

TD4 Exercice 1

1) Symétrie $\Rightarrow$ direction $\vec{E} \Rightarrow$ intersection de plans contenant le système + le point d'observation.

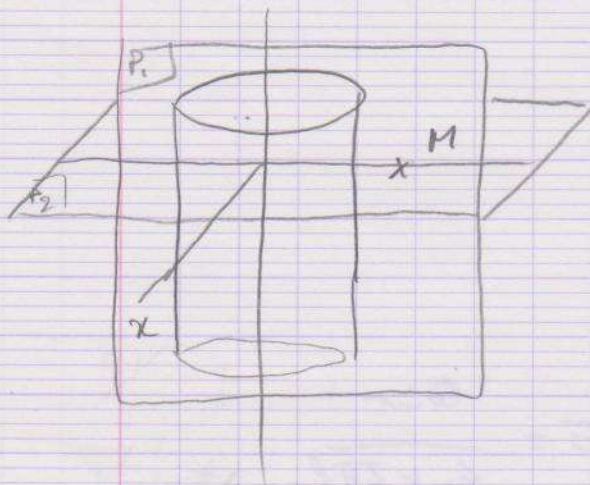
invariance $\Rightarrow E(1) \rightarrow E(2, \theta, z)$

$$E(r, \theta = \omega t, z = \omega t)$$

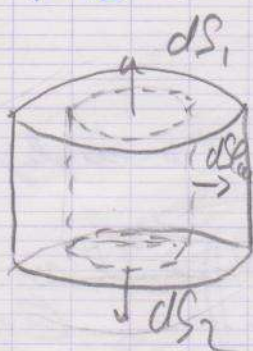
$\rightarrow$ si r et z constants E ne varie pas

$\rightarrow$ si r et θ constants E ne varie pas

$\rightarrow$ si θ et z constants E varie.
donc en fait de r

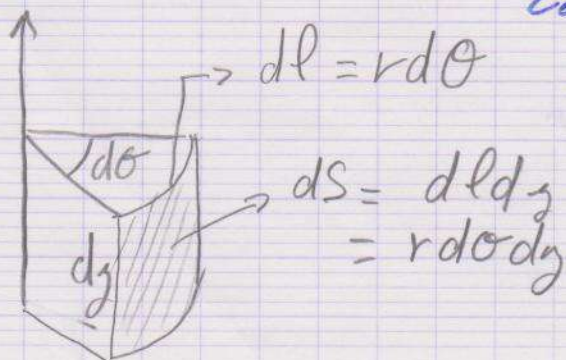


2) $r < a$



$$\oint \vec{E} \cdot d\vec{S} = \frac{|Q_{int}|_{eq}}{\epsilon_0} = 0.$$

$$a < r < b \quad \Phi(\vec{E}) = \iint_{S_{ext}} \vec{E} \cdot d\vec{S} = \frac{|Q_{int}|_{eq}}{\epsilon_0}$$



$$\iint_{\text{Sext}} \vec{E} \cdot d\vec{S}$$

$$= \frac{(Q_{\text{int}})_{\text{sg}}}{\epsilon_0}$$

$$= \iint E \vec{e}_r r d\theta dz \cdot \vec{e}_r$$

$$= \iint E \cdot r d\theta dz$$

$$= E r \iint_{\text{Sext}} d\theta dz$$

$$\Phi(\vec{E}) = E r 2\pi l$$

$$= \frac{Q_{\text{int}}}{\epsilon_0}$$

$$E = \frac{Q_{\text{int}}}{\epsilon_0 r 2\pi l}$$

$$2) \quad \vec{\Phi}(\vec{E}) = 0$$

car la distance entre a et b négligeable face à r

$$3) \quad \vec{E} = -\text{grad}(\vec{V}) \quad \left(\begin{array}{c} \partial/\partial r \\ \partial/\partial \theta \\ \partial/\partial z \end{array} \right) \rightarrow \text{Il ne reste que } \frac{\partial}{\partial r} \text{ car } E \text{ varie qu'en fonction de } r$$

$$\vec{E} \cdot \vec{e}_r = -\frac{\partial}{\partial r} V$$

$$E = -\frac{\partial V}{\partial r}$$

$$V = -\int E dr$$

$$V = -\int E dr$$

$$= -\int \frac{Q_{int}}{\epsilon_0 r 2\pi l} dr$$

$$= -\frac{Q_{int}}{\epsilon_0 2\pi l} \int \frac{1}{r} dr$$

$$= -\frac{Q_{int}}{\epsilon_0 2\pi l} \times [\ln r] + \text{cte.}$$

$$4) C = \frac{Q}{V_a - V_b}$$

$$V = \frac{-Q_{int}}{\epsilon_0 2\pi l} [\ln r]_a^b + \text{cte.}$$

$$V_b = \frac{-Q_{int}}{\epsilon_0 2\pi l} \ln b + \text{cte} \quad V_a = \frac{-Q_{int}}{\epsilon_0 2\pi l} \ln a + \text{cte.}$$

$$V_a - V_b = \frac{-Q_{int}}{\epsilon_0 2\pi l} (\ln a - \ln b)$$

$$= \frac{Q_{int}}{\epsilon_0 2\pi l} \ln \left(\frac{b}{a} \right)$$

$$C = \frac{Q}{\frac{Q}{\epsilon_0 2\pi l} \ln \left(\frac{b}{a} \right)}$$

$$= \frac{\epsilon_0 2\pi l}{\ln \left(\frac{b}{a} \right)}$$

Exercice 2

Partie A

Par symétrie

$$P_1 \cap P_2 = \{O\}$$

$$\Rightarrow \vec{E} \text{ porté par } \vec{e}_r$$

 $\vec{E}$ est invariant par rapport à θ et $\varphi \Rightarrow \vec{E}(r)$

$$r < R$$

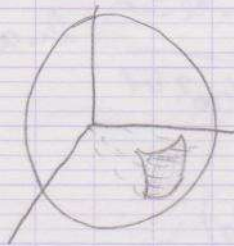


$$Q_{int} = 0 \Rightarrow \vec{E} = \vec{0}$$

$$r > R$$



$$\oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{int}}{\epsilon_0}$$



$$dl_1 = r d\varphi$$

$$dS = dl_1 dl_2 = r^2 \sin \theta d\theta d\varphi$$

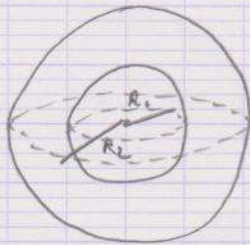
$$r \sin \theta d\theta = dl_2$$

$$\oint_S \vec{E} \cdot d\vec{S} = \oint_S E \vec{e}_r dS \vec{e}_r$$

TD4 Exercice 2 PARTIE II

$$\rho(r) = C r^2$$

3 cas à traiter $r < R_1$
 $R_1 < r < R_2$
 $r > R_2$



Théorème de Gauss

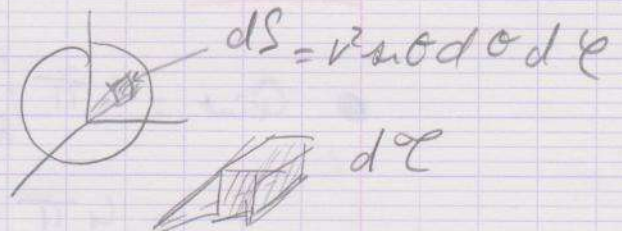
$$\oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{\text{int}}}{\epsilon_0}$$

• $r < R_1 \Rightarrow \vec{E} = \vec{0}$

• $R_1 \leq r \leq R_2 \quad d\vec{S} = dS \vec{e}_r \quad dS = r^2 \sin \theta d\theta d\varphi$

$$\begin{cases} \vec{E} = E \vec{e}_r \\ d\vec{S} = dS \vec{e}_r \end{cases} \quad \vec{E} \cdot d\vec{S} = E dS \Rightarrow \iint_S E dS = E 4\pi r^2$$

$$Q_{\text{int}} = \iiint_V \rho(r) d\mathcal{V}$$



$$d\mathcal{V} = dS dr = r^2 \sin \theta dr d\theta d\varphi$$

$$Q_{\text{int}} = \iiint C r^2 d\mathcal{V}$$

$$= \int_{R_1}^{R_2} r^2 C r^2 dr \int_0^\pi \sin \theta d\theta \int_0^{2\pi} d\varphi$$

$$\int_0^\pi \sin \theta d\theta = [-\cos \theta]_0^\pi$$

$$= -\cos \pi + \cos 0 = -(-1) + 1 = 2$$

$$= 4\pi \int_{R_1}^r r^2 C r^2 dr$$

$$= 4\pi C \int_{R_1}^r r^4 dr$$

$$= 4\pi C \left[\frac{r^5}{5} \right]_{R_1}^r$$

$$= 4\pi C \left(\frac{r^5}{5} - \frac{R_1^5}{5} \right)$$

$$= \frac{4\pi}{5} C (r^5 - R_1^5)$$

$$\frac{4\pi \epsilon r^2}{\oint E ds} = \frac{4\pi}{5\epsilon_0} C (r^5 - R_1^5) \frac{Q_{int}}{\epsilon_0}$$

$$E = \frac{C}{5\epsilon_0} \left(r^3 - \frac{R_1^5}{r^2} \right) + \text{const.}$$

$$\bullet Q_{int} = 4\pi \int_{R_1}^{R_2} r^2 C r^2 dr$$

$$= 4\pi C \left[\frac{r^5}{5} \right]_{R_1}^{R_2} = \frac{4\pi C}{5} (R_2^5 - R_1^5) + \text{const.}$$

$$4\pi \epsilon r^2 = \frac{4\pi C}{5} (R_2^5 - R_1^5) + \text{const.}$$

$$E = \frac{C}{r^2 5\epsilon_0} (R_2^5 - R_1^5)$$

TD4 Exercice 2 PARTI II

2/

$$\vec{E} = -\vec{\text{grad}} V$$

$$V = -\int dr E$$

$$= -\frac{C}{5\epsilon_0} (R_2^5 - R_1^5) \int \frac{1}{r^2} dr$$

• $r < R_1$ $E = 0$ donc $V = \text{cte.}$

• $R_1 \leq r \leq R_2$

$$V = -\frac{C}{5\epsilon_0} \int r^3 - \frac{R_1^5}{r^2} dr + \text{cte.}$$

$$= -\frac{C}{5\epsilon_0} \int r^3 dr - \int \frac{R_1^5}{r^2} dr + \text{cte.}$$

$$= -\frac{C}{5\epsilon_0} \left[\frac{r^4}{4} - R_1^5 \int \frac{1}{r^2} dr \right]$$

$$= -\frac{C}{5\epsilon_0} \left[\frac{r^4}{4} + R_1^5 \right] + \text{cte.}$$

$$V = -\int dr E$$

$$= -\frac{C}{5\epsilon_0} (R_2^5 - R_1^5) \int \frac{1}{r^2} dr + \text{cte.}$$

$$= -\frac{C}{5\epsilon_0} (R_2^5 - R_1^5) \left[-\frac{1}{r} \right] + \text{cte.}$$

$$= \frac{C}{5\epsilon_0} \frac{(R_2^5 - R_1^5)}{r} + \text{cte.}$$

- $r < R_1$
 $V_1(r) = C_1$
- $R_1 \leq r \leq R_2$

$$V_2(r) = -\frac{C}{\epsilon_0} \left[\frac{r^4}{4} + \frac{R_1^5}{r} \right] + C_2$$

- $r > R_2$
 $V_3(r) = \frac{C}{\epsilon_0} \left[\frac{R_2^5 - R_1^5}{r} \right] + C_3$

$$V(\infty) = 0 \Rightarrow r > R_2 \Rightarrow C_3 = 0.$$

For continuity at R_2 $V_2(r=R_2) = V_3(r=R_2)$

$$V_2(r=R_2) = \frac{C}{\epsilon_0} \left[\frac{R_2^4}{4} + \frac{R_1^5}{R_2} \right] + C_2$$

$$V_3(r=R_2) = \frac{C}{\epsilon_0} \left[\frac{R_2^5 - R_1^5}{R_2} \right]$$

$$V_2(r=R_2) = V_3(r=R_2) = \frac{R_2^4}{4} + \frac{R_1^5}{R_2} + C_2 = \frac{R_2^5 - R_1^5}{R_2}$$

$$C_2 = \frac{R_2^5 - R_1^5 - R_1^5}{R_2} - \frac{R_2^4}{4}$$

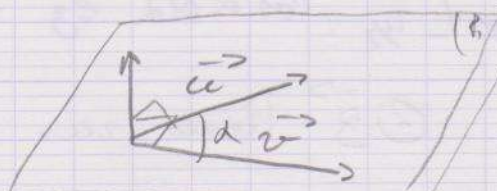
$$C_1 = R_2^4 - \frac{2R_1^5}{R_2} - \frac{R_2^4}{4}$$

$$C_2 = \frac{3R_2^4}{4} - \frac{2R_1^5}{R_2}$$

PRODUIT VECTORIEL

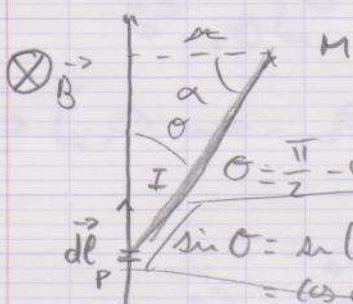
$$\vec{dB}(M) = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \wedge \vec{PM}}{PM^3}$$

$$\vec{w} = \vec{u} \wedge \vec{v}$$



$$\|\vec{w}\| = \|\vec{u}\| \|\vec{v}\| \sin \alpha$$

APPLICATION.



$$d\vec{l} \wedge \vec{PM} = dl PM \sin \theta \vec{e}_3$$

$\vec{I}$: source
 $\vec{PM}$: index
 $\vec{B}$: magnetic

$$\vec{dB}(M) = \frac{\mu_0 I}{4\pi} \frac{dl PM \sin \theta}{PM^3} \vec{e}_3$$

$$dl = dy$$

$$\tan \alpha = \frac{y}{r} \Rightarrow y = r \tan \alpha \Rightarrow \frac{dy}{d\alpha} = r \frac{1}{\cos^2 \alpha} = \frac{r}{\cos^2 \alpha}$$

$$PM^2 = \frac{r^2}{\cos^2 \alpha}$$

$$\vec{dB}(M) = \frac{\mu_0 I}{4\pi} \frac{r}{\cos^2 \alpha} \frac{\cos^2 \alpha}{r^2} \sin \theta \vec{e}_3$$

$$\vec{dB}(M) = \frac{\mu_0 I}{4\pi} \frac{1}{r} \cos \alpha \vec{e}_3$$

$$\begin{aligned}
 \vec{B}(M) &= \int_0 d\vec{B}(M) \\
 &= \int_{-\pi/2}^{\pi/2} \frac{\mu_0 I}{4\pi} \frac{1}{r} \cos \alpha \vec{e}_3 \\
 &= \frac{\mu_0 I}{4\pi r} \int_{-\pi/2}^{\pi/2} \cos \alpha d\alpha \vec{e}_3 = \frac{\mu_0 I}{2\pi r}
 \end{aligned}$$

Exercice I. ② $\vec{B}$ Pointe vers l'axe.

$$\begin{aligned}
 d\vec{B}(M) &= \frac{\mu_0 I}{4\pi} \frac{1}{h} \cos \sigma d\sigma \vec{e}_3 \\
 \vec{B}(M) &= \frac{\mu_0 I}{4\pi} \frac{1}{h} [\sin \sigma]_{\sigma_1}^{\sigma_2} \vec{e}_3 \\
 &= \frac{\mu_0 I}{4\pi} \frac{1}{h} [\sin \sigma_2 - \sin \sigma_1] \vec{e}_3
 \end{aligned}$$

Car fin
infin

$$\left. \begin{array}{l} \sigma_2 \rightarrow \frac{\pi}{2} \\ \sigma_1 \rightarrow -\frac{\pi}{2} \end{array} \right\} \vec{B}(M) = \frac{\mu_0 I}{2\pi h} \vec{e}_3$$

Exercice 2 ② $\vec{B}$ (Pour tous les axes)

Il faut additionner tous les axes.