

T D 3

Exo 1

$$1) \vec{E} = \frac{kq}{r^2}$$

$$d\vec{E} = \frac{k dQ}{PM^2}$$

$$dE^x = d\vec{E} \cdot \vec{e}_x$$

$$PM = \frac{x}{\cos \alpha} \text{ et } PM^2 = \frac{x^2}{\cos^2 \alpha}$$

$$d\vec{E} = \frac{k dQ \cos^2 \alpha}{x^2}$$

$$dQ = \lambda \cdot dl = \lambda \cdot dy$$

$$\tan \alpha = \frac{y}{x} \Rightarrow y = x \tan \alpha$$

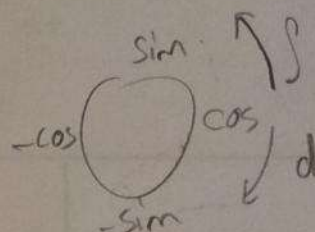
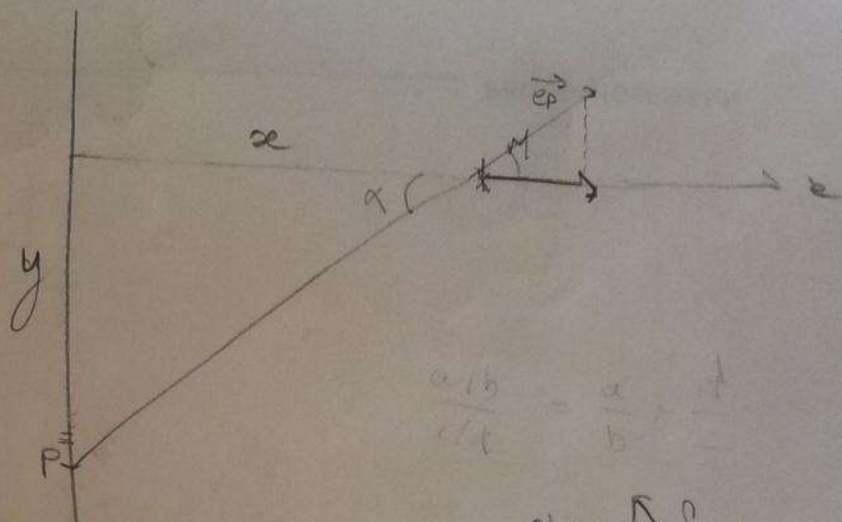
$$\frac{dy}{d\alpha} = \frac{x}{\cos^2 \alpha} \quad (\tan' \alpha = \frac{1}{\cos^2 \alpha})$$

$$\Rightarrow dy = \frac{x d\alpha}{\cos^2 \alpha}$$

$$d\vec{E} = \frac{k \lambda \frac{x d\alpha}{\cos^2 \alpha} \cos^2 \alpha}{x^2} = \frac{k \lambda d\alpha}{x}$$

$$\cos \alpha = \frac{dE^x}{dE} \Rightarrow dE^x = dE \cos \alpha = \frac{k \lambda d\alpha}{x} \cos \alpha$$

$$2) E^x = k \frac{\lambda}{x} \int_{-\pi/2}^{\pi/2} \cos \alpha d\alpha = k \frac{\lambda}{x} [\sin \alpha]_{-\pi/2}^{\pi/2} = 2k \frac{\lambda}{x}$$



Exo 3

$$E(M) = \frac{kq}{r^2}$$

$$dE(M) = \frac{k dQ}{r^2} \quad \text{avec } dQ = \sigma \cdot dS$$

$$= \frac{k \sigma dS}{r^2} = \frac{k \sigma R d\theta dR}{\sqrt{R^2 + z^2}^3}$$

$$dE(M) = \frac{k \sigma R d\theta dR}{R^2 + z^2}$$

$$\cos \alpha = \frac{z}{PM} \Rightarrow PM = \sqrt{R^2 + z^2} \Rightarrow \cos \alpha = \frac{z}{\sqrt{R^2 + z^2}}$$

$$\Rightarrow dE(M) = \frac{k \sigma R d\theta dR}{R^2 + z^2}$$

$$\cos \alpha = \frac{dE^z}{dE} \Rightarrow dE^z = \cos \alpha dE$$

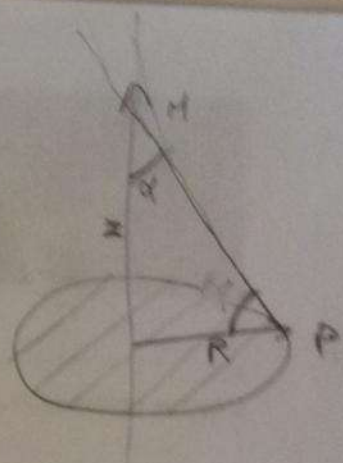
$$\Rightarrow dE^z = \frac{k \sigma R}{R^2 + z^2} \cos \alpha d\theta dR$$

$$E(M) = \iint k \frac{\sigma R}{R^2 + z^2} \times \frac{z}{\sqrt{R^2 + z^2}} d\theta dR = \iint k \frac{\sigma R z}{(R^2 + z^2)^{3/2}} d\theta dR$$

$$= k \sigma z \int_0^R \int_0^{2\pi} \frac{R}{(R^2 + z^2)^{3/2}} d\theta dR$$

$$= k \sigma z \int_0^R \frac{R}{(R^2 + z^2)^{3/2}} \int_0^{2\pi} d\theta dR$$

$$= k \sigma z \int_0^R \frac{R}{(R^2 + z^2)^{3/2}} dR$$



TD 6

Exo 2) Partie A

1) $dS = r^2 \sin \theta d\theta d\varphi$
 r variant θ et φ invariants

2) $r < R$

$$\Phi = \oint \vec{E} \cdot d\vec{S} = \frac{(Q_{\text{int}})_{\text{sg}}}{\epsilon_0} = 0$$

$r > R$

$$\Phi = \frac{(Q_{\text{int}})_{\text{sg}}}{\epsilon_0} = \oint \vec{E} \cdot d\vec{S} = \oint \vec{E} \cdot r^2 \sin \theta d\theta d\varphi$$

$$= \oint \vec{E} \cdot \vec{e}_r r^2 \sin \theta d\theta d\varphi$$

$$= E r^2 \oint \sin \theta d\theta d\varphi = E r^2 \int_0^{2\pi} d\varphi \int_0^\pi \sin \theta d\theta$$

$$= E r^2 \times 2\pi \times [-\cos \theta]_0^\pi = 2\pi E r^2 \times (1 + 1) = 4\pi E r^2$$

$$\Rightarrow \frac{(Q_{\text{int}})_{\text{sg}}}{\epsilon_0} = 4\pi E r^2$$

$$\Rightarrow E = \frac{(Q_{\text{int}})_{\text{sg}}}{4\pi r^2 \epsilon_0}$$

3) $\vec{E} = -\vec{\text{grad}}(V)$

$$V = \int \frac{(Q_{\text{int}})_{\text{sg}}}{4\pi r^2 \epsilon_0} dr$$

$$= \frac{(Q_{\text{int}})_{\text{sg}}}{4\pi \epsilon_0} \int_R^\infty \frac{1}{r^2} dr$$

$$\left(\begin{array}{c} \frac{\partial}{\partial r} \\ \frac{\partial}{\partial \theta} \\ \frac{\partial}{\partial \varphi} \end{array} \right)$$