

Les fonctions d'ondes peuvent s'additionner donc des sommes de fonctions d'ondes forment un espace vectoriel. La condition de normalisation permet donc de mathématiser / formaliser ce fait en posant que l'onde se situe dans notre espace à coup sur. Donc l'intégrale de 0 à $a = 1$ nous donne une proba de 1 pour que notre onde soit dans notre espace

$$\int_0^a |\Psi(x)|^2 dx \quad \text{or} \quad \Psi(x) = N x(a-x) \quad \text{donc}$$

$$\int_0^a |N x(a-x)|^2 dx = |N|^2 \int_0^a |x(a-x)|^2 dx = 1 = |N|^2 \int_0^a (x^2 a^2 - 2x^3 a + x^4) dx$$

~~$$|N|^2 \int_0^a (x^2 a^2 - 2x^3 a + x^4) dx = 1$$~~

$$|N|^2 \int_0^a (x^2 a^2 - 2x^3 a + x^4) dx$$

$$= |N|^2 \left[\frac{a^2 x^3}{3} - \frac{a x^4}{2} + \frac{x^5}{5} \right]_0^a = |N|^2 \left[\frac{a^5}{3} - \frac{a^5}{2} + \frac{a^5}{5} \right]$$

$$= \frac{|N|^2 a^5}{30} = 1$$

$$N^2 = \frac{30}{a^5} \Rightarrow \boxed{N = \sqrt{\frac{30}{a^5}}}$$

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2) position moyenne :

$$\begin{aligned}\langle x \rangle &= \int_0^a x |\psi(x)|^2 dx = \int_0^a x |N x(a-x)|^2 dx \\&= |N|^2 \int_0^a x^3 (a^2 - 2ax - x^2) dx = |N|^2 \int_0^a (x^3 a^2 - 2a x^4 + x^5) dx \\&= |N|^2 \left[\frac{x^4 a^2}{4} - \frac{2a x^5}{5} + \frac{x^6}{6} \right]_0^a = |N|^2 \left(\frac{a^6}{4} - \frac{2a^6}{5} + \frac{a^6}{6} \right) \\&\quad \langle x \rangle = \frac{|N|^2 a^6}{60} \quad \text{or } N = \sqrt{\frac{30}{a^5}}\end{aligned}$$

$$\langle x \rangle = \frac{\frac{30}{a^5} a^6}{60} = \frac{a}{2}$$

impulsion moyenne :

$$\begin{aligned}\langle p \rangle &= \int_{-\infty}^{+\infty} \psi^*(x) \left[-\frac{\hbar}{i} \right] \frac{\partial \psi}{\partial x} dx \\&= \int_0^a (N x(a-x)) \left[-\frac{\hbar}{i} \right] (-N(x-a) - Nx) dx \\&= -\frac{\hbar}{i} \int_0^a N^2 x(a-x)(2x-a) dx \\&= N^2 \frac{\hbar}{i} \left[2x^2 a - 2x^3 \right]_0^a = 0\end{aligned}$$

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3) incertitude de position :

$$\begin{aligned}\langle x^2 \rangle &= \int_0^a \psi^*(x) \hat{x}^2 \psi(x) dx \quad \text{avec } \hat{x}^2 = x^2 \\&= \int_0^a \psi(x^2) x^2 dx \\&= N^2 \int_0^a x^2 (a-x)^2 x^2 dx = N^2 \int_0^a x^4 (a-x)^2 dx \\&= N^2 \int_0^a x^4 a^2 - 2ax^5 + x^6 dx \\&= N^2 \left[\frac{x^5 a^2}{5} - \frac{2ax^6}{6} + \frac{x^7}{7} \right]_0^a = N^2 \left(\frac{a^7}{5} - \frac{2a^7}{6} + \frac{a^7}{7} \right) \\&= \frac{30a^7}{a^5 \times 105} = \frac{2a^2}{7}\end{aligned}$$

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{2a^2}{7} - \frac{a^2}{4}} = \sqrt{\frac{8a^2 - 7a^2}{28}}$$

$$\Delta x = \sqrt{\frac{a^2}{28}} = \frac{a}{\sqrt{28}}$$

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incertitude de l'impulsion :

$$\langle p^2 \rangle = \int_0^a \psi^*(x, t) \hat{p}^2 \psi(x, t) dx$$

$$= \int_0^a (Nx(x-a)) \hat{p}^2 \psi(x, t) dx$$

$$\hat{p}^2 \psi(x, t) = \left(-i\hbar \frac{\partial}{\partial x} \right)^2 \psi(x, t)$$

$$\hat{p}^2 \psi(x, t) = \left(-i\hbar \frac{\partial}{\partial x} \right)^2 \psi(x, t) = -\hbar^2 \frac{\partial^2}{\partial x^2} \psi(x, t)$$

$$\text{or } \frac{\partial}{\partial x} \psi(x, t) = -N(2x - a)$$

$$\hat{p}^2 \psi(x, t) = \hbar^2 \times 2N$$

$$\langle p^2 \rangle = \hbar^2 2N^2 \int_0^a x(a-x) dx = \hbar^2 2N^2 \int_0^a x^2 - xa dx$$

$$= \hbar^2 2N^2 \left[\frac{x^3}{3} - \frac{xa^2}{2} \right]_0^a = \hbar^2 2N^2 \left(\frac{a^3}{3} - \frac{a^3}{2} \right)$$

$$= \hbar^2 2N^2 \left(-\frac{1}{6} a^3 \right) = \hbar^2 2 \times \frac{30}{6a^2} = \frac{\hbar^2 10}{a^2}$$

$\nearrow$
car $N = \frac{30}{a^3}$

$$\Delta p = \sqrt{\frac{\hbar^2 10}{a^2}} = \sqrt{10} \frac{\hbar}{a}$$