

Chazal Baptiste

Exercice 1:

On lance 2D6. Soit X le nombre de 6 obtenus.

$$N = 6^2 = 36$$

X	0	1	2
$P(X)$	$\frac{25}{36}$	$\frac{10}{36}$	$\frac{1}{36}$
$F(X)$	$\frac{25}{36}$	$\frac{35}{36}$	$\frac{36}{36} = 1$

$$P(2) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

$$P(0) = \frac{5}{6} \cdot \frac{5}{6} = \frac{25}{36}$$

$$P(1) = \frac{10}{36}$$

$$2 \left(\frac{1}{6} \cdot \frac{5}{6} \right) = \frac{10}{36}$$

Exercice 2:

On lance 3 pièces. Soit X le nombre de piles.

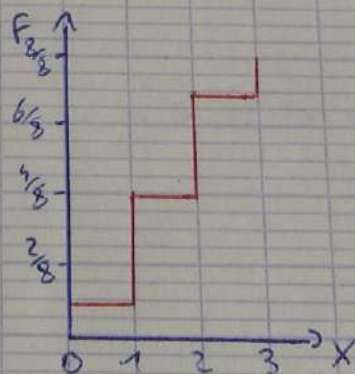
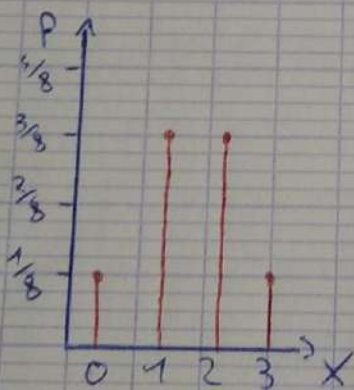
$$N = 2^3 = 8$$

X	0	1	2	3
$P(X)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$
$F(X)$	$\frac{1}{8}$	$\frac{4}{8}$	$\frac{7}{8}$	1

$$P(0) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

$$P(3) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$$

$$P(1) = \left(\frac{1}{2} \right)^3 \times 3 = \frac{3}{8}$$



Exercice 3 :

3N et 2B

$$N=10$$

R rang de la 1^{ère} boule B tirée

R	1	2	3	4
P(R)	$\frac{4}{10}$	$\frac{3}{10}$	$\frac{2}{10}$	$\frac{1}{10}$
F(R)	$\frac{4}{10}$	$\frac{7}{10}$	$\frac{9}{10}$	1

$$\begin{aligned}\langle R \rangle &= \sum P_i R_i \\ &= 2\end{aligned}$$

Exercice 4 :

$$\log_2 x = \frac{\ln x}{\ln 2}$$

a)

$$P_1 = \frac{1}{64}$$

$$I_1 = \log_2 \frac{1}{P_1} = \log N = \log(2^6) = 6 \text{ bits}$$

b)

$$P_2 = \frac{2}{64}$$

$$I_2 = \log \frac{1}{P_2} = \log \frac{N}{2} = \log 2^{6-1} = 5 \text{ bits}$$

Exercice 5 :

Exercice 7:

$$I = \log 2 = 1 \text{ bit}$$

1 bit : quantité d'informations nécessaire pour discerner deux symboles équiprobables.

Exercice 8:

1 dé = 6 faces équiprobables de proba p

a)

$$I(1 \text{ face}) = \log \frac{1}{p} = -\log p$$

$$\langle I \rangle = \log \frac{1}{p} = -\log p = \log 3 + \log 2$$

b)

$$\langle I \rangle = 0$$

$$\langle I \rangle = \sum p_i I_i = p_1 I_1 + p_2 I_2 \neq 0$$

c)

Exercice 1 :

$$1 \text{ mg d'eau} \Rightarrow S = S_{\text{molaire}} \cdot n = S_{\text{molaire}} \cdot \frac{m}{M}$$

$$S = R \ln \Omega$$

$$\Rightarrow \frac{S}{R} = \ln \Omega$$

$$\Rightarrow \Omega = e^{\frac{S}{R}}$$

$$\Rightarrow \Omega = e^{S_{\text{molaire}} \cdot \frac{m}{M}}$$

Exercice 2 :

$$I_{\text{BUF}} = R \ln K^N = \underbrace{NR \ln K}_{\text{info d'un symbole}}$$

$$= 3,5 \cdot 10^{12} \cdot 1,38 \cdot 10^{-23} \cdot \ln 26$$

$$= 1,6 \cdot 10^{-10} \text{ J/K}$$

Exercice 3 :

$$I_{\text{cerveau}} = R \ln \Omega = R \ln (K^N) = NR \ln K$$

$$K = 2$$

$$I_{\text{cerveau}} = 10^{13} \cdot 1,38 \cdot 10^{-23} \cdot \ln 2 = 4,8 \cdot 10^{-11} \text{ J/K}$$

entropie de la source
associée au message

Exercice 4:

$$H = I_{\text{information}} = \sum p_i I_i = \sum p_i \log \frac{1}{p_i} \leq \log 8 = 3 \text{ bits/symbole}$$

$$H = 5 \cdot \frac{1}{11} \log 11 + 3 \cdot \frac{2}{11} \log \frac{11}{2} \\ = \left(\frac{5}{11} + \frac{6}{11} \right) \log 11 - \frac{6}{11} \log 2$$

$$H = \log 11 - \frac{6}{11} = 2,91 \text{ bits/symbole.}$$

Exercice 5:

$$H = 2 \text{ bits/symbole}$$

$$\text{Equiprobabilité } H = \log N = \log 4 = 2 \text{ bits/symbole}$$

Exercice 6:

$$H_{\text{atlante}} = 2,7 \text{ bits/symbole}$$

$$I_{\text{atlante}} =$$

$$H_{\text{phenicien}} = \log 14 = \log 2 + \log 7 \\ = 3,8 \text{ bits/symbole}$$

Exercice 7 :

a)

Source de n symboles

1 bit = la quantité d'information pour désigner 2 symboles équiprobables

b)

$$I(\text{symbole}) = \log \frac{1}{P} \text{ or } P = \frac{1}{n} \text{ donc } I(\text{symbole}) = \log n$$

$$\langle I \rangle = H(S) = \sum p_i I_i = \sum p_i \log n = \log n \sum p_i = \log n$$

$$\text{Si } n = 4 \text{ alors } \langle I \rangle = H(S) = \log 4 = 2 \text{ bits/symbole}$$

c)

$$n = 4$$

$$P_1 = \frac{1}{2}$$

$$P_2 = \frac{1}{5}$$

$$P_3 = \frac{1}{5}$$

$$P_4 = \frac{1}{10}$$

$$\begin{aligned} H(S) = \langle I \rangle &= \sum p_i \log \frac{1}{P_i} = \frac{1}{2} \log 2 + \frac{1 \times 2}{5} \log 5 + \frac{1}{10} \log 10 \\ &= \log 2 \left(\frac{1}{2} + \frac{1}{10} \right) + \log 5 \left(\frac{2}{5} + \frac{1}{10} \right) \\ &= \frac{3}{5} \log 2 + \frac{1}{2} \log 5 \\ &= 0,6 + 1,16 \\ &= 1,76 \text{ bits/symbole} \leq H_{\max}(S) \end{aligned}$$

Exercice 8 :

$$N = 27 \text{ symboles}$$

a)

$$H(S) = \log N = \log 27 = 4,74 \text{ bits/symbole}$$

Exercice 1:

$L_1 \ L_2 \ L_3$
on on on
~~off~~ ~~off~~ ~~off~~

a) $N = 2^3 = 8$ symboles

b) $H(S) = \log N = \log 2^3 = 3$ bits/symbole.

c) 3 symboles équiprobables

$H(S) = \log 3 = 1,58$ bits/symbole

$$R = 1 - \frac{H(x)}{H_{\max}}$$

• Si $H(x) = H_{\max} \rightarrow$ équiprobabilité alors $R = 0$

• Si $H(x) = 0$ alors $R = 1 \rightarrow$ redondance maximale

$$R = 1 - \frac{1,58}{3} = 47,2\%$$

Exercice 3:

entropie pendant 1s

a)

$$H(1s) = n H(S)$$

$n = 10$ symboles

$$\Rightarrow H(1s) = 100 \text{ bits}$$

$$\rightarrow H(S) = \log 1024 = \log 2^{10} = 10 \text{ bits/symbole}$$

b)

$$H(1s) = n H(S)$$

$$n = 8000 \rightarrow H(1s) = 64000$$

$$\rightarrow H(S) = \log 256 = 8 \text{ bits/symbole}$$

c)

$$H(1s) = n H(S)$$

$$H(1s) = 24,9 \text{ bits}$$

$\rightarrow$

$$H(S) = \log 10 = \log 2 + \log 5 = 3,32 \text{ bits/symbole}$$

Exercice 4 :

a)

$$H(S) = \sum_i p_i \log \frac{1}{p_i} = \frac{2}{5} \log \left(\frac{5}{2} \right) + \frac{3}{5} \log \left(\frac{5}{3} \right) = 0,972 \text{ bits/symbole.}$$

b)

$$\langle T \rangle = 1 \cdot \frac{2}{5} + 2 \cdot \frac{3}{5} = 1,6 \text{ ms/symbole.}$$

$$D_S = \frac{1}{\langle T \rangle} = \frac{1}{1,6 \cdot 10^{-3}} = 6,25 \cdot 10^2 \text{ symboles/s.}$$

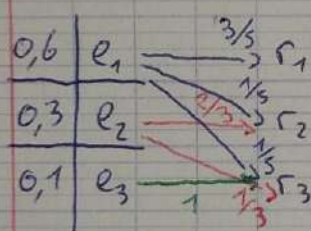
c)

$$T_S = H(S) \cdot D_S = 0,97 \cdot 625 = 606 \text{ bits/s.}$$

Exercice 5 :

a)

E $\xrightarrow{\text{canal}}$ R

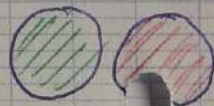


Information transmise par le canal = $I(E, R)$ Information mutuelle

Canal parfait $H(R) = H(E) = I(E, R)$



Canal totalement defectueux $I(E, R) = 0$



$$\begin{aligned} \text{Cas g n ral : } I(E, R) &= H(R) - H(R/E) \\ &= H(E) - H(E/R) \end{aligned}$$

$$\begin{aligned} p(r_1) &= p(r_1/e_1)p(e_1) + p(r_1/e_2)p(e_2) + p(r_1/e_3)p(e_3) \\ &= p(r_1/e_1)p(e_1) + 0 + 0 \\ &= \frac{3}{5} \cdot 0,6 = 0,36. \end{aligned}$$

$$\begin{aligned} p(r_2) &= p(r_2/e_1)p(e_1) + p(r_2/e_2)p(e_2) + p(r_2/e_3)p(e_3) \\ &= \frac{1}{5} \cdot 0,6 + \frac{2}{3} \cdot 0,3 + 0 = 0,32. \end{aligned}$$

$$\begin{aligned} p(r_3) &= 0,6 \cdot \frac{1}{5} + \frac{1}{3} \cdot 0,3 + 1 \cdot 0,1 \quad (\text{ou } 1 - p(r_2) - p(r_1)) \\ &= 0,32. \end{aligned}$$

b)

$$H(R) = \sum p(r_i) \log \frac{1}{p(r_i)}$$

$$= 0,36 \log \frac{1}{0,36} + 2 \cdot 0,32 \log \frac{1}{0,32}$$

$$= 1,58 \text{ bits/symbole}$$

$$< \log 3 = 1,58 \text{ (entropie max)}$$

c)

$$H(R/E) = \sum_i p(e_i) H(R/e_i) \text{ avec } H(R/e_i) = \sum_j p(r_j/e_i) \log \frac{1}{p(r_j/e_i)}$$

$$H(R/e_1) = \frac{3}{5} \log \frac{5}{3} + 2 \cdot \frac{1}{5} \log 5 = \log 5 \left(\frac{3}{5} + \frac{2}{5} \right) - \frac{3}{5} \log 3$$

$$= \log 5 - \frac{3}{5} \log 3$$

$$= 1,37 \text{ bits/symbole}$$

$$H(R/e_2) = \frac{2}{3} \log \left(\frac{3}{2} \right) + \frac{1}{3} \log 3 = \log 3 - \frac{2}{3} \log 2$$

$$= 1,58 - 0,66$$

$$= 0,92 \text{ bits/symbole}$$

$$H(R/e_3) = 1 \log \left(\frac{1}{1} \right) = 0 \text{ pas d'incertitude}$$

$$H(R/E) = 0,6 \cdot 1,37 + 0,3 \cdot 0,92 + 0,1 \cdot 0$$

$$= 1,1 \text{ bit/symbole} < H(R) = 1,58 \text{ bit/symbole}$$

d)

$$I(E, R) = H(R) - H(R/E) = 1,58 - 1,1 = 0,48 \text{ bit/symbole}$$

Exercice 6 :

a)

	π	F	Total
B	50	10	60
N	30	10	40
Total	80	20	100

$$b) P(B, \pi) = \frac{50}{100}$$

$$P(B, F) = \frac{10}{100}$$

$$P(N, \pi) = \frac{30}{100}$$

$$P(N, F) = \frac{10}{100}$$

c)

$$P(x, y) = P(x/y) P(y) = P(y/x) P(x) = P(y, x).$$

$$P(B/\pi) = \frac{P(B, \pi)}{p(\pi)} = \frac{50/100}{80/100} = \frac{5}{8}$$

$$P(N/\pi) = \frac{P(N, \pi)}{p(\pi)} = \frac{30/100}{80/100} = \frac{3}{8}$$

d)

$$p\left(\frac{6}{100}\right)^6 = (0,06)^6.$$

$H(s)$ en bits/symbole

Débit de source D_s en symboles/s

Taux d'émission T_s en bits/s

C cadence du canal en bit/s

$$T_s = H(s) \cdot D_s$$

Il faut que $T_s \leq C$ sinon pas possible

Taux d'émission après codeur en bit/s : $T_c = L_{\text{moy}} \cdot D_s$

3 codages :

- codages de longueur fixe
- codages de Sh-F
- codages de Huffman
- codages par bloc

Exercice 1 :

a)

$$\begin{aligned} H(s) &= \frac{6}{10} \log\left(\frac{10}{6}\right) + \frac{3}{10} \log\left(\frac{10}{3}\right) + \frac{1}{10} \log(10) \\ &= \log(10) - 0,6 \log(6) - 0,3 \log(3) \\ &= \log(5) + \log(2) - 0,6 \log(3) - 0,6 \log(2) - 0,3 \log(3) \\ &= \log(5) + 0,4 \log(2) - 0,9 \log(3) \\ &= 1,3 \text{ bit/symbole} \end{aligned}$$

B :	0,6
V :	0,3
D :	0,1

b)

$$T_s = D_s \cdot H(s) = \frac{180\,000}{3\,600} \cdot 1,3 = 65 \text{ bits/s}$$

c)

$T_s \leq C$ donc transmission possible.

d) 1^{er} théorème de Shannon

codage de longueur fixe $L_{\text{fixe}} = \text{ent}[H(s)] + 1$ (si $2^k < H(s) < 2^{k+1}$)
 $L_{\text{fixe}} = \text{ent}[1,3] + 1 = 2 \text{ bits/symbole}$

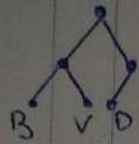
$$T_c = L \cdot D_s = 2 \cdot 50 = 100 \text{ bits/s}$$

$T_c > C$ donc non.

$$\eta = \frac{H(s)}{L} = \frac{1,3}{2} = 65\%$$

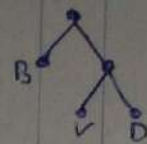
$$\text{redondance } R = 1 - \frac{H(s)}{H(s_{\text{eq}})}$$

0,6	B	0	0
0,3	✓	0	1
0,1	D	1	0



longueur fixe

≠



Sh-F

0,6	B	0
0,3	✓	1
0,1	D	1

e)

$$L_{\text{moy}} = \sum p_i L_i = 1 \cdot 0,6 + 2 \cdot (0,3 + 0,1) = 1,4 \text{ bit/symbole}$$

$$\eta = \frac{H(S)}{L_{\text{moy}}} = \frac{1,3}{1,4} = 93\%$$

$$T_c = L_{\text{moy}} \cdot D_s = 50 \cdot 1,4 = 70 \text{ bits/s}$$

$$T_c < c$$

1)

Codage par bloc de K

{ secondaire

Source d'extension d'ordre K : $N' = N^K$

$$N = 3$$

$$K = 2$$

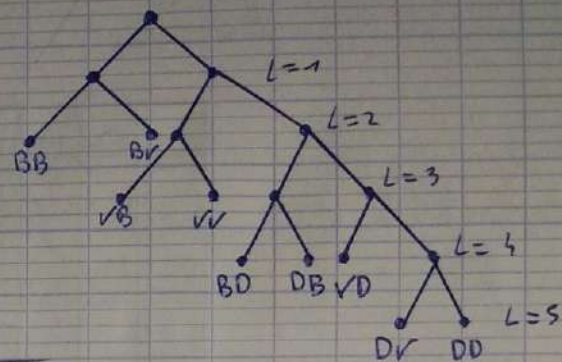
$$N' = N^K = 9 \text{ symboles}$$

BB	0,36
B✓	0,18
BD	0,06
✓B	0,18
✓✓	0,09
✓D	0,03
DB	0,06
D✓	0,03
DD	0,01

→ on classe par ordre décroissant →

BB	0,36
B✓	0,18
✓B	0,18
✓✓	0,09
BD	0,06
DB	0,06
✓D	0,03
D✓	0,03
DD	0,01

0,36	BB	0 0
0,18	BV	0 1
0,18	VB	1 0 0
0,09	VV	1 0 1
0,06	BD	1 1 0 0
0,06	DB	1 1 0 1
0,03	VD	1 1 1 0
0,03	DV	1 1 1 1 0
0,01	DD	1 1 1 1 1



$$\begin{aligned}
 L_{\text{bloc}} &= 2,7 \text{ bits/symbole} \\
 &= \sum p_i L_i \\
 &= 2(0,36 + 0,18) + 3(0,18 + 0,09) \\
 &\quad + 4(0,06 + 0,06 + 0,03) \\
 &\quad + 5(0,03 + 0,01)
 \end{aligned}$$

$$L_{\text{moy}} = \frac{L_{\text{bloc}}}{2} = \frac{2,7}{2} = 1,35 \text{ bit/symbole.}$$

$$\eta = \frac{1,3}{1,35} = 96\%$$

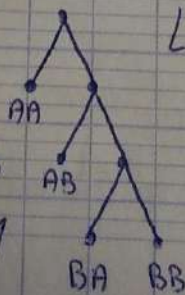
Exercice 2 :

$\begin{aligned}
 "0" &= "A" \\
 "1" &= "B"
 \end{aligned}
 \left. \vphantom{\begin{aligned} "0" &= "A" \\ "1" &= "B" \end{aligned}} \right\} \text{pour ne pas confondre}$

$$H(5) = \frac{7}{10} \log \frac{10}{7} + \frac{3}{10} \log \frac{10}{3} = 0,8 \text{ bit/symbole.}$$

Source { d'extension d'ordre 2 : $N^k = 2^2 = 4$ symboles.
secondaire

0,49	AA	0
0,21	AB	1 0
0,21	BA	1 1 0
0,09	BB	1 1 1



$$\begin{aligned}
 L_{\text{bloc}} &= 1 \cdot 0,49 + 2 \cdot 0,21 + 3 \cdot 0,09 \\
 &= 1,79 \text{ bit/symbole.}
 \end{aligned}$$

$$L_{\text{moy}} = \frac{L_{\text{bloc}}}{2} = 0,9 \text{ bit/symbole}$$

Exercice 3 :

	O	A	B	AB	Tot
R+	37%	32,1%	6,2%	2,8%	84,2
R-	7%	7,2%	1,2%	0,5%	15,8
Tot	44	45,3	7,4	3,3	100

a)

O: 44%

A: 45,3%

B: 7,4%

AB: 3,3%

b)

R+: 84,2%

R-: 15,8%

c)

Si: R+, le groupe le plus probable est A.

d)

$$p(x,y) = p(x)p(y)$$

$$p(O)p(+) = 0,44 \cdot 0,842$$

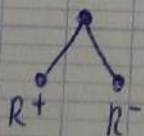
$$p(O,+) = 0,37$$

e)

Codage Sh-F séparément :

	Rhésus
0,842	R+ 0
0,158	R- 1

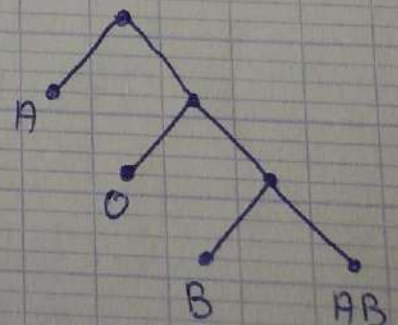
$$L_R = 1 \text{ bit/symbole}$$



Groupe sanguin

0,453	A	0
0,44	O	1 0
0,074	B	1 1 0
0,033	AB	1 1 1

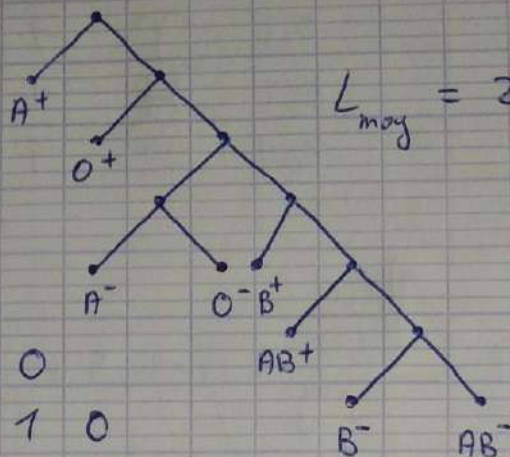
$$L_{GS} = 1,654 \text{ bit/symbole}$$



$$L_{GST} = L_R + L_{GS} = 2,65 \text{ bits/symbole}$$

Codage Sh-F non séparés :

0,381	A ⁺	0					
0,37	O ⁺	1	0				
0,072	A ⁻	1	1	0	0		
0,04	O ⁻	1	1	0	1		
0,062	B ⁺	1	1	1	0		
0,028	AB ⁺	1	1	1	1	0	
0,012	B ⁻	1	1	1	1	1	0
0,005	AB ⁻	1	1	1	1	1	1



$$L_{\text{moy}} = 2,18 \text{ bits/symbole}$$

Exercice 4 :

a)

$$\begin{aligned}
 H(s) &= 0,5 \log\left(\frac{1}{0,5}\right) + 0,2 \log\left(\frac{1}{0,2}\right) + 0,2 \log\left(\frac{1}{0,2}\right) + 0,1 \log\left(\frac{1}{0,1}\right) \\
 &= 0,5 + 0,4 \log(5) + 0,1 \log(10) \\
 &= 0,5 + 0,4(2,3) + 0,1(\log(2) + \log(5)) \\
 &= 0,5 + 0,92 + 0,1 + 0,1(2,3) \\
 &= 1,52 + 0,23 \\
 &= 1,75 \text{ bit/symbole.}
 \end{aligned}$$

b)

$$R = 1 - \frac{H(s)}{H(s_{\text{eqv}})} = 1 - \frac{1,75}{\log(4)} = 1 - \frac{1,75}{2} = \frac{0,25}{2} = 0,125$$

soit 12,5%.

c)

$$T_s = H(s) \cdot D_s = 1,75 \cdot 100 = 175 \text{ bits/s.}$$

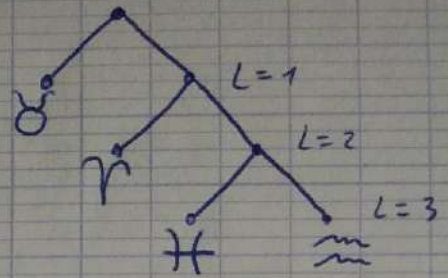
d)

$$c = 200 \text{ bits/s.}$$

$T_s < c$ donc transmission possible.

e)

0,5	0		
0,2	1	0	
0,2	1	1	0
0,1	1	1	1



f)

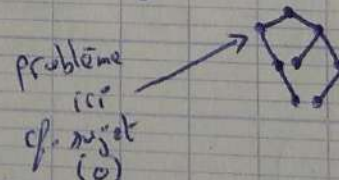
$$L_{\text{moy}} = 1 \cdot 0,5 + 2 \cdot 0,2 + 3 \cdot 0,3 = 1,8 \text{ bit/symbole.}$$

g)

$$\eta = \frac{H(s)}{L} = \frac{1,75}{1,8} \approx 97\%.$$

Exercice 5 :

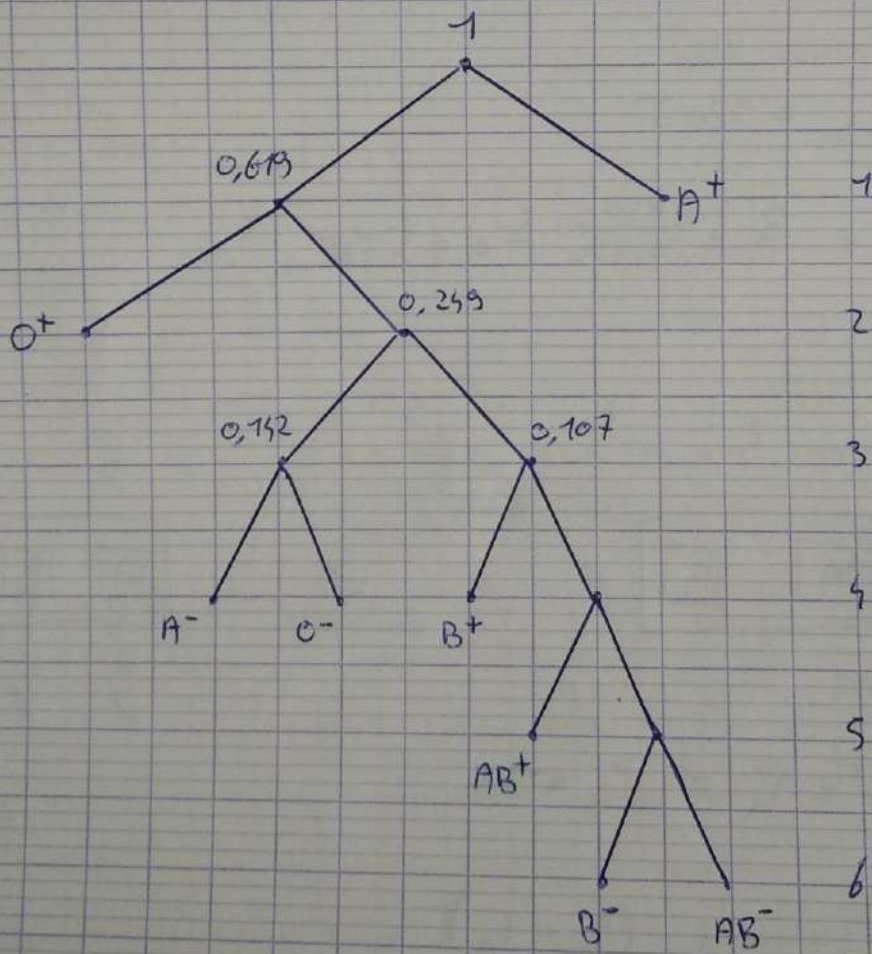
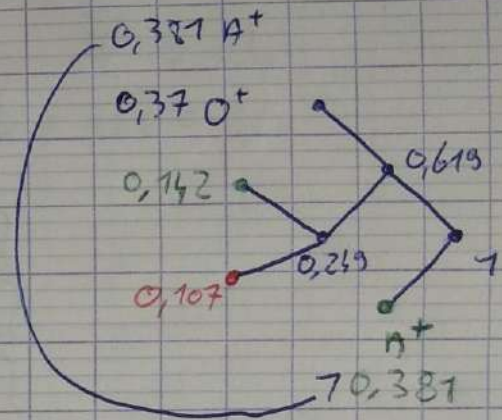
- instantanées -
- a) Oui
 - b) Non, un nœud est inutile.
 - c) Non, deux nœuds sont inutiles.
 - d) Non, pas instantané (un symbole (en 0) n'est pas sur une feuille (car il y a un 011)).



TD4
Energie, entropie
& information

Exercice 6 :

0,381	A^+
0,37	O^+
0,107	
0,072	A^-
0,07	O^-
0,062	B^+
0,028	AB^+
0,012	B^-
0,005	AB^-



$$L_{\text{moy}} = 1 \cdot 0,381 + 2 \cdot 0,37 + 4 \cdot 0,202 = 2,18 \text{ bits/symbole.}$$

TD 5
Energie, entropie
& information

$$Q = m c \Delta T$$

$\downarrow$ $\downarrow$ $\downarrow$ $\searrow$
 J J/kg $J/kg/K$ K

$$Q = m L$$

$\downarrow$ $\downarrow$ $\downarrow$
 J J/kg J/kg

$$dS = \frac{\partial Q}{T}$$

$$\Delta S = \int dS = \int \frac{\partial Q}{T} = \frac{1}{T} \int \partial Q = \frac{Q}{T}$$

$$\int_1^2 dx = \Delta x = x_2 - x_1$$

$$\int_1^2 dS = \Delta S = S_2 - S_1$$

$$\int \partial Q = Q$$

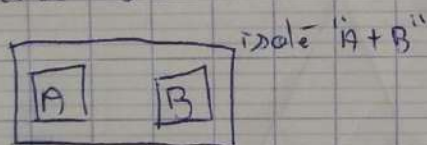
Exercice 1:

Aquarium 20°

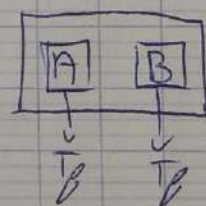
$Q_{\text{aquarium}} = 120 J$

$$\Delta S = \frac{Q}{T} = \frac{120}{273+20} \approx 0,45/K.$$

Exercice 3:



$\Rightarrow$
évolution



$$10^\circ = T_A \neq T_B = 100^\circ$$

$$a) Q_A = m_A C_A (T_f - T_A)$$

$$Q_B = m_B C_B (T_f - T_B)$$

$$Q_{A+B} = 0 = Q_A + Q_B$$

$$Q_A + Q_B = 0$$

$$m_A C_A (T_f - T_A) + m_B C_B (T_f - T_B) = 0$$

$$\Rightarrow T_f = \frac{m_A C_A T_A + m_B C_B T_B}{m_A C_A + m_B C_B} = 324 K$$

(51°C)

Si $m_A = m_B = m$
alors $T_f = \frac{T_A + T_B}{2}$

10 TDS
Energie, entropie
Information

b)

$$\Delta S_B < 0 \text{ car } T_B \searrow T_f \quad | \quad \Delta S_A > 0 \text{ car } T_A \nearrow T_f$$

$$\Delta S_A + \Delta S_B > 0 \quad | \quad \Delta S_B = m_B c_B \ln\left(\frac{T_f}{T_B}\right) = -5,45/\text{K}.$$

$$\Delta S_A = 6,25/\text{K}.$$

irréversible