

TAI n°10

$$\Psi(z,t) = \frac{1}{\sqrt{2}} \left[\Phi_0 e^{\frac{-iE_0 t}{\hbar}} + \Phi_1 e^{\frac{-iE_1 t}{\hbar}} \right]$$

$$E_n = \left(n + \frac{1}{2}\right) \hbar \omega$$

$$\text{donc } E_0 = \frac{1}{2} \hbar \omega \text{ et } E_1 = \frac{3}{2} \hbar \omega$$

$$\text{donc } \Psi(z,t) = \frac{1}{\sqrt{2}} \left[\Phi_0 e^{\frac{-i\omega t}{2}} + \Phi_1 e^{\frac{-3i\omega t}{2}} \right]$$

$$\Psi(z,t) \Psi^*(z,t) = \frac{1}{2} \left[\Phi_0^* e^{\frac{i\omega t}{2}} + \Phi_1^* e^{\frac{3i\omega t}{2}} \right] \left[\Phi_0 e^{\frac{-i\omega t}{2}} + \Phi_1 e^{\frac{-3i\omega t}{2}} \right]$$

On sait que $\Phi_0 \in \mathbb{R}$ et $\Phi_1 \in \mathbb{R}$

donc :

$$\Psi(z,t) \Psi^*(z,t) = \frac{1}{2} \left[\Phi_0 e^{\frac{i\omega t}{2}} + \Phi_1 e^{\frac{3i\omega t}{2}} \right] \left[\Phi_0 e^{\frac{-i\omega t}{2}} + \Phi_1 e^{\frac{-3i\omega t}{2}} \right]$$

$$\Psi(z,t) \Psi^*(z,t) = \frac{1}{2} \left[\Phi_0^2 + \Phi_0 \Phi_1 e^{-i\omega t} + \Phi_1 \Phi_0 e^{i\omega t} + \Phi_1^2 \right]$$

$$\Psi(2t)\Psi^*(2t) = \frac{1}{2} [\Phi_0^2 + \Phi_0\Phi_1 e^{-i\omega t} + \Phi_1\Phi_0 e^{i\omega t} + \Phi_1^2]$$

$$\Psi(2t)\Psi^*(2t) = \frac{1}{2} [\Phi_0^2 + \Phi_1^2 + \Phi_0\Phi_1 (e^{-i\omega t} + e^{i\omega t})]$$

$$\Psi(2t)\Psi^*(2t) = \frac{1}{2} [\Phi_0^2 + \Phi_1^2 + \Phi_0\Phi_1 2\cos(\omega t)]$$

Donc $\psi(x,t)$ normalisée

$$2) \quad \langle x \rangle = \int_{-\infty}^{+\infty} \psi^* x \psi dx$$

$$= \frac{1}{2} \int_{-\infty}^{+\infty} \phi_0^2 - \phi_1^2 - 2\phi_0\phi_1 \cos(\omega t) dx$$

$$= \frac{1}{2} \int_{-\infty}^{+\infty} \phi_0^2 x dx + \frac{1}{2} \int_{-\infty}^{+\infty} \phi_1^2 x dx + \cos(\omega t) \int_{-\infty}^{+\infty} \phi_0 x \phi_1 dx$$

$$= \cos(\omega t) \int_{-\infty}^{+\infty} \phi_0(x) x \phi_1(x) dx$$

$$\Rightarrow \underline{\underline{\langle x \rangle = A \cos(\omega t)}}.$$

1) On intègre $\Psi(x,t)\Psi^*(x,t)$

$$\int_{-\infty}^{+\infty} \Psi(x,t)\Psi^*(x,t) dx = \frac{1}{2} \int_{-\infty}^{+\infty} \Phi_0^2 dx + \frac{1}{2} \int_{-\infty}^{+\infty} \Phi_1^2 dx + \frac{1}{2} 2 \cos(ut) \int_{-\infty}^{+\infty} \Phi_0 \Phi_1 dx$$

$$= \underbrace{\frac{1}{2} \int_{-\infty}^{+\infty} \Phi_0^2 dx}_1 \text{ car normalisée,} + \underbrace{\frac{1}{2} \int_{-\infty}^{+\infty} \Phi_1^2 dx}_1 \text{ car normalisée,} + \underbrace{\cos(ut) \int_{-\infty}^{+\infty} \Phi_0 \Phi_1 dx}_{\text{produit scalaire} \equiv 0 \text{ car } \Phi_0 \text{ et } \Phi_1 \text{ orthogonales}}$$

$$= \frac{1}{2} + \frac{1}{2} = \boxed{1}$$

Donc $\Psi(x,t)$ normalisée