

1) Condition de normalisation: $\int_0^a |\Psi(x,t)|^2 dx = 1$

$$|\Psi(x,t)|^2 = (N x(a-x))^2$$

$$= N^2 x^4 - 2N^2 a x^3 + N^2 a^2 x^2$$

$$\int_0^a (N^2 x^4 - 2N^2 a x^3 + N^2 a^2 x^2) dx = N^2 \left[\frac{x^5}{5} - 2a \frac{x^4}{4} + a^2 \frac{x^3}{3} \right]_0^a$$

$$= N^2 \left(\frac{a^5}{5} - \frac{a^5}{2} + \frac{a^5}{3} \right)$$

$$= N^2 a^5 \left(\frac{1}{30} \right)$$

D'où: $\Rightarrow N^2 a^5 \left(\frac{1}{30} \right) = 1$

$$N^2 = 30 \times \frac{1}{a^5}$$

$$N = \sqrt{30 \times \frac{1}{a^5}}$$

2) Position moyenne:

$$\langle \hat{x} \rangle = \int_0^a x |\Psi(x)|^2 dx$$

$$= \int_0^a x (N^2 x^4 - 2N^2 a x^3 + N^2 a^2 x^2) dx$$

$$= N^2 \int_0^a (x^5 - 2a x^4 + a^2 x^3) dx$$

$$= N^2 \left[\frac{x^6}{6} - 2a \frac{x^5}{5} + a^2 \frac{x^4}{4} \right]_0^a$$

$$= N^2 \left(\frac{a^6}{6} - 2 \frac{a^6}{5} + \frac{a^6}{4} \right)$$

$$= \frac{1}{60} N^2 a^6$$

$$= \frac{1}{60} \times 30 \times \frac{1}{a^5} \times a^6$$

$$= \frac{1}{2} a$$

Impulsion moyenne:

$$\langle \hat{p} \rangle = -i\hbar \int_0^a (\psi^* \frac{d}{dx} \psi) dx$$

$$= -i\hbar \int_0^a (Nxa - Nx^2)(-2Nx + Na) dx$$

$$= -i\hbar \int_0^a (2N^2x^3 - N^2ax^2 - 2N^2ax^2 + N^2a^2x) dx$$

$$= -i\hbar N^2 \int_0^a (2x^3 - 3ax^2 + a^2x) dx$$

$$= -i\hbar N^2 \left[\frac{2x^4}{4} - 3a \frac{x^3}{3} + a^2 \frac{x^2}{2} \right]_0^a$$

$$= -i\hbar N^2 \left(\frac{1}{2}a^4 - a^4 + \frac{1}{2}a^4 \right)$$

$$= 0$$

$\psi^* = \psi \Rightarrow$ pas de position imaginaire

$$\frac{d}{dx} \psi = -2Nx + Na$$

3) Incertitude de la position: $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$

$$\langle x^2 \rangle = \int_0^a x^2 |\psi(x)|^2 dx$$

$$= \int_0^a x^2 (N^2x^4 - 2N^2ax^3 + N^2a^2x^2) dx$$

$$= N^2 \int_0^a (x^6 - 2ax^5 + a^2x^4) dx$$

$$= N^2 \left[\frac{x^7}{7} - 2a \frac{x^6}{6} + a^2 \frac{x^5}{5} \right]_0^a$$

$$= N^2 \left(\frac{a^7}{7} - \frac{a^7}{3} + \frac{a^7}{5} \right)$$

$$= N^2 a^7 \left(\frac{1}{105} \right)$$

$$= \frac{2}{7} a^2$$

$$\Rightarrow \Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{2}{7}a^2 - \frac{1}{4}a^2} = a \sqrt{\frac{2}{7} - \frac{1}{4}} = a \frac{\sqrt{7}}{14}$$

Incertitude minimum sur l'impulsion:

$$\Delta p = \frac{\hbar}{2\Delta x} = \frac{1}{2} \times \frac{\hbar}{a \frac{\sqrt{7}}{14}} = \frac{1}{2} \times \frac{\hbar 14}{a \sqrt{7}} = \frac{7}{a \sqrt{7}} = \frac{7\sqrt{7}}{a} = \frac{\sqrt{7}}{a}$$

TAI 7:

$$\text{Soit } \phi(x) = \begin{cases} N x e^{-\alpha x/2} & \text{si } 0 \leq x \leq \infty \\ 0 & \text{ailleurs} \end{cases}$$

$$\text{avec } \alpha \in \mathbb{R}^+ \text{ et } N = \sqrt{\frac{\alpha^3}{2}}$$

1) Probabilité de trouver la particule dans l'espace dx :

$$dP(x) = |\phi(x)|^2 dx$$

Probabilité de trouver la particule entre x et x+dx :

$$P(x) = \int_x^{x+dx} dP(x) (=1)$$

$$\text{Soit } dP(x) = |\phi(x)|^2$$

$$= \phi^*(x) \wedge \phi(x)$$

$$= N^2 x^2 e^{-\alpha x}$$

$$= \sqrt{\frac{\alpha^3}{2}}^2 x^2 e^{-\alpha x}$$

$$= \frac{\alpha^3}{2} x^2 e^{-\alpha x}$$

$$\text{ici } \phi^*(x) = \phi(x)$$

$$\text{donc } \phi^*(x) \wedge \phi(x) = \phi^2(x)$$

conjugue complexe de $\phi(x)$.

Position :

$$\langle x \rangle = \int_0^\infty x |\phi(x)|^2 dx = \frac{\alpha^3}{2} \int_0^\infty x^3 e^{-\alpha x} dx$$

$$= \frac{\alpha^3}{2} \times \frac{3!}{\alpha^4} = \frac{3}{\alpha}$$

Carré de la position :

$$\langle x^2 \rangle = \int_0^\infty x^2 |\phi(x)|^2 dx = \frac{\alpha^3}{2} \times x^4 e^{-\alpha x} dx$$

$$= \frac{\alpha^3}{2} \times \frac{4!}{\alpha^5} = \frac{12}{\alpha^2}$$

2) Impulsion:

$$\langle p \rangle = -i\hbar \int_0^{\infty} \left(\phi(x) \frac{d\phi(x)}{dx} \right) dx$$

$$\frac{d\phi(x)}{dx} = \left(N x e^{-\frac{\alpha x}{2}} \right) = N \left(e^{-\frac{\alpha x}{2}} + \left(-\frac{\alpha}{2} \right) x e^{-\frac{\alpha x}{2}} \right)$$

$$= -i\hbar \int_0^{\infty} N^2 x e^{-\frac{\alpha x}{2}} \left(e^{-\frac{\alpha x}{2}} + \left(-\frac{\alpha}{2} \right) x e^{-\frac{\alpha x}{2}} \right) dx$$

$$= -i\hbar \int_0^{\infty} N^2 x e^{-\frac{\alpha x}{2}} \left(e^{-\frac{\alpha x}{2}} + \left(-\frac{\alpha}{2} \right) x e^{-\frac{\alpha x}{2}} \right) dx$$

$$= -i\hbar N^2 \left[\int_0^{\infty} x e^{-\alpha x} dx - \frac{\alpha}{2} \int_0^{\infty} x^2 e^{-\alpha x} dx \right] = -i\hbar N^2 \left(\frac{1}{\alpha^2} - \frac{\alpha}{2} \times \frac{2}{\alpha^3} \right)$$

$$= -i\hbar N^2 \left(\frac{1}{\alpha^2} - \frac{1}{\alpha^2} \right) = \underline{0}$$

Carré de la position:

$$\langle p^2 \rangle = \int_0^{\infty} \phi^* \left(-i\hbar \frac{d}{dx} \right)^2 \phi dx = -\hbar^2 \int_0^{\infty} \phi^* \frac{d^2 \phi}{dx^2} dx$$

$$\frac{d^2 \phi(x)}{dx^2} = N \left(-\frac{\alpha}{2} \right) \left[e^{-\frac{\alpha x}{2}} + e^{-\frac{\alpha x}{2}} - \frac{\alpha}{2} x e^{-\frac{\alpha x}{2}} \right]$$

$$= -\hbar^2 \int_0^{\infty} N^2 \left(-\frac{\alpha}{2} \right) x e^{-\frac{\alpha x}{2}} \left(2e^{-\frac{\alpha x}{2}} - \frac{\alpha}{2} x e^{-\frac{\alpha x}{2}} \right) dx$$

$$= -\hbar^2 N^2 \left[-\alpha \int_0^{\infty} x e^{-\alpha x} dx + \frac{\alpha^2}{4} \int_0^{\infty} x^2 e^{-\alpha x} dx \right]$$

$$= -\hbar^2 N^2 \left(-\alpha \frac{1}{\alpha^2} + \frac{\alpha^2}{4} \frac{2}{\alpha^3} \right)$$

$$= -\hbar^2 N^2 \left(-\frac{1}{\alpha} + \frac{1}{2\alpha} \right)$$

$$= -\hbar^2 N^2 \left(-\frac{1}{2\alpha} \right)$$

$$= N^2 \times \frac{1}{2\alpha} \times \hbar^2 = \frac{\alpha^3}{2} \times \frac{1}{2\alpha} \times \hbar^2 = \underline{\underline{\frac{\alpha^2}{4} \hbar^2}}$$

3) Principe d'incertitude : $\Delta x \Delta p \geq \frac{\hbar}{2}$

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

$$= \sqrt{\frac{12}{\alpha^2} - \frac{9}{\alpha^2}}$$

$$= \sqrt{\frac{3}{\alpha^2}}$$

$$= \frac{\sqrt{3}}{\alpha}$$

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$$

$$= \sqrt{\frac{\alpha^2 \hbar^2}{4} - 0}$$

$$= \frac{\hbar \alpha}{2}$$

Ainsi : $\Delta x \Delta p = \frac{\alpha \hbar}{2} \frac{\sqrt{3}}{\alpha} = \frac{\hbar \sqrt{3}}{2} \geq \frac{1}{2} \hbar$

Le principe d'incertitude d'Heisenberg vérifié.

$$E = -\frac{\hbar^2 \alpha^2}{2m}$$

$$\phi(x) = N x e^{-\alpha x}$$

1)

Equation de Schrödinger: $-\frac{\hbar^2}{2m} \phi''(x) + V(x) \phi(x) = E \phi(x)$

$$V(x) = \frac{E \phi(x) + \frac{\hbar^2}{2m} \phi''(x)}{\phi(x)}$$

$$= \frac{E \times N x e^{-\alpha x} + \frac{\hbar^2}{2m} \times (-\alpha N e^{-\alpha x} (2 - \alpha x))}{N x e^{-\alpha x}}$$

$$= \frac{-\frac{\hbar^2 \alpha^2}{2m} \times \cancel{N x e^{-\alpha x}} + \frac{\hbar^2}{2m} \times (-\alpha \cancel{N e^{-\alpha x}} (2 - \alpha x))}{\cancel{N x e^{-\alpha x}}}$$

$$= \frac{-\hbar^2 \alpha^2 x - \hbar^2 \alpha (2 - \alpha x)}{2m x}$$

$$= \frac{-\cancel{\hbar^2 \alpha^2 x} - \hbar^2 \alpha x + \cancel{\hbar^2 \alpha^2 x}}{2m x}$$

$$= -\frac{\hbar^2 \alpha}{m x} \Rightarrow \text{Pour } x \neq 0$$

$$; \text{ Pour } x = \infty \Rightarrow V(x) = \frac{0}{0}$$

La forme indéterminée

2) $P(x \leq \text{particule} \leq x + dx) = \int_x^{x+dx} |\phi(x)|^2 dx$

$$= \int_x^{x+dx} N^2 x^2 e^{-2\alpha x} dx$$

$$f(x) = N^2 x^2 e^{-2\alpha x}$$

$$f'(x) = 2N^2 e^{-2\alpha x} (x - \alpha x^2)$$

$$\Rightarrow f'(x) = 0$$

$$\begin{cases} 2N^2 e^{-2\alpha x} = 0 \\ x - \alpha x^2 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ 1 - \alpha x = 0 \end{cases} \Leftrightarrow \begin{cases} x = 0 \\ x = \frac{1}{\alpha} \end{cases}$$

Soit f définie et dérivable sur $] -\infty; +\infty[$, donc sur $[0; +\infty[$

x	0	$1/\alpha$	$+\infty$
$1-\alpha x$	+	0	-
$N^2 x e^{-2\alpha x}$	+		+
$f'(x)$	+	0	-
$f(x)$			

$$1 - \alpha x < 0$$

$$\Leftrightarrow \alpha x > 1$$

$$\Leftrightarrow x > \frac{1}{\alpha}$$

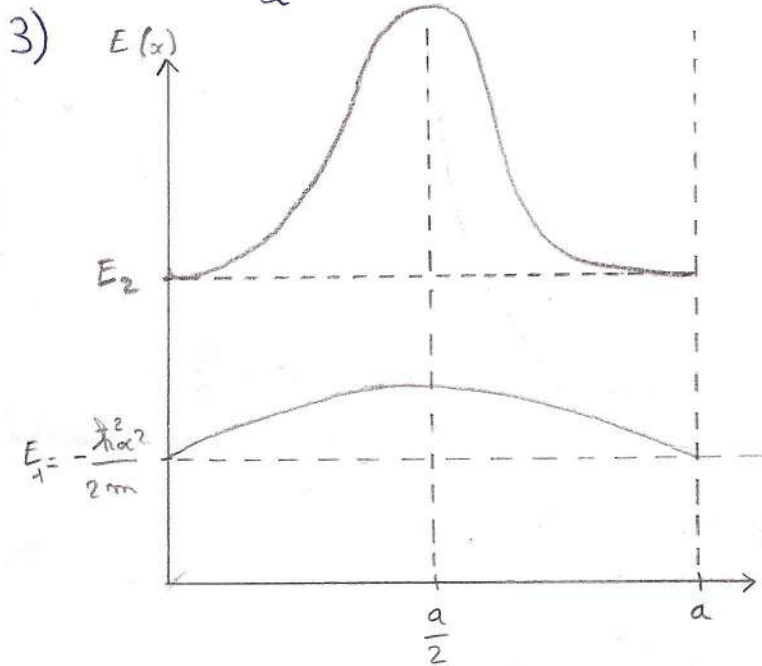
$$N^2 2x e^{-2\alpha x} > 0$$

$$\Leftrightarrow 2x > 0$$

$$\Leftrightarrow x > 0$$

$$N^2 \times \frac{1}{\alpha^2} e^{-2}$$

$$x_{\max} = \frac{1}{\alpha}$$



E_1 : état fondamental

E_2 : 1^{er} état excité

↳ fonction Gaussienne

TAI 9:

$$1) \langle E \rangle = \int_{-\infty}^{+\infty} \Psi(x, 0) \times E \times \Psi^*(x, 0) dx \quad (x, 0) \text{ car } t=0$$

$$= \int_{-\infty}^{+\infty} E(A\phi_1(x) + B\phi_2(x)) \times \Psi^*(x, 0) dx$$

$$= \int_{-\infty}^{+\infty} (AE_1\phi_1(x) + BE_2\phi_2(x)) \times (A\phi_1^*(x) + B\phi_2^*(x)) dx$$

$$= \int_{-\infty}^{+\infty} |A|^2 E_1 \phi_1^2(x) + AB E_2 \phi_1(x) \phi_2(x) + AB E_1 \phi_1(x) \phi_2(x) + |B|^2 E_2 \phi_2^2(x) dx$$

$$= |A|^2 E_1 \int_{-\infty}^{+\infty} \phi_1^2(x) dx + |B|^2 E_2 \int_{-\infty}^{+\infty} \phi_2^2(x) dx$$

$$+ AB \int_{-\infty}^{+\infty} \phi_1(x) \phi_2(x) dx \quad (E_2 + E_1)$$

Fonction d'onde normée et ortho~~normée~~

$$\Rightarrow \int \phi_1^2(x) dx = 1$$

$$\int \phi_2^2(x) dx = 1$$

$$\int \phi_1(x) \phi_2(x) dx = 0$$

$$\Leftrightarrow \langle E \rangle = |A|^2 E_1 + |B|^2 E_2$$

$$\langle E^2 \rangle = \int_{-\infty}^{+\infty} \Psi(x,0) \times E^2 \times \Psi^*(x,0) dx$$

$$= \int_{-\infty}^{+\infty} E^2 (A\phi_1(x) + B\phi_2(x)) \times \Psi^*(x,0) dx$$

$$= \int_{-\infty}^{+\infty} (AE_1^2\phi_1(x) + BE_2^2\phi_2(x)) (A\phi_1^*(x) + B\phi_2^*(x)) dx$$

$$= |A|^2 E_1^2 \int_{-\infty}^{+\infty} \phi_1^2(x) dx + |B|^2 E_2^2 \int_{-\infty}^{+\infty} \phi_2^2(x) dx + AB \int_{-\infty}^{+\infty} \phi_1(x) \phi_2(x) dx (E_1^2 + E_2^2)$$

Fonction d'onde normée et orthonormée:

$$\Rightarrow \int \phi_1^2(x) dx = 1$$

$$\int \phi_2^2(x) dx = 1$$

$$\int \phi_1(x) \phi_2(x) dx = 0$$

$$\Rightarrow \langle E^2 \rangle = |A|^2 E_1^2 + |B|^2 E_2^2$$

2) Par identification: $|A|^2 = \frac{1}{4} \Rightarrow |A| = \pm \frac{1}{2}$

$$|B|^2 = \frac{3}{4} \Rightarrow |B| = \pm \frac{\sqrt{3}}{2}$$

D'où: $\Psi(x,t) = \frac{1}{2} \Psi_1(x,t) + \frac{\sqrt{3}}{2} \Psi_2(x,t)$

3) Incertitude sur l'énergie de la particule

$$\Delta E = \sqrt{\langle E^2 \rangle - \langle E \rangle^2}$$

$$= \sqrt{|A|^2 E_1^2 + |B|^2 E_2^2 - (|A|^2 E_1 + |B|^2 E_2)^2}$$

$$= \sqrt{|A|^2 E_1^2 + |B|^2 E_2^2 - |A|^4 E_1^2 - |B|^4 E_2^2 - 2|A|^2 E_1 |B|^2 E_2}$$

$$= \sqrt{\frac{1}{4} E_1^2 + \frac{3}{4} E_2^2 - \frac{1}{16} E_1^2 - \frac{9}{16} E_2^2 - E_1 \frac{3}{8} E_2}$$

$$= \sqrt{\frac{3}{16} E_1^2 + \frac{3}{16} E_2^2 - \frac{3}{8} E_1 E_2}$$

$$= \sqrt{\frac{3}{16} (E_1^2 + E_2^2 - 2 E_1 E_2)}$$

$$= \sqrt{\frac{3}{16} (E_1 - E_2)^2}$$

$$= \sqrt{3} \times \frac{E_1 - E_2}{4}$$

TAI 10:

$$E_m = (m + \frac{1}{2}) \hbar \omega$$

$$E_0 = \frac{1}{2} \hbar \omega$$

$$E_1 = \frac{3}{2} \hbar \omega$$

1) Ajout d'une dimension temporelle à $\Psi(x, 0)$

$$\Psi(x, t) = \frac{1}{\sqrt{2}} \left(\phi_0(x) e^{-\frac{i E_0 t}{\hbar}} + \phi_1(x) e^{-\frac{i E_1 t}{\hbar}} \right)$$

$$= \frac{1}{\sqrt{2}} \left(\phi_0(x) e^{-\frac{i \frac{1}{2} \hbar \omega t}{\hbar}} + \phi_1(x) e^{-\frac{i \frac{3}{2} \hbar \omega t}{\hbar}} \right)$$

$$= \frac{1}{\sqrt{2}} \left(\phi_0(x) e^{-\frac{i \omega t}{2}} + \phi_1(x) e^{-\frac{3 i \omega t}{2}} \right)$$

Fonction d'onde normalisée: $\int_{-\infty}^{+\infty} \Psi(x, t) \Psi^*(x, t) dx = 1$

$$\Psi(x, t) \times \Psi^*(x, t) = \frac{1}{2} \left[\left(\phi_0(x) e^{-\frac{i \omega t}{2}} + \phi_1(x) e^{-\frac{3 i \omega t}{2}} \right) \left(\phi_0^*(x) e^{+\frac{i \omega t}{2}} + \phi_1^*(x) e^{+\frac{3 i \omega t}{2}} \right) \right]$$

Or, $\phi_0(x)$ et $\phi_1(x)$ réels:

$$= \frac{1}{2} \underbrace{e^{-\frac{i \omega t}{2}}}_{=1} \left(\phi_0(x) + \phi_1(x) e^{-i \omega t} \right) \underbrace{e^{+\frac{i \omega t}{2}}}_{=1} \left(\phi_0(x) + \phi_1(x) e^{+i \omega t} \right)$$

$$= \frac{1}{2} \left(\phi_0(x) + \phi_1(x) e^{-i \omega t} \right) \left(\phi_0(x) + \phi_1(x) e^{+i \omega t} \right)$$

$$= \frac{1}{2} \left(\phi_0^2(x) + \phi_0(x) \phi_1(x) e^{i \omega t} + \phi_0(x) \phi_1(x) e^{-i \omega t} + \phi_1^2(x) \right)$$

$$= \frac{1}{2} \left(\phi_0^2(x) + \phi_1^2(x) + \phi_0(x) \phi_1(x) (e^{i \omega t} + e^{-i \omega t}) \right) \quad \begin{array}{l} \text{Formule d'Euler:} \\ e^{i \omega t} = \cos \omega t + i \sin \omega t \\ e^{-i \omega t} = \cos \omega t - i \sin \omega t \end{array}$$

$$= \frac{1}{2} \left(\phi_0^2(x) + \phi_1^2(x) + \phi_0(x) \phi_1(x) \times 2 \cos(\omega t) \right)$$

D'où:

$$\begin{aligned} \int_{-\infty}^{+\infty} \Psi(x, t) \times \Psi^*(x, t) dx &= \frac{1}{2} \underbrace{\int_{-\infty}^{+\infty} \phi_0^2(x) dx}_{=1 \text{ car normalisée}} + \frac{1}{2} \underbrace{\int_{-\infty}^{+\infty} \phi_1^2(x) dx}_{=1 \text{ car normalisée}} \\ &\quad + \cos(\omega t) \underbrace{\int_{-\infty}^{+\infty} \phi_0(x) \phi_1(x) dx}_{=0 \text{ car orthonormalisée}} \\ &= \frac{1}{2} + \frac{1}{2} + 0 = 1 \rightarrow \text{normalisée} \end{aligned}$$

$$2) \langle x \rangle = \int_{-\infty}^{+\infty} \Psi(x, t) \times x \times \Psi^*(x, t) dx \quad x \text{ pas opérateur}$$

$$= \frac{1}{2} \int_{-\infty}^{+\infty} x \phi_0^2(x) dx + \frac{1}{2} \int_{-\infty}^{+\infty} x \phi_1^2(x) dx + \int_{-\infty}^{+\infty} x \phi_0(x) \phi_1(x) \cos(\omega t) dx$$

$$= \frac{1}{2} \underbrace{\langle x \rangle}_{=0} \phi_0(x) + \frac{1}{2} \underbrace{\langle x \rangle}_{=0} \phi_1(x) + \cos(\omega t) \int_{-\infty}^{+\infty} x \phi_0(x) \phi_1(x) dx$$

$$= \cos(\omega t) \int_{-\infty}^{+\infty} x \phi_0(x) \phi_1(x) dx$$

$$= A \cos(\omega t)$$

$$E_m = (m + \frac{1}{2}) \hbar \omega$$

$$\Rightarrow \omega = \frac{E_m}{(m + \frac{1}{2}) \hbar} \quad \text{Pour } m=1: \quad \omega = \frac{E_1 - E_0}{\hbar}$$

$$3) E_m = (m + \frac{1}{2}) \hbar \omega \quad \text{et } E = h \nu = h \frac{c}{\lambda} \quad \text{et } \hbar = \frac{h}{2\pi}$$

$$\hookrightarrow \lambda = \frac{hc}{E} = \frac{\cancel{h} 2\pi c}{\hbar \omega} = \frac{2\pi c}{\omega} \quad \text{avec } \omega = \sqrt{\frac{k}{\mu}}$$

$$\text{D'où: } \lambda = 2\pi c \times \sqrt{\frac{\mu}{k}} \Rightarrow k = \frac{(2\pi c)^2 \mu}{\lambda^2}$$

Masse réduite: $\mu = \frac{m_N \times m_O}{m_N + m_O} = \frac{M_N \times M_O}{M_N + M_O} \times \frac{1}{N_A} \rightarrow \text{nombre d'Avogadro}$
 (problème à 2 corps) ↳ $6,022 \times 10^{23} \text{ mol}^{-1}$

$$= \frac{14 \times 10^{-3} \times 16 \times 10^{-3}}{14 \times 10^{-3} + 16 \times 10^{-3}} \times \frac{1}{6,022 \times 10^{23}}$$

$$= 1,24 \times 10^{-23} \quad \text{sans unité}$$

$$\text{Donc: } k = \frac{(2\pi c)^2 \mu}{\lambda^2} = \frac{(2\pi \times 3 \times 10^8)^2 \times 1,24 \times 10^{-23}}{5330 \times 10^{-9}}$$

$$k = 8,268$$

1) Equation de Schrödinger:

$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V(r) \psi = E \psi$$

$$\text{or } \nabla^2 = \frac{1}{r} \frac{\partial^2 (r \psi)}{\partial r^2}$$

$$\text{Donc } -\frac{\hbar^2}{2m} \times \frac{1}{r} \frac{\partial^2 (r \psi)}{\partial r^2} + V(r) \psi(r) = E \psi(r)$$

$$\text{or } \psi(r) = \frac{u(r)}{r}$$

$$-\frac{\hbar^2}{2m} \frac{\partial^2 u(r)}{\partial r^2} + V(r) u(r) = E u(r)$$

$$-\frac{\hbar^2}{2m} \underbrace{u''(r)}_a + V(r) \underbrace{u(r)}_c = E \underbrace{u(r)}_c$$

$$b=0$$

2)

$$\Delta = b^2 - 4ac$$

$$\Delta = -4 \times \left(-\frac{\hbar^2}{2m} \times (V(r) - E) \right)$$

$$\boxed{r < R}$$

$$\Delta = -4 \times \frac{+\hbar^2 \times (V_0 + E)}{2m} \rightarrow > 0$$

$$\underline{\Delta < 0}$$

$$\text{Solution explosive: } u(r) = A e^{ikr} + B e^{-ikr}$$

$$\boxed{r > R}$$

$$\Delta = -4 \times \frac{\hbar^2 \times E}{2m} \rightarrow E > 0$$

$$\underline{\Delta \leq 0}$$

$$\text{Solution de la forme: } u(r) = A e^{kr}$$

3)

$$r < R$$

$$\lim_{r \rightarrow 0} \psi(r) = \lim_{r \rightarrow 0} \frac{U(r)}{r} = 0$$

$$U(0) = 0$$

$$A e^{+ik_0 r} + B e^{-ik_0 r} = 0$$

$$A + B = 0 \Rightarrow A = -B$$

$$\text{Donc } U(r) = A e^{+ik_0 r} - A e^{-ik_0 r}$$

$$= A (e^{+ik_0 r} - e^{-ik_0 r})$$

$$= 2i A \sin(k_0 r) = C \sin(k_0 r)$$

$$U'(r) = C k_0 \cos(k_0 r)$$

$$U''(r) = -C k_0^2 \sin(k_0 r)$$

D'après le 1):

$$-\frac{\hbar^2}{2m} U''(r) + (V(r) - E) \cdot U(r) = 0$$

$$\text{or: } r < R$$

$$-\frac{\hbar^2}{2m} U''(r) - (V(0) + E) U(r) = 0$$

$$+\frac{\hbar^2}{2m} C k_0^2 \sin(k_0 r) - C (V_0 + E) \sin(k_0 r) = 0$$

$$\frac{\hbar^2}{2m} k_0^2 - (V_0 + E) = 0$$

$$k_0^2 = \frac{V_0 + E}{\frac{\hbar^2}{2m}} = \frac{(V_0 + E) 2m}{\hbar^2}$$

$$k_0 = \frac{\sqrt{(V_0 + E) 2m}}{\hbar}$$

$$\psi(r) = \frac{U(r)}{r} = C \sin(k_0 r) / r$$

$$r > R$$

$$u(r) = A e^{k_1 r} + B e^{-k_1 r}$$

$$u(\infty) = 0$$

$$r \rightarrow +\infty \quad \underbrace{A' e^{k_1 r}}_{\neq 0} + \underbrace{B' e^{-k_1 r}}_{= 0 \text{ lorsque } r \rightarrow +\infty} = 0$$

$$\underbrace{A'}_{\neq 0} e^{k_1 r} = 0$$

$$A' = 0$$

$$u(r) = B' e^{-k_1 r}$$

$$u'(r) = -B' k_1 e^{-k_1 r}$$

$$u''(r) = B' k_1^2 e^{-k_1 r}$$

$$-\frac{\hbar^2}{2m} u''(r) - E u(r) = 0$$

$$-\frac{\hbar^2}{2m} B' k_1^2 e^{-k_1 r} - E B' e^{-k_1 r} = 0$$

$$-\frac{\hbar^2}{2m} k_1^2 - E = 0$$

$$-\frac{\hbar^2}{2m} k_1^2 - \frac{\hbar^2 \alpha^2}{2m} = 0$$

$$k_1^2 = \alpha^2 \Rightarrow k_1 = \alpha$$

Donc :

$$u(r) = B' e^{-\alpha r}$$

$$\text{or } \varphi(r) = \frac{B'}{r} e^{-\alpha r} \quad \left(\text{car } \varphi(r) = \frac{u(r)}{r} \right)$$

4) Nous avons condition de continuité : $V_0 = 0$

$$\phi_n(r) = A \sin(k_1 r)$$

$$\phi_n(r) = A \sin\left(\frac{n\pi}{a} r\right)$$

$$k_n = \frac{\sqrt{2m(E_1 + V_0)}}{\hbar}$$

$$= \frac{\sqrt{2m E_n}}{\hbar}$$

or selon le cours :

$$E_n = \frac{\hbar^2 \pi^2 n^2}{2m a^2} \quad E_1$$

$$k_n = \frac{\sqrt{2m \frac{\hbar^2 \pi^2 n^2}{2m a^2}}}{\hbar}$$

$$k_n = \frac{n\pi}{a}$$

Exemple $n=3$:

$$\phi_3(r) = A \sin\left(\frac{3\pi}{a} r\right)$$

