Exercice n°8

1) $E = \frac{-\hbar^2\alpha^2}{2m}$ $\quad \phi = Nxe^{-\alpha x}$

Equation de Schrödinger : $-\frac{\hbar^2}{2m} \times \frac{\partial^2}{\partial x^2} \Psi(x,t) + V(x,t) \times \Psi(x,t) = i\hbar \frac{\partial}{\partial t} \Psi(x,t)$

Comme nous avons une energie définie E, alors l'equation de Schrödinger se simplifie par est donne :

$E\phi(x) = \frac{-\hbar^2\alpha^2}{2m} Nxe^{-\alpha x} = -\frac{\hbar^2}{2m} \times \frac{\partial^2}{\partial x^2} Nxe^{-\alpha x} + V(x) \times Nxe^{-\alpha x}$

On isole $V(x)$:

$$\Rightarrow V(x) = \frac{-\frac{\hbar^2\alpha^2}{2m} Nxe^{-\alpha x} + \frac{\hbar^2}{2m} \times \frac{\partial^2}{\partial x^2} Nxe^{-\alpha x}}{Nxe^{-\alpha x}}$$

Dérivons $Nxe^{-\alpha x}$ 2 fois : $\underbrace{Nx}_{u}\underbrace{e^{-\alpha x}}_{v}$

De la forme $(uv)' = u'v + uv'$

$u = Nx \quad v = e^{-\alpha x}$

$u' = N \quad v' = -\alpha e^{-\alpha x}$

$\Rightarrow Ne^{-\alpha x} + Nx(-\alpha e^{-\alpha x})$

$\phi'(x) = \underbrace{Ne^{-\alpha x}}_{u} \underbrace{(1-\alpha x)}_{v}$

$u = Ne^{-\alpha x} \quad v = 1 - \alpha x$

$u' = -\alpha Ne^{-\alpha x} \quad v' = -\alpha$

$\Rightarrow (-\alpha Ne^{-\alpha x})(1-\alpha x) + (Ne^{-\alpha x}) \times (-\alpha)$

$= -\alpha Ne^{-\alpha x} + \alpha^2 x Ne^{-\alpha x} - \alpha Ne^{-\alpha x}$

$= -2\alpha Ne^{-\alpha x} + \alpha^2 x Ne^{-\alpha x}$

$\phi''(x) = \alpha Ne^{-\alpha x}(\alpha x - 2)$

On revient à $V(x)$:

$$V(x) = \frac{-\frac{\hbar^2\alpha^2}{2m} Nxe^{-\alpha x} + \frac{\hbar^2}{2m} \times \alpha Ne^{-\alpha x}(\alpha x - 2)}{Nxe^{-\alpha x}}$$

$$= \frac{-\hbar^2\alpha^2 Nxe^{-\alpha x} + \hbar^2 \alpha Ne^{-\alpha x}(\alpha x - 2)}{2mNxe^{-\alpha x}}$$

$$= \frac{-\hbar^2\alpha^2 x + \hbar^2\alpha(\alpha x - 2)}{2mx}$$

$$= \frac{-\hbar^2\alpha^2 x + \hbar^2\alpha^2 x - 2\hbar^2\alpha}{2mx}$$

$$= \frac{-2\hbar^2\alpha}{2mx} = \boxed{-\frac{\hbar^2\alpha}{mx}}$$

2) Probabilité de trouver la particule entre x et $x+dx$:

$$dP(x) = |\phi(x)|^2 dx$$

$$= |N x e^{-\alpha x}|^2 dx$$

$$= \underbrace{N^2 x^2}_{u} \underbrace{e^{-2\alpha x}}_{v} dx \qquad u = N^2 x^2 \qquad v = e^{-2\alpha x}$$

$$u' = 2N^2 x \qquad v' = -2\alpha e^{-2\alpha x}$$

$$= 2N^2 x e^{-2\alpha x} + N^2 x^2 \times (-2\alpha e^{-2\alpha x})$$

$$= N^2 (2x e^{-2\alpha x} + x^2 \times (-2\alpha e^{-2\alpha x}))$$

$$= N^2 (2x e^{-2\alpha x} - 2x^2 \alpha e^{-2\alpha x})$$

D'où : $N^2 (2x e^{-2\alpha x} - 2x^2 \alpha e^{-2\alpha x}) = 0$

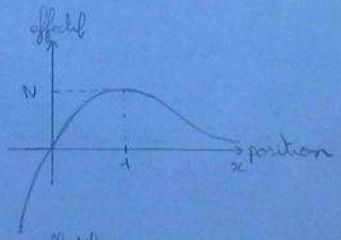
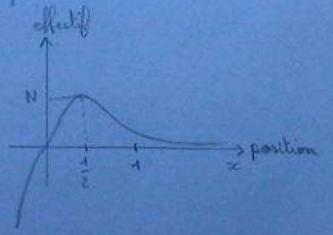
~~$2x e^{-2\alpha x} - 2x^2 \alpha e^{-2\alpha x} = 0$~~

$$2N^2 e^{-2\alpha x} (x - \alpha x^2) = 0$$

$$\Rightarrow \begin{cases} 2N^2 e^{-2\alpha x} = 0 \\ x - \alpha x^2 = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ 1 - \alpha x = 0 \end{cases} \Rightarrow \begin{cases} x = 0 \\ x = \frac{1}{\alpha} \end{cases}$$

Donc $x_{max} = \frac{1}{\alpha}$

3) Pour $\alpha = 1$: $x_{max} = \frac{1}{1} = 1$



Pour $\alpha = 2$: $x_{max} = \frac{1}{2}$