y = 0$$

f résultante.

$$\begin{aligned} \vec{F}^x &= \vec{F}_{1/3}^x + \vec{F}_{u/3}^x = \frac{q^2 k}{a^2} \frac{\sqrt{2}}{2} - \frac{4q^2 k}{a^2} \\ &= \frac{q^2 k}{a^2} \left(\frac{\sqrt{2}}{2} - 4 \right) \end{aligned}$$

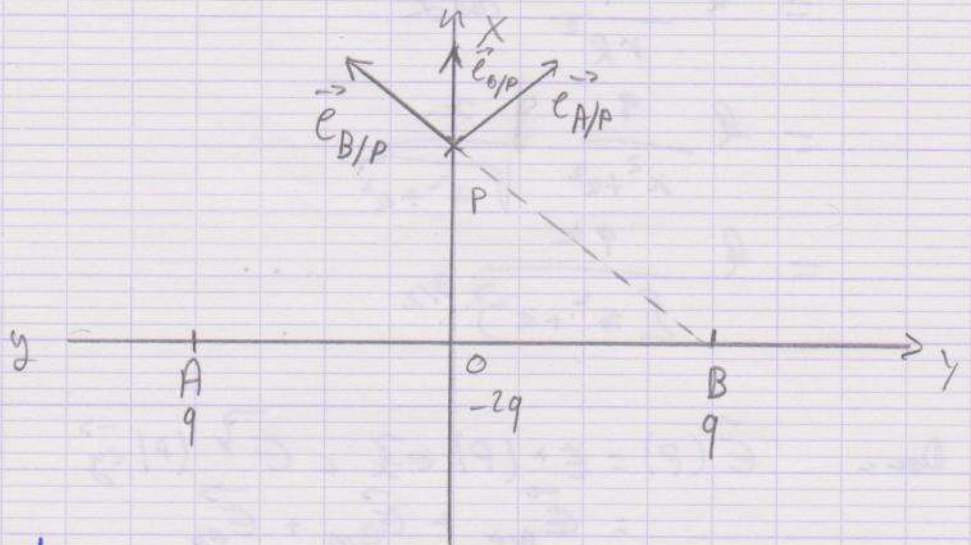
$$\vec{F}^y = \vec{F}_{1/3}^y + \vec{F}_{u/3}^y = \frac{kq^2}{a^2} \left(2 - \frac{\sqrt{2}}{2} \right)$$

Intensité de la résultante $\|\vec{F}\|$

$$\|\vec{F}\| = \sqrt{F_x^2 + F_y^2}$$

$$= 0,13 \text{ N.}$$

Exercice 2



Champs créés par A sur P

$$\vec{E}_{A/P} = k \frac{q_A}{r_{AP}^2} \vec{e}_{A/P}$$

$$\vec{E}_{B/P} = k \frac{q_B}{r_{BP}^2} \vec{e}_{B/P} \quad \text{champ créé par B sur P.}$$

$$\vec{E}_{O/P} = k \frac{q_O}{r_{OP}^2} \vec{e}_{O/P}$$

Champ résultant $\vec{E} = E_x \vec{e}_x + E_y \vec{e}_y$

Projeté $\vec{E}_{A/P}$ sur $\vec{e}_x$ et $\vec{e}_y$

$$\vec{E}_{A/P} = E_{A/P}^x \vec{e}_x + E_{A/P}^y \vec{e}_y$$

$$\vec{E}_{B/P} = E_{B/P}^x \vec{e}_x + E_{B/P}^y \vec{e}_y$$

$$\vec{E}_{O/P} = E_{O/P}^x \vec{e}_x + E_{O/P}^y \vec{e}_y$$

27 min

21 min

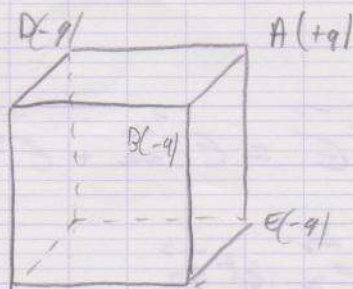
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$$\begin{aligned}
 E_{A/P}^x &= \vec{E}_{A/P} \cdot \vec{e}_x = \|\vec{E}_{A/P}\| \|\vec{e}_x\| \cos \alpha \\
 &= k \frac{q}{r^2} \cos \alpha \\
 &= k \frac{q}{x^2 + a^2} \frac{x}{\sqrt{x^2 + a^2}} \\
 &= k \frac{qx}{(x^2 + a^2)^{3/2}} \dots
 \end{aligned}$$

Donc $\vec{E}(P) = E^x(P) \vec{e}_x + E^y(P) \vec{e}_y$
 $= \vec{E}_{A/P} + \vec{E}_{B/P} + \vec{E}_{C/P}$

$$\begin{aligned}
 &= \vec{E}_{A/P}^x \vec{e}_x + \vec{E}_{A/P}^y \vec{e}_y \\
 &+ \vec{E}_{B/P}^x \vec{e}_x + \vec{E}_{B/P}^y \vec{e}_y \\
 &+ \vec{E}_{C/P}^x \vec{e}_x
 \end{aligned}
 \quad \text{car } \vec{E}_{B/P}^y \vec{e}_y = -\vec{E}_{A/P}^y \vec{e}_y$$

Exercice 3.



Potentiel créé par B, D, C

A $\xrightarrow{r}$ B $\vec{u}$

+q

$$\vec{E}_{A/B} = k \frac{q}{r^2} \vec{u}$$

relation champ - potentiel

$$\vec{E} = - \text{grad } V$$

si $V(x, y, z) \Rightarrow \text{grad} = \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \end{pmatrix}$

$$\vec{E} = - \frac{\partial}{\partial x} V(x, y, z) \vec{e}_x - \frac{\partial}{\partial y} V(x, y, z) \vec{e}_y - \frac{\partial}{\partial z} V(x, y, z) \vec{e}_z$$

$$\vec{E}(r) = k \frac{q}{r^2} \vec{u} = - \frac{\partial}{\partial r} V(r)$$

$$V(r) = - \int dr \vec{E}(r) \vec{u} = - \int dr k \frac{q}{r^2} = \frac{kq}{r} + C$$

(s=0)
car $V(r=0) = 0$.

$$V_1(a) = V_D(a) + V_B(a) + V_C(a) = -3 \frac{kq}{a}$$

Potential due to C, F et H

$$V_2(a) = V_C(a) + V_F(a) + V_H(a) = \frac{3kq}{a\sqrt{2}}$$

Potential due to G

$$V_3(a) = V_G(a) = -\frac{kq}{a\sqrt{3}}$$

Potential total

$$\begin{aligned} V_T(a) &= V_1(a) + V_2(a) + V_3(a) \\ &= -\frac{3kq}{a} + \frac{3kq}{a\sqrt{2}} - \frac{kq}{a\sqrt{3}} \\ &= -\frac{kq}{a\sqrt{6}} \left[3\sqrt{6} - 3\sqrt{3} + \sqrt{2} \right] \end{aligned}$$