

EX 6

$$0 \leq x \leq a$$

NB: les fonctions les plus courantes sont donc l'ensemble des fonctions définies sur un espace vectoriel $\rightarrow$ condition de normalisation

$$\psi(x) = Nx(a-x) \quad \text{avec } N \in \mathbb{R}$$

1) Normalisation $\Rightarrow$ c'est trouver la condition qui nous assure de trouver la particule dans l'espace $[0, a]$ l'intégrale de 0 à a = 1

$$\int_0^a \psi(x) dx = N \int_0^a x(a-x) dx = N \int_0^a (ax - x^2) dx$$

$$= N \left[\frac{ax^2}{2} - \frac{x^3}{3} \right]_0^a = 1$$

$$= N \left(\frac{a^3}{2} - \frac{a^3}{3} \right) = 1$$

$$= N \frac{a^3}{6} = 1$$

$$\Rightarrow N = \frac{6}{a^3}$$

$$\text{donc } \psi(x) = \frac{6x}{a^3}(x-a)$$

② ~~calculer la valeur moyenne~~ = position moyenne

$$\langle x \rangle = \int_{-\infty}^{+\infty} x |\psi(x)|^2 dx$$

$$= \int_0^a x \left(\frac{6}{a^3} x(a-x) \right)^2 dx$$

$$(2) \langle x \rangle = \int_{-\infty}^{+\infty} x |\psi(x)|^2 dx$$

$$= \int_0^a x \left(\frac{6x^2}{a^3} - \frac{6x}{a^2} \right)^2 dx$$

$$= \int_0^a x \left(\frac{36x^4}{a^6} + \frac{36x^2}{a^4} - \frac{2 \cdot 36x^3}{a^5} \right) dx$$

$$= \int_0^a \frac{36x^5}{a^6} - \frac{2 \cdot 36x^4}{a^5} + \frac{36x^5}{a^6} dx$$

$$= 36 \left[\frac{x^6}{6a^6} - \frac{2x^5}{5a^5} + \frac{x^6}{6a^6} \right]_0^a$$

$$= 36 \left(\frac{a^6}{4a^6} - \frac{2a^5}{5a^5} + \frac{a^6}{6a^6} \right) = 36 \left(\frac{1}{4} - \frac{2}{5} + \frac{1}{6} \right) = 36 \left(\frac{5}{12} - \frac{2}{5} \right) = 36 \times \frac{13}{60}$$

$$\frac{6 \times 13}{10} = \frac{3 \times 13}{5} \rightarrow \text{position moyenne}$$

$$= \frac{36}{60} = \frac{6}{10} = \frac{3}{5}$$

$$= N \int$$

$$= N \int$$

$$= N \int$$

$$\Rightarrow$$

donc ψ

② valeur

$\psi = 0$

$$= \frac{N^2 a^6}{4} - \frac{2 N^2 a^6}{5}$$

$$N = \frac{6}{a^3}$$

$$\text{or } \psi = \frac{6}{a^3} \left(\frac{x^2}{a^3} - \frac{x}{a^2} \right)$$

impulsion moyenne:

$\hat{p}$, impulsion est égale à $\hat{p}$ si que de comme les autres

la val moyenne d'un générateur = N mesures de $\hat{A}$ (ici $\hat{p}$)
avec N particules ds le m état

$$\text{donc } \langle \hat{p} \rangle_{\Psi} = \langle \Psi | \hat{p} | \Psi \rangle = \int_0^a \Psi^*(x, t) (\hat{p} \Psi(x, t)) dx$$

on ne dépend pas du temps et $\Psi \in \mathbb{R}$

$$\hat{p} = \hbar k \Psi$$

$$\text{donc } \langle \hat{p} \rangle_{\Psi} = \int_0^a \hbar k \Psi^2(x) dx = \hbar k \int_0^a \left(\frac{6x}{a^3} (x-a) \right)^2 dx$$

$$= \hbar k \int_0^a \left(\frac{6x^2}{a^3} - \frac{6x}{a^2} \right)^2 dx$$

$$= \hbar k \int_0^a \frac{36x^4}{a^6} + \frac{36x^2}{a^4} - \frac{36x^3}{a^5} dx$$

$$= \hbar k \left[\frac{x^5}{5a^6} + \frac{x^3}{3a^4} - \frac{36x^4}{4a^5} \right]_0^a$$

$$= 36 \hbar k \left(\frac{1}{5a} + \frac{1}{3a} - \frac{2a}{4a} \right) = 36 \hbar k \left(\frac{1}{5a} + \frac{1}{3a} - \frac{1}{2a} \right)$$

$$= 36 \hbar k \left(\frac{2}{10a} + \frac{3}{10a} - \frac{5}{10a} \right) = 36 \hbar k \left(\frac{2+3-5}{10a} \right) = 36 \hbar k \left(\frac{0}{10a} \right)$$

$$= 36 \hbar k \left(\frac{1}{5a} + \frac{1}{3a} - \frac{1}{2a} \right) = 36 \hbar k \left(-\frac{3}{10a} + \frac{1}{3a} \right) = 36 \hbar k \left(\frac{1}{30a} \right)$$

$$= \frac{18 \hbar k}{15a}$$

Ex 6.3

uncertainty position:

$$\text{uncertainty } \langle x \rangle = \Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

$$\langle x \rangle^2 = \left(\frac{39}{5}\right)^2 = \frac{1521}{25}$$

$$\langle x^2 \rangle = \int_0^a x^2 \left(\frac{6x^4}{a^3} - \frac{6x^3}{a^2} \right)^2 dx$$

$$= \int_0^a x^2 \left(\frac{36x^8}{a^6} + \frac{36x^6}{a^4} - \frac{36 \times 2 x^7}{a^5} \right) dx$$

$$= 36 \int_0^a \left(\frac{x^8}{a^4} - \frac{2x^9}{a^5} + \frac{x^{10}}{a^6} \right) dx$$

$$= 36 \left[\frac{x^9}{9a^4} - \frac{2x^{10}}{10a^5} + \frac{x^{11}}{11a^6} \right]_0^a = 36 \left(\frac{a^5}{9} - \frac{2a^5}{10} + \frac{a^5}{11} \right)$$

$$= 36 \left(\frac{20a^5}{495} - \frac{a^5}{5} \right) = 36 \left(\frac{a^5}{495} \right)$$

$$\Delta x = \sqrt{\frac{36}{495} a^5 - \frac{1521}{25}}$$

uncertainty: impulse:

$$\Delta p = \sqrt{\langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2}$$

$$\langle \hat{p} \rangle^2 = \left(\frac{18}{150} \hbar h \right)^2$$

$$\langle \hat{p}^2 \rangle = \int_0^a \psi(x) \hat{p}^2 (\psi(x)) dx$$

$$= \int_0^a \hbar^2 \psi'(x) \psi'(x) dx = \hbar^2 \int_0^a \psi'^2(x) dx$$

$$= \hbar^2 \int_0^a 36 \left(\frac{x^4}{a^6} + \frac{x^6}{a^4} - \frac{2x^3}{a^5} \right) \left(\frac{6x^2}{a^3} - \frac{6x}{a^2} \right) dx$$

$$= \hbar^2 \int_0^a 36 \left(\frac{6x^6}{a^9} - \frac{6x^5}{a^8} + \frac{6x^4}{a^7} - \frac{6x^3}{a^6} - \frac{12x^5}{a^8} + \frac{12x^4}{a^7} \right) dx$$

$$= \hbar^2 \int_0^a 36 \left(\frac{6x^6}{a^9} - \frac{18x^5}{a^8} + \frac{18x^4}{a^7} - \frac{6x^3}{a^6} \right) dx$$

$$= \hbar^2 36 \left[\frac{6x^7}{7a^9} - \frac{18x^6}{6a^8} + \frac{18x^5}{5a^7} - \frac{6x^4}{4a^6} \right]_0^a$$

$$= 36 \hbar^2 \left(\frac{6}{7a^2} - \frac{18}{6a^2} + \frac{18}{5a^2} - \frac{6}{4a^2} \right)$$

$$= 216 \hbar^2 \left(\frac{1}{7a^2} - \frac{3}{6a^2} + \frac{3}{5a^2} - \frac{1}{4a^2} \right)$$

$$= 216 \hbar^2 \left(\frac{-5}{14a^2} + \frac{7}{20a^2} \right) = 108 \hbar^2 \left(\frac{-5}{7a^2} + \frac{7}{10a^2} \right) = 108 \hbar^2 \left(\frac{-1}{70a^2} \right)$$

$$= -\frac{54}{35} \frac{\hbar^2}{a^2}$$

$$\Delta p = \sqrt{\left(-\frac{54}{35} \frac{\hbar^2}{a^2} \right) - \frac{324}{225} \frac{\hbar^2}{a^2}} = \frac{\hbar}{a} \sqrt{\left(-\frac{54}{35} \right) - \frac{324}{225}}$$

$$= \frac{\hbar}{a} \sqrt{-\frac{528}{175}}$$

$$= \frac{\hbar}{a} \times \frac{3\sqrt{406}}{35}$$

$$u(r) = \psi(r) r$$

$$\psi'(r) = \frac{\partial (r \psi)}{\partial r}$$

$$\psi''(r) = \frac{\partial^2 (r \psi)}{\partial r^2}$$

$$\nabla^2 = \frac{\psi''(r)}{r} \Rightarrow \psi''(r) r = \frac{u''(r)}{r}$$

$$-\frac{\hbar^2}{2m} \nabla^2 \psi + V(r) \psi = E \psi$$

~~$$u''(r) = \nabla^2 r^2$$~~

$$-\frac{\hbar^2}{2m} \psi''(r) r + V(r) \psi = E \psi$$

on remplace tout !!

$$-\frac{\hbar^2}{2m} \frac{u''(r)}{r} + V(r) \frac{u(r)}{r} = E \frac{u(r)}{r}$$

on simplifie par r
et c'est bon

Exercise 6 3 impulsion

$$\langle p^2 \rangle = -i\hbar \int \psi^2 \frac{d\psi^2}{dx} dx$$

$$\psi^2 = \frac{36}{a^2} x^2 (a^2 - 2ax + x^2)$$

$$\frac{d\psi^2}{dx} = \frac{2 \times 36 a^2}{a^2} x - \frac{36 \times 2}{a^2} x^2 + \frac{36 \times 2}{a^2} x^3$$

$$\langle p^2 \rangle = (-i\hbar) \int \frac{36}{a^2} x^2 (a^2 - 2ax + x^2) \times \frac{72a^2 x - 216ax^2 + 144x^3}{a^2} dx$$

$$= (-i\hbar) \frac{36}{a^4} \int (a^2 x^2 - 2ax^3 + x^4) (72a^2 x - 216ax^2 + 144x^3) dx$$

$$= (-i\hbar) \frac{36}{a^4} \int (72a^4 x^3 - 216a^3 x^4 + 144a^2 x^5 - 144a^3 x^4 + 432a^2 x^5 - 288ax^6 + 72a^2 x^5 - 216ax^6 + 144x^7) dx$$

$$= (-i\hbar) \frac{36}{a^4} \left(\frac{72a^4 a^4}{4} - \frac{216a^3 a^5}{5} + \frac{144a^2 a^6}{6} + \frac{144a^3 a^5}{5} + \frac{432a^2 a^6}{6} - \frac{288aa^7}{7} + \frac{72a^2 a^6}{6} - \frac{216aa^7}{7} + \frac{144a^8}{8} \right)$$

$$= (-i\hbar) \frac{36}{a^4} \left(\frac{72a^8}{4} - \frac{216a^8}{5} + \frac{144a^8}{6} + \frac{144a^8}{5} + \frac{432a^8}{6} - \frac{288a^8}{7} + \frac{72a^8}{6} - \frac{216a^8}{7} + \frac{144a^8}{8} \right)$$

$$= (-i\hbar) \frac{36}{a^4} \left(\frac{208a^8}{5} \right)$$

$$= (-i\hbar) \frac{10368a^4}{5}$$

$$\Rightarrow \Delta = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$$

$$= \sqrt{(-i\hbar)^2 \frac{10368a^4}{5} - 0} = +i\hbar a^2 \sqrt{\frac{10368}{5}}$$