TAI n°8

Equation de Schrödinger :

$$-\frac{\hbar^2}{2m}\psi''(x) + V(x)\psi(x) = E\cdot\psi(x)$$

D'après l'énoncé : $\psi(x) = Nxe^{-\alpha x}$

$$E = -\frac{\hbar^2\alpha^2}{2m}$$

$$\hbar = \frac{h}{mv} \Rightarrow h = p\lambda$$

$$V(x) = \frac{E\cdot\psi(x) - \frac{\hbar^2}{2m}\psi''(x)}{\psi(x)}$$

$$V(x) = \frac{ENxe^{-\alpha x} - \frac{\hbar^2}{2m}\left(-\alpha Ne^{-\alpha x}(2-x\alpha)\right)}{Nxe^{-\alpha x}}$$

$$V(x) = \frac{Ne^{-\alpha x}\left(Ex - \frac{\hbar^2}{2m}(\alpha(2-x\alpha)\right)}{Nxe^{-\alpha x}}$$

$$V(x) = \frac{Ex - \frac{\hbar^2}{2m}(\alpha(2-x\alpha))}{x}$$

$$V(x) = \frac{-\frac{\hbar^2\alpha^2}{2m}x - \frac{\hbar^2}{2m}(\alpha(2-x\alpha))}{x}$$

$$V(x) = \frac{\frac{1}{2m}\left(-\hbar^2\alpha^2 x - \hbar^2(\alpha(2-x\alpha)\right)}{x}$$

$$V(x) = \frac{-\hbar^2 x^2 \alpha - \hbar^2(\alpha(\ell - x\alpha))}{2mx}$$

$$V(x) = \frac{-\hbar^2 \alpha(\alpha x - 2 - \alpha x))}{2mx}$$

$$V(x) = \frac{2\hbar^2 \alpha}{2mx}$$

$$\boxed{V(x) = \frac{\hbar^2 \alpha}{mx}}$$ Donc la formule est vérifié sur $0 \leq x \leq \infty$

Vérifions maintenant ~~sur~~ pour $\psi(x) = 0$

D'après l'équation de Schrödinger :

si $\psi(x) = 0$ alors $V(x) <=> \frac{0}{0}$ " "

$V(x) <=> \infty$

2- $P(x \leq \text{particule} \leq x + d(x)) = \int_x^{x+dx} |\psi(x)|^2 dx$

$$= \int_x^{x+dx} N^2 x^2 e^{-2\alpha x} dx$$

Observons les variations de cette probabilité

$f(x) = N^2 x^2 e^{-2\alpha x}$

$f'(x) = 2x N^2 e^{-2\alpha x} - N^2 x^2 2\alpha e^{-2\alpha x}$

$f'(x) = 0 \Leftrightarrow \ldots$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0$ ou $x = 2$

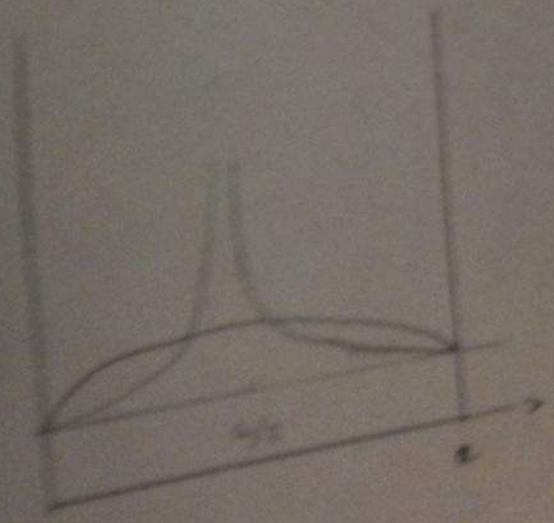
x	$-\infty$		0		2		$+\infty$
$f'(x)$		$+$	0	$-$	0	$+$	
$f(x)$		↗	$\frac{4}{e^2}$	↘	0	↗	

$f\left(\frac{1}{2}\right) = N\left(\frac{1}{2}\right)^2 e^{-\frac{1}{2}}$

Donc $x_{max} = \frac{N}{4}e^{-\frac{1}{2}}$

$f\left(\frac{1}{2}\right) = N\frac{1}{4}e^{-\frac{1}{2}}$

3.



x_0 correspond au premier état excité