

x 7: Soit:
$$\Psi(x) = \begin{cases} N x e^{-\alpha x/2} & \text{si } 0 \leq x \leq \infty \\ 0 & \text{ailleurs} \end{cases}$$

avec $\alpha \in \mathbb{R}^+$ et $N = \sqrt{\alpha^3/2}$

1). Probabilité de trouver la particule dans l'espace dx :

$$dP(x) = |\Psi(x)|^2 dx$$

Probabilité de trouver la particule entre x et $x+dx$:

$$P(x) = \int_x^{x+dx} dP(x) (= 1)$$

$$\begin{aligned} \text{Soit } dP(x) &= |\Psi(x)|^2 \\ &= \Psi^*(x) \wedge \Psi(x) \\ &= N^2 x^2 e^{-\alpha x} \\ &= \frac{\alpha^3}{2} (x^2 e^{-\alpha x}) \end{aligned}$$

ici $\Psi^*(x) = \Psi(x)$
donc $\Psi^*(x) \wedge \Psi(x)$
 $= \Psi^2(x)$

$$\begin{aligned} \bullet \langle x \rangle &= \int_0^\infty x |\Psi(x)|^2 dx \\ &= \frac{\alpha^3}{2} \int_0^\infty x^3 e^{-\alpha x} dx \\ &= \frac{\alpha^3}{2} \times \frac{3!}{\alpha^4} \\ &= \frac{3}{\alpha} \end{aligned}$$

$$\begin{aligned} \bullet \langle x^2 \rangle &= \int_0^\infty x^2 |\Psi(x)|^2 dx \\ &= \frac{\alpha^3}{2} \times x^4 e^{-\alpha x} dx \\ &= \frac{\alpha^3}{2} \times \frac{4!}{\alpha^5} \\ &= \frac{12}{\alpha^2} \end{aligned}$$

car $\int x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}} \rightarrow \text{énoncé}$

NB: Les formules sont dans le cours: 6. Exemples: $\langle x \rangle$, $\langle P \rangle$, $\langle E \rangle$...

$$\langle p \rangle = i\hbar \int_0^\infty \left(\psi^* + \frac{d\psi}{dx} \right) dx$$

$$= i\hbar \int_0^\infty \left(\psi^* \times \frac{d}{dx} (N\alpha e^{-\frac{\alpha x}{2}}) \right) dx$$

$$= i\hbar \int_0^\infty \left(\psi^* \times \left[N \times \left(e^{-\frac{\alpha x}{2}} - \frac{\alpha x}{2} e^{-\frac{\alpha x}{2}} \right) \right] \right) dx$$

$$= i\hbar \int_0^\infty \left(\psi^* \times \left[N \times e^{-\frac{\alpha x}{2}} \left(1 - \frac{\alpha x}{2} \right) \right] \right) dx$$

$$= i\hbar \int_0^\infty \left(N^2 \times \alpha e^{-\alpha x} \left(1 - \frac{\alpha x}{2} \right) \right) dx$$

$$= i\hbar N^2 \int_0^\infty \left(\alpha e^{-\alpha x} - \frac{\alpha}{2} \alpha x^2 e^{-\alpha x} \right) dx$$

$$= i\hbar N^2 \left[\int_0^\infty \alpha e^{-\alpha x} dx - \frac{\alpha}{2} \int_0^\infty \alpha x^2 e^{-\alpha x} dx \right]$$

$$= i\hbar N^2 \times \left(\frac{1}{\alpha^2} - \frac{2\alpha}{2\alpha^3} \right)$$

$$= i\hbar N^2 \times 0$$

$$= 0$$

ici $\phi^* = \phi$ car ϕ est
réel donc il n'a pas
de conjugué

REPONSE

?

$$\langle p \rangle = \int_{-\infty}^{+\infty} \phi^*(x,t) \hat{p} \psi(x,t) dx$$

$$\langle p^2 \rangle = \int_{-\infty}^{+\infty} \psi^*(x,t) \hat{p}^2 \psi(x,t) dx$$

← voir cours
(partie 6. exemple)

$$\hat{p}^2 = -\hbar^2 \frac{\partial^2}{\partial x^2}$$

$$\hat{p} = -i\hbar \frac{\partial}{\partial x}$$

$$\langle \hat{p}^2 \rangle = \int_0^{+\infty} \Psi^*(x,t) \hat{p}^2 \Psi(x,t) dx$$

$$\hat{p}^2 = -\hbar^2 \frac{\partial^2}{\partial x^2}$$

$$= \int_0^{+\infty} \left[N \alpha e^{-\frac{\alpha x}{2}} (-\hbar^2) \frac{d^2}{dx^2} (N x e^{-\frac{\alpha x}{2}}) \right] dx$$

$$g = \frac{h^2}{m^3} = \frac{m}{V}$$

$$n = \frac{m}{n}$$

$$m = g \times V$$

$$= \int_0^{+\infty} \left[\Psi^* (-\hbar^2) N \times \frac{d}{dx} (e^{-\frac{\alpha x}{2}} (1 - \frac{\alpha x}{2})) \right] dx$$

$$= \int_0^{+\infty} \left[\Psi^* (-\hbar^2) N \times \left[-\frac{d}{dx} \left(e^{-\frac{\alpha x}{2}} - \frac{\alpha x}{2} e^{-\frac{\alpha x}{2}} \right) \right] \right] dx$$

$$= \int_0^{+\infty} \left[\Psi^* (-\hbar^2) N \times \left[-\frac{\alpha}{2} e^{-\frac{\alpha x}{2}} - \frac{\alpha}{2} \left(e^{-\frac{\alpha x}{2}} - \frac{\alpha x}{2} e^{-\frac{\alpha x}{2}} \right) \right] \right] dx$$

$$= \int_0^{+\infty} \left[\Psi^* (-\hbar^2) N \times \left[-\alpha e^{-\frac{\alpha x}{2}} + \frac{\alpha^2}{4} x e^{-\frac{\alpha x}{2}} \right] \right] dx$$

$$= \int_0^{+\infty} \left[\Psi^* (-\hbar^2) N \times \left[e^{-\frac{\alpha x}{2}} (-\alpha + \frac{\alpha^2}{4} x) \right] \right] dx$$

$$= (-\hbar^2) N^2 \int_0^{+\infty} \left[(-\alpha) x e^{-\frac{\alpha x}{2}} + \frac{\alpha^2}{4} x^2 e^{-\frac{\alpha x}{2}} \right] dx$$

$$= (-\hbar^2) N^2 \left[\left(-\frac{\alpha}{2} \right) \int_0^{+\infty} x e^{-\frac{\alpha x}{2}} dx + \frac{\alpha^2}{4} \int_0^{+\infty} x^2 e^{-\frac{\alpha x}{2}} dx \right]$$

$$= (-\hbar^2) N^2 \left(-\alpha \left(\frac{1!}{\alpha^2} \right) + \frac{\alpha^2}{4} \left(\frac{2!}{\alpha^3} \right) \right)$$

$$= (-\hbar^2) N^2 \left(-\frac{1}{\alpha} + \frac{1}{2\alpha} \right)$$

$$= (-\hbar^2) N^2 \left(-\frac{1}{2\alpha} \right)$$

$$= (-\hbar^2) \frac{\alpha^3}{2} \times \left(-\frac{1}{2\alpha} \right)$$

$$= \frac{\hbar^2 \alpha^2}{4}$$

$$\begin{aligned}
 &= i\hbar \left(\frac{1}{2} \right) \int_{-\infty}^{\infty} x e^{-\alpha x^2} \left(1 - \frac{\alpha}{2} x^2 \right) dx \\
 &= i\hbar N \int_{-\infty}^{\infty} x e^{-\alpha x^2} \left(1 - \frac{\alpha}{2} x^2 \right) dx \\
 &= i\hbar N \left(\int_{-\infty}^{\infty} x e^{-\alpha x^2} dx - \frac{\alpha}{2} \int_{-\infty}^{\infty} x^3 e^{-\alpha x^2} dx \right) \\
 &= i\hbar N \left(\int_{-\infty}^{\infty} x e^{-\alpha x^2} dx - \frac{\alpha}{2} \int_{-\infty}^{\infty} x^3 e^{-\alpha x^2} dx \right) \\
 &= i\hbar N \left(\frac{1}{\alpha^2} - \frac{\alpha}{2} \cdot \frac{2}{\alpha^3} \right)
 \end{aligned}$$

$$= \frac{\hbar}{\alpha^2}$$

3)

Incertitude

$$\begin{aligned}
 \Delta x &= \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \\
 &= \sqrt{\frac{12}{\alpha^2} - \frac{9}{\alpha^2}} \\
 &= \sqrt{\frac{3}{\alpha^2}} \\
 &= \frac{\sqrt{3}}{\alpha}
 \end{aligned}$$

$$\begin{aligned}
 \Delta p &= \sqrt{\langle p^2 \rangle - \langle p \rangle^2} \\
 &= \sqrt{\frac{\hbar^2 \alpha^2}{4} - 0} \\
 &= \frac{\hbar \alpha}{2}
 \end{aligned}$$

$$\Delta x \Delta p = \frac{\sqrt{3}}{2} \hbar \gg \frac{1}{2} \hbar \text{ donc principe de Heisenberg respecté!}$$