

Equation de Schrödinger $-\frac{\hbar^2}{2m} \nabla^2 \Psi(r) + V(r) \Psi(r) = E \Psi(r) \quad (1)$
 avec $\nabla^2 = \frac{1}{r} \frac{\partial^2}{\partial r^2} r$

1) $-\frac{\hbar^2}{2m} u''(r) + V(r) u(r) = E u(r) \quad (2)$

On sait que l'onde stationnaire respecte l'équation 1

$\frac{1}{r} \left(-\frac{\hbar^2}{2m} \nabla^2 u(r) + V(r) u(r) \right) = E u(r)$

$\Rightarrow -\frac{\hbar^2}{2m} \nabla^2 u(r) + V(r) u(r) = E u(r)$

$\Rightarrow -\frac{\hbar^2}{2m} \left(\frac{1}{r} \frac{\partial^2}{\partial r^2} r \right) u(r) + V(r) u(r) = E u(r)$

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$\Rightarrow -\frac{\hbar^2}{2m} u''(r) \left(\frac{1}{r} u(r) \right) + V(r) u(r) = E u(r)$
 $\Psi(r)$

2) $E = -\frac{\hbar^2 \alpha^2}{2m}$ avec $0 < E < V_0$

$r < R: \Rightarrow V(r) = -V_0$

donc (2) $\Rightarrow -\frac{\hbar^2}{2m} u''(r) - V_0 u(r) = -\frac{\hbar^2 \alpha^2}{2m} u(r)$

$\Rightarrow \frac{\hbar^2}{2m} (-u''(r) + u(r) \alpha^2) = V_0 u(r)$

$\Rightarrow \frac{\hbar^2}{2m V_0} = \frac{u(r)}{u(r) \alpha^2 - u''(r)}$

$\Rightarrow A \sin(k_0 r)$

solution 1 $|n=3|$

$A \sin(k_0 r)$
 $\alpha^2 A \sin(k_0 r) - (A \sin(k_0 r))''$
 $= \sin(k_0 r)$
 $\alpha^2 \sin(k_0 r) - (-\sin(k_0 r))$
 $= \alpha^2 + 1$
 $\alpha^2 + k^2$
 $\frac{\hbar^2}{2m} (\alpha^2 + k^2) = \frac{\hbar^2}{2m} \alpha^2$

solution 2

$[B \exp(-\alpha r)]'$
 $B \exp(-\alpha r)$

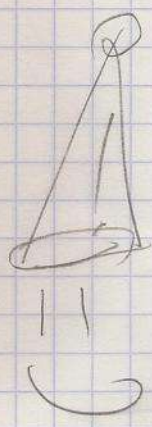
$\Rightarrow \frac{(e^{-\alpha r})'}{e^{-\alpha r}} = -\alpha$
 $\Rightarrow \alpha^2$

$r > R \Rightarrow V(r) = 0$

$-\frac{\hbar^2}{2m} u''(r) = -\frac{\hbar^2 \alpha^2}{2m} u(r)$
 $\Rightarrow u''(r) = \alpha^2 u(r)$

$\Rightarrow \frac{u''(r)}{u(r)} = \alpha^2$

$\Rightarrow B \exp(-\alpha r)$



Exercice 11

3) $\Psi(r) \xrightarrow{r \rightarrow 0} 0 \Rightarrow$ fonction de type $\exp(-\alpha r)$

$\Psi(r) \xrightarrow{r \rightarrow \infty} 0 \Rightarrow$ fonction de type $\sin(\alpha r)$

Cas $r < R$

co solution $\frac{1}{r} \sin(k_0 r) = \Psi(r)$

On a $\Psi(r) = \frac{u(r)}{r}$ d'où $u(r) = A \sin(k_0 r)$

$\rightarrow$ On remplace dans l'équation $\frac{u(r)}{u(r)\alpha^2 - u''(r)} = \frac{\hbar^2}{2mV_0}$

$$\hookrightarrow \frac{A \sin(k_0 r)}{A \sin(k_0 r) - [A \sin(k_0 r)]''}$$

$$= \frac{\sin(k_0 r)}{\alpha^2 \sin(k_0 r) - [\sin(k_0 r)]''}$$

$$\sin(k_0 r)'' = k_0 [\cos(k_0 r)]' = -k_0^2 \sin(k_0 r)$$

$$= \frac{\sin(k_0 r)}{\alpha^2 \sin(k_0 r) (1 - (-k_0^2))}$$

$$= \frac{1}{\alpha^2 + k_0^2} \quad \text{or } k_0 = \frac{\sqrt{2m(V_0 + E)}}{\hbar}$$

$$= \frac{1}{\alpha^2 + \frac{2m(V_0 + E)}{\hbar^2}}$$

$$= \frac{\hbar^2}{\hbar^2 \alpha^2 + 2m(V_0 + E)} \quad \text{or } E = \frac{\hbar^2 \alpha^2}{2m}$$

$$= \frac{\hbar^2}{\hbar^2 \alpha^2 + 2mV_0 - \hbar^2 \alpha^2}$$

$$= \frac{\hbar^2}{2mV_0}$$

L'équation est vérifiée.

Cas $r > R$

co solution $\frac{1}{r} \exp(-\alpha r) = \Psi(r)$

On a donc $u(r) = \Psi(r) r = B \exp(-\alpha r)$

On remplace dans l'équation $\frac{u''(r)}{u(r)} = \alpha^2$

$$\frac{[B \exp(-\alpha r)]''}{B \exp(-\alpha r)} = \frac{(e^{-\alpha r})''}{e^{-\alpha r}} = \frac{-\alpha(e^{-\alpha r})'}{e^{-\alpha r}} = \alpha^2 \frac{e^{-\alpha r}}{e^{-\alpha r}} = \alpha^2$$

L'équation est vérifiée.