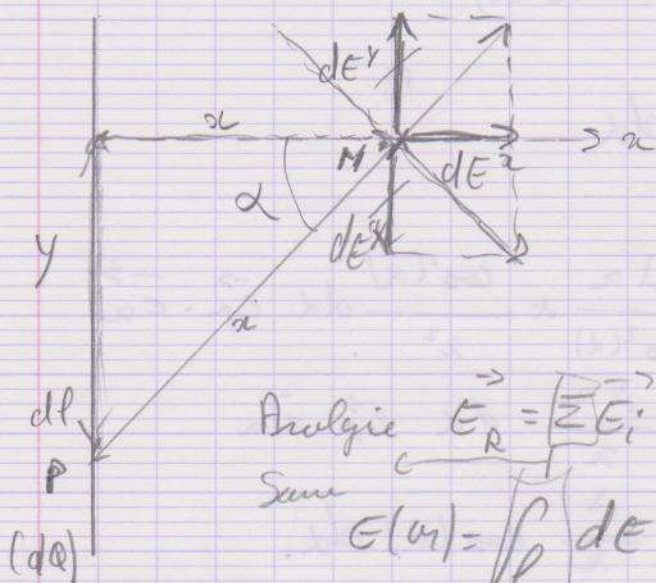


⑦

SERIE 3 Exercice 1



La symétrie par rapport de l'axe y veut s'annuler

Dans on se considère pour les composantes selon x.

Analogue $\vec{E}_R = \sum \vec{E}_i$ Dans le cas de charge ponctuelle

$E(x) = \int \frac{k dq}{r^2}$ Dans le cas d'une infinité de charge

Pour éviter de travailler sur $+\infty/-\infty$ la longueur du fil on ne travaille sur α qui prendra valeur de $-\frac{\pi}{2}$ à $\frac{\pi}{2}$

Ce calcul

$$\vec{dE} = k \frac{dq}{r^2} \vec{e}_r$$

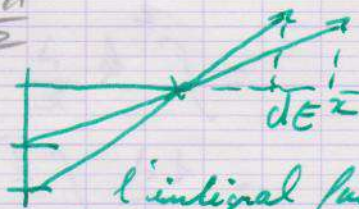
$$dE^x = \vec{dE} \cdot \vec{e}_x = k \frac{dq \times x^2}{(\cos \alpha)^2} \vec{e}_x$$

$$(\cos \alpha)^2 = \left(\frac{x}{r} \right)^2 \quad r^2 = \frac{x^2}{\cos^2 \alpha}$$

$$dq = \lambda dy$$

$$\tan \alpha = \frac{y}{x} \Rightarrow y = x \tan \alpha \quad (\text{on fait varier } y \text{ selon } \alpha \text{ de } \frac{dy}{dx})$$

$$\frac{dy}{dx} = x \frac{d}{dx} \tan \alpha$$



l'intégral fait la somme des éléments différentiel (infinité)

$$\frac{dy}{dx} = x \frac{d}{dx} \tan x = \frac{x}{\cos^2(x)}$$

$$(\tan x)' = \frac{1}{\cos^2 x}$$

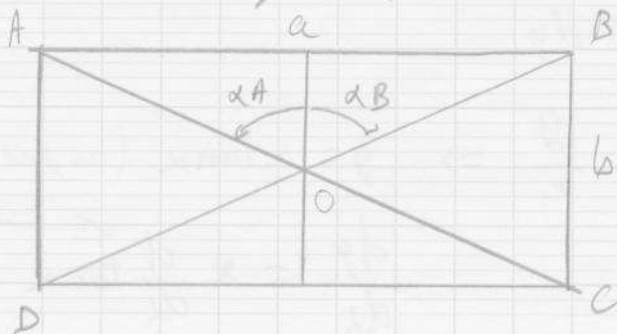
$$dy = \frac{x}{\cos^2 x} dx$$

$$\begin{aligned} d\vec{E} \cdot \vec{e}_x &= k \frac{\lambda x}{\cos^2(x)} \times \frac{\cos^2(x)}{x^2} dx \vec{e}_p \cdot \vec{e}_x \\ &= k \frac{\lambda}{x} dx \vec{e}_p \cdot \vec{e}_x \\ &= k \frac{\lambda}{x} \cos(x) dx. \end{aligned}$$

Maintenant on calcule l'ensemble
(addition)

$$\begin{aligned} E(M) &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dE \cdot x \\ &= \int k \frac{\lambda}{x} \cos(x) dx \\ &= \frac{k\lambda}{x} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos(x) dx = \frac{k\lambda}{x} \left[\sin(x) \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\ E(M) &= \frac{k\lambda}{x} \left(\sin \frac{\pi}{2} - \sin \left(-\frac{\pi}{2} \right) \right) \\ E(M) &= 2 \frac{k\lambda}{x} = 1 + (-1) = 2 \end{aligned}$$

2- On fait la même chose pour AB et BC
puis on lit partir de la symétrie.



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$$\tan \alpha_A = \frac{1}{2} a \times \frac{2}{b} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\alpha_A = -\frac{\pi}{3}$$

$$E_0^{AB} = \int_{\alpha_A}^{\alpha_B} \frac{k \lambda}{r} \cos \alpha \, d\alpha$$

$$b = \frac{\sqrt{3}}{3} a$$

$$E_0^{AB} = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{k \lambda}{r} \cos(\alpha) \, d\alpha$$

$$= \frac{k \lambda}{r} \left[\sin \alpha \right]_{-\frac{\pi}{3}}^{\frac{\pi}{3}}$$

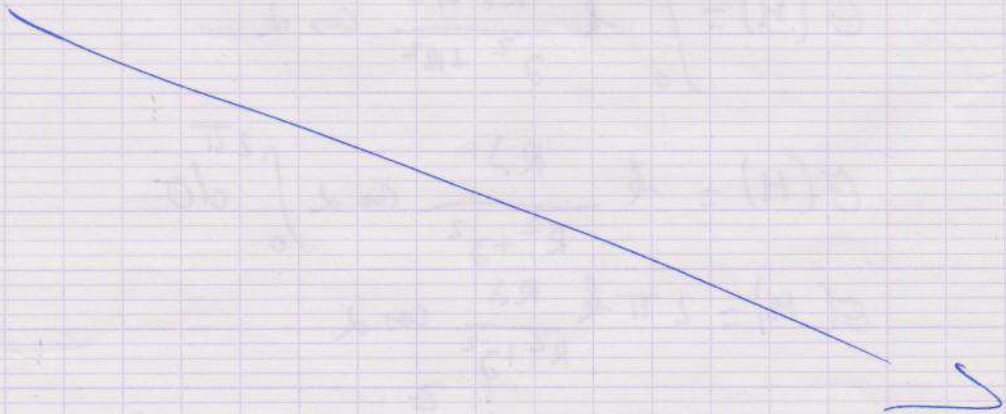
$$E_0^{AB} = \frac{k \lambda}{r} \left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \right)$$

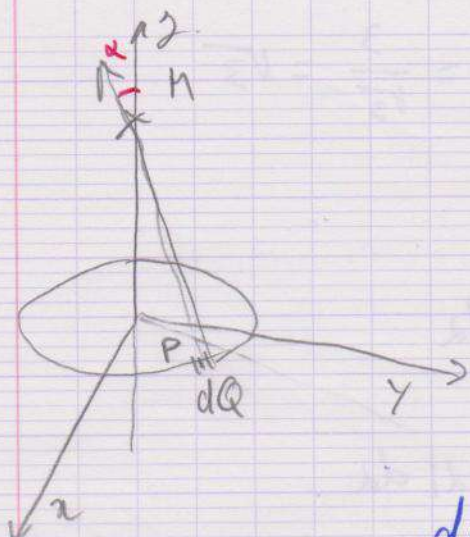
$$= \frac{2\sqrt{3} \lambda \lambda}{b}$$

Même logique pour le segment BC.

b - A la fin le champ est nul car tout s'annule par symétrie.

Exercice 2



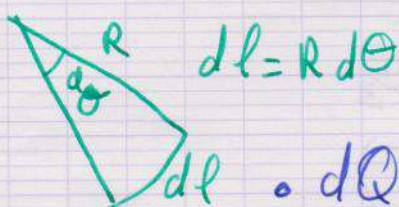


Une fois de plus seulement
la composante en z nous
intéresse est pes x n'y y .

$$E(M) = \int_e dE^z$$

$$dE^z = d\vec{E} \cdot \vec{e}_z$$

$$d\vec{E} = k \frac{dQ}{PM^2} \vec{e}_Q$$



$$dl = R d\theta$$

$$dQ = \lambda dl = \lambda R d\theta$$

$$PM^2 = z^2 + R^2$$

$$d\vec{E} = k \frac{R \lambda d\theta}{z^2 + R^2} \vec{e}_Q$$

$$d\vec{E} \cdot \vec{e}_z = k \frac{R \lambda d\theta}{z^2 + R^2} \vec{e}_Q \cdot \vec{e}_z$$

$$= k \frac{R \lambda d\theta}{z^2 + R^2} \cos \alpha$$

$$E(M) = \int_0^{2\pi} k \frac{R \lambda d\theta}{z^2 + R^2} \cos \alpha$$

$$E(M) = k \frac{R \lambda}{R^2 + z^2} \cos \alpha \int_0^{2\pi} d\theta$$

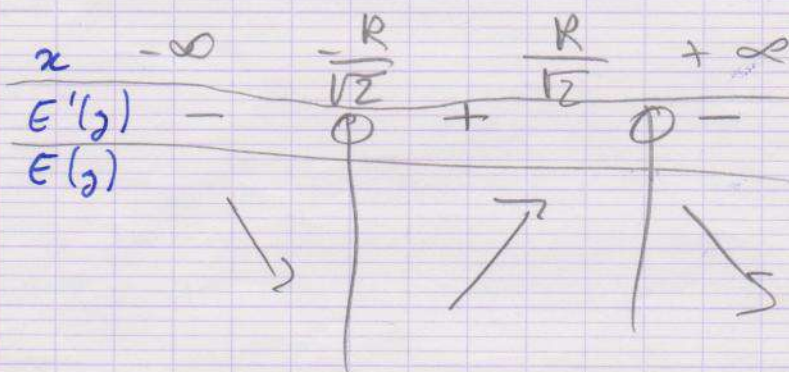
$$E(M) = 2\pi k \frac{R \lambda}{R^2 + z^2} \cos \alpha$$

$$\cos \alpha = \frac{z}{\sqrt{R^2 + z^2}}$$

(9)

$$\begin{aligned}
 E'(z) &= \frac{d}{dz} E(z) \\
 &= 2\pi k \Delta R \frac{d}{dz} \frac{z}{(R^2 + z^2)^{3/2}} \\
 &\quad \downarrow \\
 &= 2\pi k \Delta R \frac{R^2 - 2z^2}{(R^2 + z^2)^{3/2}}
 \end{aligned}$$

$$E'(z) = 0 \quad z = \pm \frac{R}{\sqrt{2}}$$



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9 bis

$$E(M) = \int_0^{2\pi} k \frac{R\lambda d\theta}{z^2 + R^2} \cos \alpha = k \frac{R\lambda}{R^2 + z^2} \cos \alpha \int_0^{2\pi} d\theta$$

$$= 2\pi k \frac{R\lambda}{R^2 + z^2} \cos \alpha$$

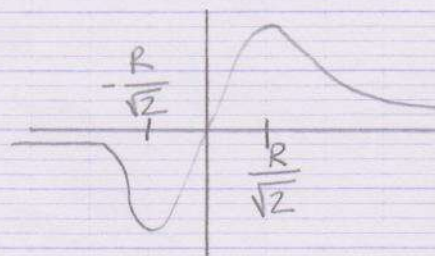
$$\cos \alpha = \frac{z}{\sqrt{R^2 + z^2}}$$

$$E'(z) = \frac{d}{dz} E(z) = 2\pi k \lambda R \frac{d}{dz} \frac{z}{(R^2 + z^2)^{3/2}}$$

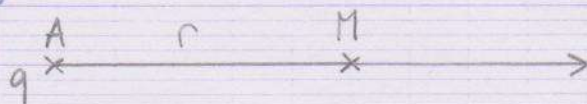
$$= 2\pi k \lambda R \frac{R^2 - z^2}{(R^2 + z^2)^{3/2}}$$

$$E'(z) = 0 \quad z = \pm \frac{R}{\sqrt{2}} \quad (R^2 - z^2 = 0)$$

z	$-\infty$	$-\frac{R}{\sqrt{2}}$	$\frac{R}{\sqrt{2}}$	$+\infty$	
$E'(z)$	-	0	+	0	-
$E(z)$					



$E(M)$: le champ
 $V(M)$: le potentiel
 dE : élém^e dif-
 érentiel de champ



$$\vec{E} = \sum \vec{E}_i = \int d\vec{E}$$

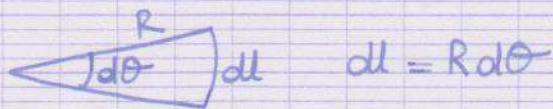
$$\vec{E}(M) = k \frac{q}{r^2} \vec{e}_r$$

$$V(M) = k \frac{q}{r}$$

$$dE^2 = d\vec{E} \cdot \vec{e}_z$$

$$dV = k \frac{dQ}{r}$$

$$dQ = \lambda dl$$

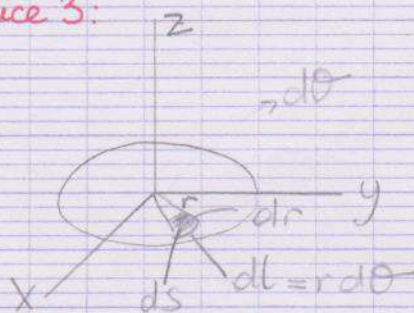


$$V(M) = \int dV$$

$$dV = k \frac{dQ}{PM} = k \frac{\lambda R d\theta}{\sqrt{R^2 + z^2}} \quad PM = \sqrt{R^2 + z^2}$$

$$V(M) = \int_0^{2\pi} k \frac{\lambda R d\theta}{\sqrt{R^2 + z^2}} = k \frac{\lambda R}{\sqrt{R^2 + z^2}} \int_0^{2\pi} d\theta = k \frac{\lambda R}{\sqrt{R^2 + z^2}} \times 2\pi$$

Exercise 3:



$$E(M) = \int d\vec{E}$$

$$d\vec{E} = k \frac{dQ}{PM^2} \vec{e}_q$$

$$dQ = \sigma ds$$

$$ds = r d\theta dr$$

Donc $dQ = \sigma r d\theta dr$

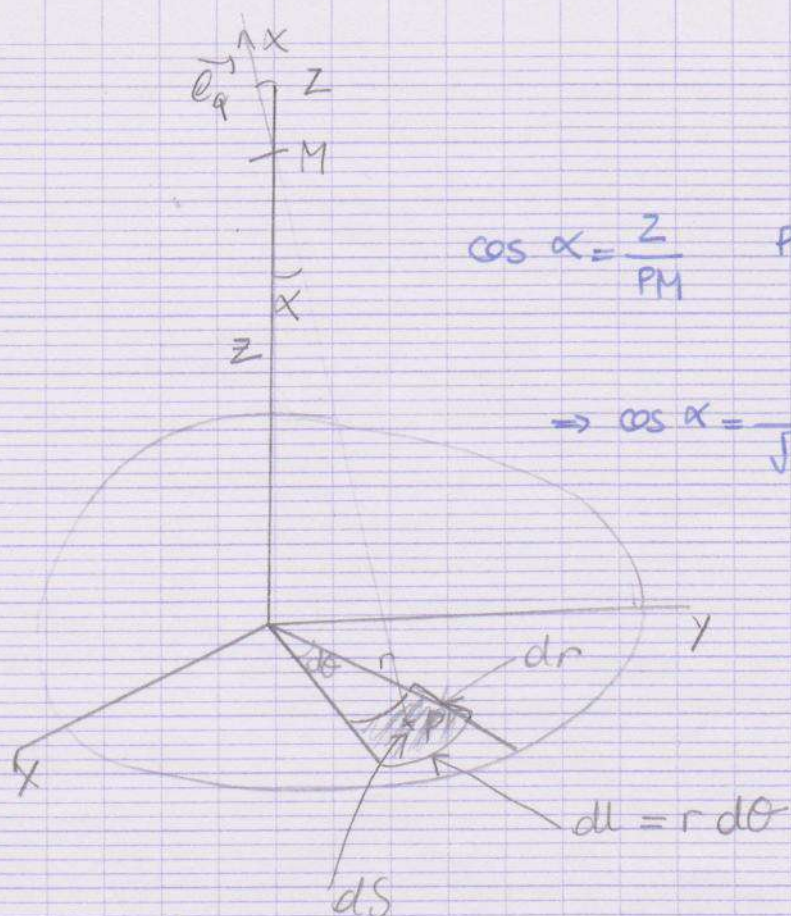
$$d\vec{E} = k \frac{\sigma r d\theta dr}{PM^2} \vec{e}_q$$

$$d\vec{E} \cdot \vec{e}_z = k \frac{\sigma r d\theta dr}{PM^2} \vec{e}_q \cdot \vec{e}_z$$

$$dE^z = k \frac{\sigma r d\theta dr}{PM^2} \cos \alpha$$

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$$\cos \alpha = \frac{z}{PM} \quad PM = \sqrt{r^2 + z^2}$$

$$\Rightarrow \cos \alpha = \frac{z}{\sqrt{r^2 + z^2}}$$

$$E(M) = \iiint_0^{2\pi} \int_0^R k \frac{\sigma r d\theta dr}{PM^2} \cos \alpha$$

$$PM^2 = z^2 + r^2 \quad \text{et donc} \quad \cos \alpha = \frac{z}{\sqrt{z^2 + r^2}}$$

$$E(M) = \sigma k z \int_0^{2\pi} \int_0^R \frac{r d\theta dr}{(z^2 + r^2)^{3/2}}$$

$$\begin{cases} u = r^2 \\ du = 2r dr \end{cases}$$

$$\begin{cases} r=0 \rightarrow u=0 \\ r=R \rightarrow u=R^2 \end{cases}$$

$$\frac{r dr}{(z^2 + r^2)^{3/2}} = \frac{1}{2} \frac{du}{(z^2 + u)^{3/2}}$$

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(12)

$$\int_0^R \frac{r dr}{(r^2 + z^2)^{3/2}} = \int_0^{R^2} \frac{1}{2} \frac{du}{(u + z^2)^{3/2}}$$

$$= \frac{1}{2} \left[-2 (u + z^2)^{-1/2} \right]$$

$$= - (u + z^2)^{-1/2}$$

Donc $E(M) = -\sigma k z 2\pi \left[(u + z^2)^{-1/2} \right]_0^{R^2}$

$$= -\sigma k z 2\pi \left((R^2 + z^2)^{-1/2} - \frac{1}{|z|} \right)$$

$$(z^2)^{-1/2} = \frac{1}{(z^2)^{1/2}} = \frac{1}{\sqrt{z^2}} \quad \begin{array}{ll} \text{si } z > 0 & |z| = z \\ \text{si } z < 0 & |z| = -z \end{array}$$

2.b) Plan infini $\Rightarrow R \rightarrow +\infty$

$$\lim_{R \rightarrow \infty} -\sigma k z 2\pi \left(\underbrace{(R^2 + z^2)^{-1/2}}_{R \rightarrow \infty = 0} - \frac{1}{|z|} \right) = \sigma k 2\pi$$

3. $V(M) = \int_S dV$

$$dV = k \frac{dQ}{PM}$$

$$dQ = \sigma r dr d\theta$$

$$PM = \sqrt{z^2 + r^2}$$

$$V(M) = \int_0^R \int_0^{2\pi} k \frac{\sigma r dr d\theta}{\sqrt{z^2 + r^2}} = 2\pi k \sigma \int_0^R \frac{r dr}{\sqrt{z^2 + r^2}}$$

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$$\int dr \frac{r}{\sqrt{r^2 + z^2}} = (r^2 + z^2)^{1/2}$$

$$V(M) = 2\pi k \sigma \left[(r^2 + z^2)^{1/2} \right]_0^R$$

$$= 2\pi k \sigma ((R^2 + z^2)^{1/2} - |z|)$$

$$\int_S ds = \int_0^{2\pi} d\theta \int_0^R r dr$$