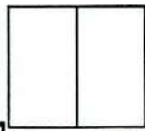
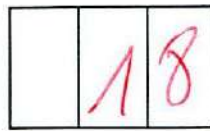


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PL2 - 2014

MATIÈRE Mathématiques du réel.

Exercice 1:

$$f(x, y) = \begin{cases} \frac{(x+y)^2}{x^2+y^2} & \text{Si } (x, y) \neq (0, 0) \\ 0 & \text{sinon.} \end{cases} \quad \text{Soit } f: \mathbb{R}^2 \rightarrow \mathbb{R}$$

$$\bullet f(x, 0) = \frac{(x+0)^2}{x^2+0} = \frac{x^2}{x^2} = 1 \neq 0.$$

$$\bullet f(0, y) = \frac{(0+y)^2}{0+y^2} = \frac{y^2}{y^2} = 1 \neq 0.$$

donc $f(x, y)$ n'est pas continue. ✓

Exercice 2:

$$f(x, y, z) = \frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}z^2 - xy - xz + yz + 1.$$

$$\frac{\partial f}{\partial x}(x, y, z) = x - y - z \quad \begin{cases} \frac{\partial^2 f}{\partial x^2}(x, y, z) = 1 \\ \frac{\partial^2 f}{\partial x \partial y}(x, y, z) = -1 \\ \frac{\partial^2 f}{\partial x \partial z}(x, y, z) = -1 \end{cases}$$

$$\frac{\partial f}{\partial y}(x, y, z) = y - x + z \quad \begin{cases} \frac{\partial^2 f}{\partial y \partial x}(x, y, z) = -1 \\ \frac{\partial^2 f}{\partial y^2}(x, y, z) = 1 \\ \frac{\partial^2 f}{\partial y \partial z}(x, y, z) = 1 \end{cases}$$

$$\frac{\partial f}{\partial z}(x, y, z) = z - x + y \quad \begin{cases} \frac{\partial^2 f}{\partial z \partial x}(x, y, z) = -1 \\ \frac{\partial^2 f}{\partial z \partial y}(x, y, z) = 1 \\ \frac{\partial^2 f}{\partial z^2}(x, y, z) = 1 \end{cases}$$

On fait la matrice: $M = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix}$

On étudie les mineurs principaux

$$M_1 = 1 > 0 \quad M_1' = 1 \geq 0 \quad M_1'' = 1 \geq 0$$

$$M_2 = 0 \geq 0 \quad M_2' = 0 \geq 0$$

$$M_3 = 0 \geq 0.$$

~~On peut donc conclure que la fonction est convexe~~
~~et qu'elle admet un minimum en 1~~

Comme les mineurs principaux sont tous supérieurs ou égaux à zéro $f(x, y, z)$ est convexe et admet un minimum en 1 car $f(0, 0, 0) = 1$.

Exercice 3: $f \rightarrow \mathbb{R}^2$

$$f(x, y) = x^2 + y^2 + z^2 - 2xyz.$$

$$\frac{\partial f}{\partial x}(x, y, z) = 2x - 2yz \begin{cases} \frac{\partial^2 f}{\partial x^2}(x, y, z) = 2 \\ \frac{\partial^2 f}{\partial x \partial y}(x, y, z) = -2z \\ \frac{\partial^2 f}{\partial x \partial z}(x, y, z) = -2y \end{cases}$$

$$\frac{\partial f}{\partial y}(x, y, z) = 2y - 2xz \begin{cases} \frac{\partial^2 f}{\partial y \partial x}(x, y, z) = -2z \\ \frac{\partial^2 f}{\partial y^2}(x, y, z) = 2 \\ \frac{\partial^2 f}{\partial y \partial z}(x, y, z) = -2x \end{cases}$$

$$\frac{\partial f}{\partial z}(x, y, z) = 2z - 2xy \begin{cases} \frac{\partial^2 f}{\partial z \partial x}(x, y, z) = -2y \\ \frac{\partial^2 f}{\partial z \partial y}(x, y, z) = -2x \\ \frac{\partial^2 f}{\partial z^2}(x, y, z) = 2 \end{cases}$$

$$\text{grad } f = \vec{0} \Leftrightarrow \begin{cases} 2x - 2yz = 0 \\ 2y - 2xz = 0 \\ 2z - 2xy = 0 \end{cases} \Leftrightarrow \begin{cases} x = yz \\ 2y - 2yz^2 = 0 \\ 2y - 2y^2z = 0 \end{cases}$$

$$\begin{cases} x = yz \\ y(1 - z^2) = 0 \\ z(1 - y^2) = 0 \end{cases} \Leftrightarrow \begin{cases} x = yz \\ 1 = z^2 \\ 1 = y^2 \end{cases} \Leftrightarrow \begin{cases} x = 1 \text{ ou } x = -1 \\ z = 1 \text{ ou } z = -1 \\ y = 1 \text{ ou } y = -1 \end{cases}$$

les Valeurs limites de f sont $(1, 1, 1)$ et $(-1, -1, -1)$.

On fait la Hessienne :

$$H = \begin{pmatrix} 2 & -2z & -2y \\ -2z & 2 & -2x \\ -2y & -2x & 2 \end{pmatrix}$$

$$H_1 = 2 \geq 0.$$

$$H_2 = (2 \times 2 - 2 \times 1) = 0 \geq 0.$$

$$H_3 = (2 \times 2 \times 2 + 2 \times 2 \times 2 + 2 \times 2 \times 2) - (2 \times 2 \times 1 + 2 \times 1 \times 2 + 2 \times 1 \times 1) = 0 \geq 0.$$

Pour $f(1, 1, 1)$ et $f(-1, -1, -1)$, $f(x, y)$ admet un minimum local ~~global~~. mm.

Exercice 4: $D = \{(x, y) \in \mathbb{R}^2 \mid 1 \leq x \leq 2 \text{ et } 1 \leq y \leq 2\}.$

$$\int_1^2 \int_1^2 \frac{1}{1+x+y} dx dy = \int_1^2 \left[\ln(1+x+y) \right]_1^2 dx.$$

$$= \int_1^2 \left[\ln(3+x) - \ln(2+x) \right] dx$$

$$= \left[(3+x) \ln(3+x) - (3+x) \right] - \left[(2+x) \ln(2+x) - (2+x) \right] \Big|_1^2$$

$$= [(5 \ln 5 - 5) - (4 \ln 4 - 4)] - [(4 \ln 4 - 4) - (3 \ln 3 - 3)].$$

$$= 5 \ln 5 - 8 \ln 4 + 3 \ln 3 + 6.$$

Exercise 5:

$$D = \{ (x, y, z) \in \mathbb{R}^3 \mid 0 \leq x \leq 1, 0 \leq y \leq 1, \text{ et } 0 \leq z \leq 1 \}.$$

$$\int_0^1 \int_0^1 \int_0^1 (x + y + z) \, dx \, dy \, dz = \int_0^1 \int_0^1 \left[xz + yz + \frac{z^2}{2} \right]_0^1 \, dx \, dy.$$

$$= \int_0^1 \int_0^1 \left[\left(x + y + \frac{1}{2} \right) - (0 + 0 + 0) \right] \, dx \, dy.$$

$$= \int_0^1 \left[xy + \frac{y^2}{2} + \frac{y}{2} \right]_0^1 \, dy = \int_0^1 (x + 1) \, dx$$

$$= \left[\frac{x^2}{2} + x \right]_0^1 = \frac{1}{2} + 1 = 3/2.$$