

Mécanique Quantique

TAI

DD TAI ①

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Ex 3: 1) $\frac{mv^2}{2} = \frac{e^2}{4\pi\epsilon_0 r^2} \Leftrightarrow \frac{4\pi\epsilon_0}{e^2} = \frac{2}{2mv^2}$

$\Leftrightarrow \frac{4\pi\epsilon_0}{e^2} = \frac{1}{2mv^2}$. Or $r \gg a_0 = \frac{4\pi\epsilon_0 \hbar^2}{e^2 m_e}$ (et $\hbar = \frac{h}{2\pi}$)

$\Rightarrow a_0 = \frac{\epsilon_0 \hbar^2}{\pi e^2 m_e}$

On a $\frac{4\pi\epsilon_0}{e^2} = \frac{1}{2mv^2}$, ce qui donne $r \gg \frac{\hbar^2}{2mv^2 m_e}$

Soit, avec $m = m_e$, $r^2 m_e v^2 \gg \hbar^2 \Leftrightarrow r m v \gg \hbar$

Or $L = r m v \Rightarrow \boxed{L \gg \hbar}$

2) Incertitude minimum : On sait que $\Delta x \Delta p \gg \frac{\hbar}{2}$ or $\Delta x \approx a$

$\Rightarrow \Delta p \gg \frac{\hbar}{2a}$

De plus, $\epsilon_c = \frac{p^2}{2m_e}$

$\epsilon_c = \frac{\hbar^2}{8a^2 m_e}$

Impulsion : $p = m v \Rightarrow v = \frac{p}{m}$ or $p = \sqrt{2m\epsilon_c}$

$v = \frac{\sqrt{2 \times \frac{\hbar^2}{8a^2 m_e}}}{m_e} = \frac{\sqrt{\frac{\hbar^2}{4a^2}}}{m_e} \Rightarrow \boxed{v = \frac{\hbar}{2am}}$

men kg

$a_0 = 5,3 \times 10^{-11}$

$v = 1,1 \times 10^6 \text{ m.s}^{-1}$ ou $1,1 \times 10^3 \text{ m.s}^{-1}$

$\epsilon_c = \frac{mv^2}{2}$

← en eV

$e = 1,6 \times 10^{-19} \text{ C}$

$\epsilon_0 = 8,85 \times 10^{-12} \text{ F.m}^{-1}$

$\hbar = 1,05 \times 10^{-34} \text{ J.s}$

$1 \text{ eV} = 1,6 \times 10^{-19} \text{ J}$

$$3). E_n = \frac{-E_0}{n^2} \quad \text{et} \quad \Delta E = \frac{hc}{\lambda} = 12,74 \text{ eV}$$

$$\Delta E = E_n - E_0 = \frac{-E_0}{n^2} - E_0 = E_0 \left(1 - \frac{1}{n^2}\right)$$

$$\Leftrightarrow \frac{\Delta E}{E_0} = 1 - \frac{1}{n^2} \Leftrightarrow \frac{\Delta E}{E_0} - 1 = -\frac{1}{n^2}$$

$$\Leftrightarrow n^2 = \frac{-1}{\frac{\Delta E}{E_0} - 1} \approx 16$$

$$\Rightarrow \boxed{n = 4}$$

$$15 = 6,24 \times 10^{18} \text{ eV}$$

Ex 4: 1) Une fonction d'onde $\Psi(x,t)$ est solution de l'équation de Schrödinger si et seulement si:

$$\frac{-\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} = i\hbar \frac{\partial \Psi}{\partial t} = \hat{E} \Psi$$

• On a $\Psi_A(x,t) = A \cos(kx - \omega t)$

$$\begin{aligned} \frac{\hbar^2}{2m} \frac{\partial^2 \Psi_A}{\partial x^2} &= \frac{-A\hbar^2}{2m} \frac{\partial}{\partial x} (-k \sin(kx - \omega t)) \quad [1^{\text{e}} \text{ dérivation}] \\ &= \frac{A\hbar^2 k^2}{2m} \frac{\partial}{\partial x} \cos(kx - \omega t) \quad [2^{\text{e}} \text{ dérivation}] \end{aligned}$$

$$\text{Or } k = \frac{p}{\hbar} \rightarrow = \frac{Ap^2}{2m} \cos(kx - \omega t)$$

$$E_c = \frac{p^2}{2m} \rightarrow = A E_c \cos(kx - \omega t) \quad [1]$$

$$\begin{aligned} i\hbar \frac{\partial \Psi_A}{\partial t} &= i\hbar A \omega \sin(kx - \omega t) \stackrel{[2]}{=} i A E_c \sin(kx - \omega t) \\ E &= E_c + V = E_c \text{ (part. libre)} \\ \omega &= \frac{E}{\hbar} \end{aligned}$$

On a [1] $\neq$ [2] donc $\Psi_A(x,t)$ pas solution.

• On a $\Psi_b(x,t) = B \sin(kx - \omega t)$

$$\begin{aligned} \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi_b &= \frac{-B\hbar^2 k^2}{2m} \frac{\partial}{\partial x} \cos(kx - \omega t) \\ &= \frac{B\hbar^2 k^2}{2m} \frac{\partial}{\partial x} \sin(kx - \omega t) \\ &= \frac{Bp^2}{2m} \sin(kx - \omega t) \\ &= B E_c \sin(kx - \omega t) \quad [1] \end{aligned}$$

$$\begin{aligned} i\hbar \frac{\partial}{\partial t} \Psi_b &= -i\hbar B \omega \cos(kx - \omega t) \\ &= -iB E_c \cos(kx - \omega t) \end{aligned}$$

On a $[1] \neq [2]$ donc $\Psi_b(x,t)$ pas solution.

2) On a $\Psi(x,t) = A e^{\frac{i}{\hbar}(px - Et)}$ au lieu de
 $\Psi(x,t) = A e^{\frac{i}{\hbar}(px - Et)$ donc l'équation de
 Schrödinger doit avoir cette forme :

$$X \frac{\partial^2}{\partial x^2} \Psi = Y \frac{\partial}{\partial t} \Psi$$

$$\Leftrightarrow X A \frac{\partial^2}{\partial x^2} \left[\frac{-ip}{\hbar} e^{\frac{i}{\hbar}(px - Et)} \right] = Y A \left[\frac{iE}{\hbar} e^{\frac{i}{\hbar}(px - Et)} \right]$$

$$\Leftrightarrow -X A \frac{p^2}{\hbar^2} e^{\frac{i}{\hbar}(px - Et)} = Y A e^{\frac{i}{\hbar}(px - Et)} \cdot \frac{iE}{\hbar}$$

$$\Leftrightarrow -X \frac{p^2}{\hbar^2} = Y \frac{iE}{\hbar} \quad ; \text{ avec } E = E_c \text{ (part. libre).}$$

On pose $X = \frac{-\hbar^2}{2m}$ et $Y = i\hbar$:

$$\frac{p^2}{2m} = -E \Rightarrow \text{Il faut prendre } X = \frac{-\hbar^2}{2m} \text{ et } Y = -i\hbar$$

ainsi on retrouve bien l'équation de Schrödinger:

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} = i\hbar \frac{\partial \Psi}{\partial t}$$

$$\begin{aligned} 3) \Psi(x,t) &= A e^{i(kx - \omega t)} \Rightarrow \frac{\partial \Psi}{\partial x} = A i k e^{i(kx - \omega t)} \\ \frac{\partial^2 \Psi}{\partial x^2} &= A i^2 k^2 e^{i(kx - \omega t)} \\ &= -A k^2 e^{i(kx - \omega t)} \\ \Rightarrow \frac{\partial \Psi}{\partial t} &= -A i \omega e^{i(kx - \omega t)} \\ \frac{\partial^2 \Psi}{\partial t^2} &= A i^2 \omega^2 e^{i(kx - \omega t)} \\ &= -A \omega^2 e^{i(kx - \omega t)} \end{aligned}$$

On a donc l'équation de Klein-Gordon qui s'écrit:

$$\begin{aligned} \frac{\partial^2 \Psi}{\partial t^2} - c^2 \frac{\partial^2 \Psi}{\partial x^2} + \frac{m^2 c^4}{\hbar^2} \Psi &= 0 \quad \Leftrightarrow \\ -A \omega^2 e^{i(kx - \omega t)} + c^2 A k^2 e^{i(kx - \omega t)} + \frac{m^2 c^4}{\hbar^2} A e^{i(kx - \omega t)} &= 0 \\ \Leftrightarrow A e^{i(kx - \omega t)} \left[-\omega^2 + c^2 k^2 + \frac{m^2 c^4}{\hbar^2} \right] &= 0 \\ \Leftrightarrow A e^{i(kx - \omega t)} \left[\frac{-E^2}{\hbar^2} + c^2 \frac{p^2}{\hbar^2} + \frac{m^2 c^4}{\hbar^2} \right] &= 0 \\ \Leftrightarrow \frac{1}{\hbar^2} A e^{i(kx - \omega t)} (-E^2 + c^2 p^2 + m^2 c^4) &= 0 \\ \Leftrightarrow -E^2 + c^2 p^2 + m^2 c^4 = 0 \quad \Leftrightarrow E^2 = c^2 p^2 + m^2 c^4 \end{aligned}$$

$$\Rightarrow \boxed{E = cp + mc^2} \quad \text{et l'éq admet sol. de la forme} \\ \Psi = A e^{i(kx - \omega t)}$$

Ex 5: 1) $\Psi(x) = N x(a-x)$.

On veut $\int_0^a |\Psi(x)|^2 dx = 1 \Leftrightarrow \int_0^a |N x(a-x)|^2 dx = 1$

$\Leftrightarrow |N|^2 \int_0^a |x a - x^2|^2 dx = 1 \Leftrightarrow |N|^2 \int_0^a (x a - x^2)^2 dx = 1$

$\Leftrightarrow |N|^2 \int_0^a (x^2 a^2 - 2 a x^3 + x^4) dx = 1$

$\Leftrightarrow |N|^2 \left[\frac{x^3 a^2}{3} - \frac{2 a x^4}{4} + \frac{x^5}{5} \right]_0^a = 1$

$\Leftrightarrow |N|^2 \left(\frac{a^5}{3} - \frac{a^5}{2} + \frac{a^5}{5} \right) = 1$

$\Leftrightarrow |N|^2 \frac{a^5}{30} = 1 \Leftrightarrow |N|^2 = \frac{30}{a^5}$

$\Leftrightarrow |N| = \sqrt{\frac{30}{a^5}} \Rightarrow \boxed{N = \sqrt{\frac{30}{a^5}}}$

2) • Position moyenne:

$$\begin{aligned} \langle x \rangle &= \int_0^a x |\Psi|^2 dx = \int_0^a x |N x(a-x)|^2 dx \\ &= |N|^2 \int_0^a x^3 (a^2 - 2 a x + x^2) dx \\ &= |N|^2 \left[\frac{x^4 a^2}{4} - \frac{2 a x^5}{5} + \frac{x^6}{6} \right]_0^a \\ &= |N|^2 \frac{a^6}{60} = \frac{30}{a^5} \times \frac{a^6}{60} = \boxed{\frac{a}{2}} \end{aligned}$$

• Impulsion moyenne :

$$\langle p \rangle = \int_0^a \psi^* \left(\frac{-\hbar}{i} \frac{\partial \psi}{\partial x} \right) dx$$

$$= -i\hbar \int_0^a N x(a-x) N(a-x) dx$$

$$= -i\hbar \int_0^a (N^2 x(a-x)(a-x)) dx$$

$$= -i\hbar \int_0^a N^2 (ax^2 - 3x^2a + 2x^3) dx$$

$$= -N^2 i\hbar \left[\frac{ax^3}{3} - \frac{3a^2x^2}{2} + \frac{2x^4}{4} \right]_0^a$$

$$= -N^2 i\hbar \left(\frac{a^4}{3} - \frac{3a^3}{2} + \frac{a^4}{2} \right)$$

$$\boxed{\langle p \rangle = 0}$$

3) • Incertitude $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$

$$\text{Calcul de } \langle x^2 \rangle = \int_0^a \psi^* x^2 \psi dx$$

$$= N^2 \int_0^a x^2 (a-x)^2 x^2 dx$$

$$= N^2 \int_0^a x^4 (a-x)^2 dx$$

$$= N^2 \int_0^a x^4 (a^2 - 2ax + x^2) dx$$

$$= N^2 \int_0^a (a^2 x^4 - 2a x^5 + x^6) dx$$

$$\begin{aligned}
 &= N^2 \left[\frac{x^5 a^2}{5} - \frac{ax^6}{3} + \frac{x^7}{7} \right]_0^a \\
 &= N^2 \left(\frac{a^7}{5} - \frac{a^7}{3} + \frac{a^7}{7} \right) \\
 &= N^2 \times \frac{a^7}{105} = \frac{30}{a^5} \times \frac{a^7}{105}
 \end{aligned}$$

$$\boxed{\langle x^2 \rangle = \frac{2}{7} a^2}$$

$$\begin{aligned}
 \text{Donc } \Delta x &= \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{2a^2}{7} - \left(\frac{a}{2}\right)^2} \\
 &= \sqrt{\frac{2a^2}{7} - \frac{a^2}{4}} = \sqrt{\frac{a^2}{28}}
 \end{aligned}$$

$$\boxed{\Delta x = \frac{a}{2\sqrt{7}}}$$

Correct mais ne
répond pas à la question "en déduire"

↳ il faut utiliser :

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Rightarrow \Delta p_{\min} = \Delta x \times \frac{\hbar}{2}$$

• Incertitude sur l'impulsion : $\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$

$$\text{Or } \langle p \rangle = 0 \Rightarrow \Delta p = \sqrt{\langle p^2 \rangle}$$

$$\begin{aligned}
 \text{Calcul de } \langle p^2 \rangle &= \int_0^a \Psi^* p^2 \Psi \cdot dx \\
 &= \int_0^a N^2 (a-x) p^2 \Psi \cdot dx
 \end{aligned}$$

$$\text{Or } p^2 \Psi = \left(i\hbar \frac{\partial}{\partial x} \right)^2 \Psi = -\hbar^2 \frac{\partial^2}{\partial x^2} \Psi$$

$$\text{avec } \frac{\partial}{\partial x} \Psi = N(a-2x) \Rightarrow \frac{\partial^2}{\partial x^2} \Psi = -2N$$

$$\text{d'où } p^2 \Psi = \hbar^2 2N$$

$$\text{On a donc } \langle p^2 \rangle = \hbar^2 2N^2 \int_0^a x(a-x) \cdot dx$$

$$\begin{aligned}
 &= \hbar^2 N^2 \int_0^a (ax - x^2) \cdot dx \\
 &= \hbar^2 N^2 \left[\frac{ax^2}{2} - \frac{x^3}{3} \right]_0^a \\
 &= \hbar^2 N^2 \left(\frac{a^3}{6} \right) = \hbar^2 \times \frac{30}{a^5} \times \frac{a^3}{6}
 \end{aligned}$$

$$\boxed{\langle p^2 \rangle = \hbar^2 \frac{10}{a^2}}$$

$$\text{Donc } \Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \sqrt{\hbar^2 \frac{10}{a^2}}$$

$$\boxed{\Delta p = \frac{\hbar}{a} \sqrt{10}}$$

EX 6 : 1) Proba de trouver particule dans l'espace dx .
 $dP(x) = |\Psi|^2 \cdot dx$

Proba entre x et $x+dx$:

$$P(x) = \int_x^{x+dx} dP(x)$$

Soit $dP(x) = |\Phi|^2 = \Phi^* \cdot \Phi = \Phi^2$
 $= \left(\frac{N}{a^3} e^{-ax/2} \right)^2 = \frac{N^2}{a^3} e^{-ax}$ avec $N = \sqrt{\frac{a^3}{2}}$
 $= \frac{a^3}{2} e^{-ax}$

$$\bullet \langle x \rangle = \int_0^\infty x |\Phi|^2 \cdot dx = \int_0^\infty \frac{a^3}{2} x e^{-ax} \cdot dx$$

$$= \frac{a^3}{2} \frac{3!}{a^4} = \boxed{\frac{3}{a}}$$

NOTA ③

$$\bullet \langle x^2 \rangle = \int_0^{\infty} \phi^* x^2 \phi \cdot dx = \int_0^{\infty} x^2 |\phi|^2 \cdot dx$$

$$= \frac{\alpha^3}{2} \int_0^{\infty} x^4 e^{-\alpha x} \cdot dx = \frac{\alpha^3}{2} \frac{4!}{\alpha^5}$$

$$\boxed{\langle x^2 \rangle = \frac{12}{\alpha^2}}$$

2) $\bullet \langle p \rangle = \int_0^{\infty} \left(\phi^* i\hbar \frac{\partial \phi}{\partial x} \right) dx = i\hbar \int_0^{\infty} \frac{\partial}{\partial x} (N\alpha e^{-\alpha x/2}) \cdot dx$

$$= i\hbar \int_0^{\infty} \phi^* N \left(e^{-\frac{\alpha x}{2}} \cdot \frac{d\alpha}{2} e^{-\frac{\alpha x}{2}} \right) \cdot dx$$

$$= i\hbar \int_0^{\infty} \phi^* N e^{-\frac{\alpha x}{2}} \left(1 - \frac{\alpha x}{2} \right) \cdot dx$$

$$= i\hbar N^2 \int_0^{\infty} x e^{-\frac{\alpha x}{2}} e^{-\frac{\alpha x}{2}} \left(1 - \frac{\alpha x}{2} \right) \cdot dx$$

$$= i\hbar N^2 \int_0^{\infty} x e^{-\alpha x} \left(1 - \frac{\alpha x}{2} \right) \cdot dx$$

$$= i\hbar N^2 \int_0^{\infty} x e^{-\alpha x} - \frac{\alpha}{2} x^2 e^{-\alpha x} \cdot dx$$

$$= i\hbar N^2 \int_0^{\infty} x e^{-\alpha x} \cdot dx - \frac{\alpha}{2} \int_0^{\infty} x^2 e^{-\alpha x} \cdot dx$$

$$= i\hbar N^2 \left(\frac{1}{\alpha^2} - \frac{2\alpha}{2\alpha^3} \right) = i\hbar \left(\frac{\alpha^3}{2\alpha^2} - \frac{2\alpha^4}{4\alpha^3} \right)$$

$$= i\hbar \left(\frac{\alpha}{2} - \frac{\alpha}{2} \right)$$

$$\boxed{\langle p \rangle = 0}$$

$$\begin{aligned}
\langle p^2 \rangle &= \int_0^\infty \phi^* p^2 \phi \cdot dx = \int_0^\infty \phi^* (-\hbar^2) \frac{\partial}{\partial x} \left(N \alpha e^{-\frac{\alpha x}{2}} \right) \cdot dx \\
&= N(-\hbar^2) \int_0^\infty \phi^* \frac{\partial}{\partial x} \left(e^{-\frac{\alpha x}{2}} \left(1 - \frac{\alpha x}{2} \right) \right) \cdot dx \\
&= N(-\hbar^2) \int_0^\infty \phi^* \frac{\partial}{\partial x} \left(e^{-\frac{\alpha x}{2}} - \frac{\alpha x}{2} e^{-\frac{\alpha x}{2}} \right) \cdot dx \\
&= N(-\hbar^2) \int_0^\infty \phi^* \left(-\frac{\alpha}{2} e^{-\frac{\alpha x}{2}} - \frac{\alpha}{2} \left(e^{-\frac{\alpha x}{2}} - \frac{\alpha}{2} e^{-\frac{\alpha x}{2}} \right) \right) dx \\
&= N(-\hbar^2) \int_0^\infty \phi^* \left(-\frac{\alpha}{2} e^{-\frac{\alpha x}{2}} + \frac{\alpha^2 x}{4} e^{-\frac{\alpha x}{2}} \right) \cdot dx \\
&= N(-\hbar^2) \int_0^\infty \phi^* e^{-\frac{\alpha x}{2}} \left(-\frac{\alpha}{2} + \frac{\alpha^2 x}{4} \right) \cdot dx \\
&= N^2(-\hbar^2) \int_0^\infty \alpha e^{-\frac{\alpha x}{2}} \times e^{-\frac{\alpha x}{2}} \left(\frac{\alpha^2 x}{4} - \frac{\alpha}{2} \right) \cdot dx \\
&= N^2(-\hbar^2) \int_0^\infty \frac{\alpha^2 x^2}{4} e^{-\alpha x} - \alpha x e^{-\alpha x} \cdot dx \\
&= N^2(-\hbar^2) \left[\frac{\alpha^2}{4} \int_0^\infty x^2 e^{-\alpha x} \cdot dx - \alpha \int_0^\infty x e^{-\alpha x} \cdot dx \right]
\end{aligned}$$

$$\text{Or } \int_0^\infty x^n e^{-\alpha x} \cdot dx = \frac{n!}{\alpha^{n+1}}$$

$$\begin{aligned}
&= N^2(-\hbar^2) \left(\frac{\alpha^2}{4} \frac{2!}{\alpha^3} - \alpha \frac{1!}{\alpha^2} \right) \\
&= N^2(-\hbar^2) \left(\frac{-1}{2\alpha} \right) = -\hbar^2 \left(\frac{\alpha^3}{2} \right) \left(\frac{-1}{2\alpha} \right)
\end{aligned}$$

$$\boxed{\langle p^2 \rangle = \frac{\alpha^2}{4} \hbar^2}$$

3) . $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{12}{a^2} - \frac{9}{a^2}} = \sqrt{\frac{3}{a^2}} = \frac{\sqrt{3}}{a}$
 . $\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \sqrt{\frac{a^2 \hbar^2}{4} - 0} = \frac{\hbar a}{2}$
 $\Delta x \Delta p = \frac{\hbar \sqrt{3}}{2} > \frac{\hbar}{2} \Rightarrow$ principe de Heisenberg respecté.

Ex 7 : 1) $P(t) = \varphi^* \cdot \varphi = \left(\frac{1}{\sqrt{\tau}} e^{i\omega t} \right) \left(\frac{1}{\sqrt{\tau}} e^{-i\omega t} \right)$
 $= \frac{1}{\tau}$

On a donc $P(t) = \begin{cases} \frac{1}{\tau} & \text{si } -\tau/2 \leq t \leq \tau/2 \\ 0 & \text{ailleurs} \end{cases}$
 Sur $\Delta t = \tau$
 $\Rightarrow P(t)$ est prop à $\tau : \frac{\tau}{2} - \left(-\frac{\tau}{2}\right) = \frac{2\tau}{2} = \tau$

On a bien une proba uniforme d'un photon se manifeste dans l'intervalle de temps $\Delta t = \tau$.

2) $A(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\tau/2}^{\tau/2} \varphi(t) \cdot e^{i\omega t} \cdot dt$
 $= \frac{1}{\sqrt{2\pi}} \int_{-\tau/2}^{\tau/2} \frac{1}{\sqrt{\tau}} e^{i\omega t} \times e^{i\omega t} \cdot dt$
 $= \frac{1}{\sqrt{2\pi}} \int_{-\tau/2}^{\tau/2} e^{i(\omega - \omega_0)t} \cdot dt$
 $= \frac{1}{\sqrt{2\pi}} \left[\frac{e^{i(\omega - \omega_0)t}}{i(\omega - \omega_0)} \right]_{-\tau/2}^{\tau/2}$

$$= \frac{1}{\sqrt{8\pi\tau}} \left(\frac{e^{i(\omega-\omega_0)\tau/2}}{i(\omega-\omega_0)} - \frac{e^{i(\omega-\omega_0)\tau/2}}{i(\omega-\omega_0)} \right)$$

$$= \frac{1}{\sqrt{8\pi\tau}} \left(\frac{[\cos((\omega-\omega_0)\tau/2) + i\sin((\omega-\omega_0)\tau/2)] - [\cos((\omega-\omega_0)\tau/2) - i\sin((\omega-\omega_0)\tau/2)]}{i(\omega-\omega_0)} \right)$$

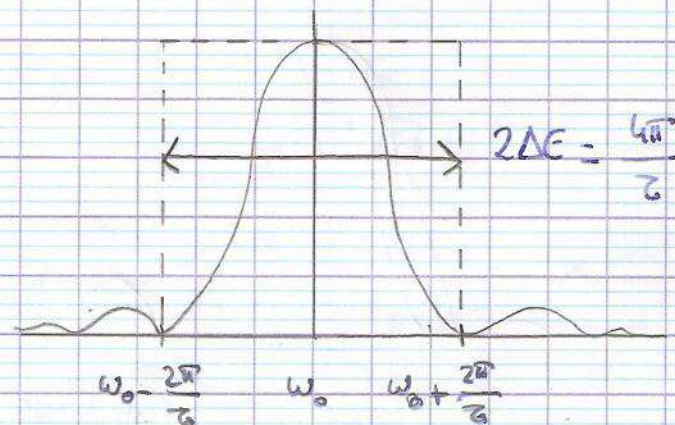
$$= \frac{1}{\sqrt{8\pi\tau}} \frac{2i\sin((\omega-\omega_0)\tau/2)}{i(\omega-\omega_0)}$$

$$= \frac{\tau}{\sqrt{8\pi\tau}} \frac{\sin((\omega-\omega_0)\tau/2)}{(\omega-\omega_0)\tau/2}$$

$$= \sqrt{\frac{\tau}{8\pi}} \frac{\sin((\omega-\omega_0)\tau/2)}{(\omega-\omega_0)\tau/2}$$

$$A(\omega) = \underbrace{\sqrt{\frac{\tau}{8\pi}}}_{C = \text{cte}} \text{sinc}((\omega-\omega_0)\tau/2)$$

3)



On trouve ces valeurs car en les réinjectant dans $A(\omega)$, on obtient $A(\omega) = 0$.

$\Rightarrow$ Quand $\tau \rightarrow \infty$, largeur de $|A(\omega)|^2 \rightarrow 0$.

Q. 7.1. ⑨

$\omega_0 - \frac{2\pi}{\delta}$	ω_0	$\omega_0 + \frac{2\pi}{\delta}$
$E = \hbar\omega_0 - \frac{\hbar}{\delta}$	$E = \hbar\omega_0$	$E = \hbar\omega_0 + \frac{\hbar}{\delta}$

$$\Rightarrow \Delta E = \frac{2\hbar}{\delta} \Rightarrow \Delta E \Delta t = \frac{2\hbar}{\delta} \cdot \delta = 2\hbar > \hbar$$

4) On a $\frac{hc}{\lambda} = h\nu = E_2 - E_1$.

$$\Delta\left(\frac{hc}{\lambda}\right) = \Delta(E_2 - E_1) = \Delta E$$

$$E_2 = \frac{hc}{\lambda}$$

$$E_1 = \frac{hc}{\lambda + \Delta\lambda}$$

$$-hc \frac{\Delta\lambda}{\lambda^2} = \frac{2\hbar}{\delta} \Rightarrow \boxed{\Delta\lambda = \frac{2\lambda^2}{c\delta}}$$

Ex 8 : 1)

$$E = \frac{\hbar^2 d^2}{2m}$$

$$\hat{H} = \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V$$

$$\hat{H}\phi = \frac{-\hbar^2}{2m} \frac{\partial^2 \phi}{\partial x^2} + V\phi \quad \text{et} \quad \hat{H}\phi = E\phi$$

$$\begin{aligned} \frac{\partial}{\partial x} \phi &= N \frac{\partial}{\partial x} \alpha e^{-\alpha x} = N(e^{-\alpha x} + \alpha(-1)e^{-\alpha x}) \\ &= N e^{-\alpha x} (1 - \alpha x) \end{aligned}$$

$$\frac{\partial^2}{\partial x^2} \phi = N(-\alpha e^{-\alpha x} (1 - \alpha x) - \alpha e^{-\alpha x}) = N\alpha e^{-\alpha x} (\alpha x - 2)$$

$$\frac{-\hbar^2}{2m} \left(N\alpha e^{-\alpha x} (\alpha x - 2) \right) + V \cdot N\alpha e^{-\alpha x} = E \phi$$

$$\Rightarrow V = \frac{\hbar^2}{m} N\alpha e^{-\alpha x} + V N\alpha e^{-\alpha x} = 0$$

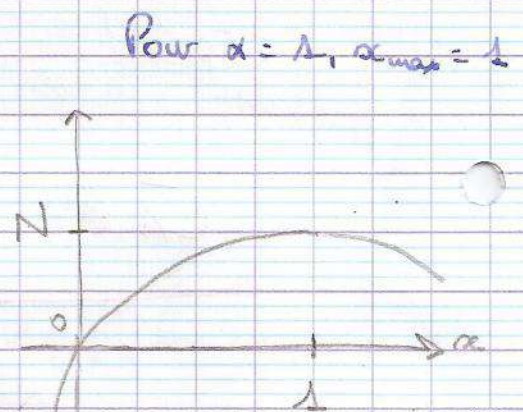
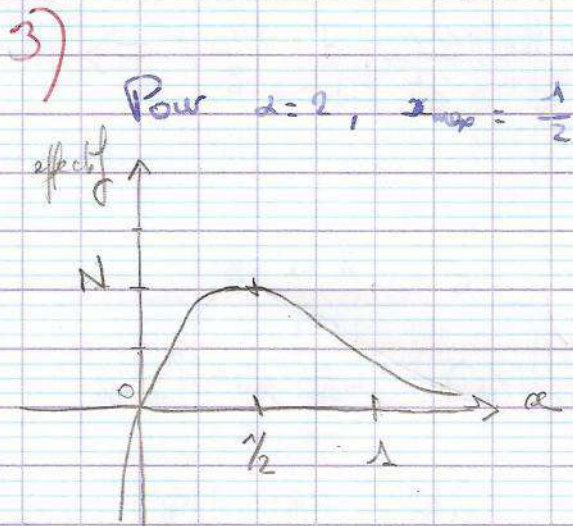
$$= N\alpha e^{-\alpha x} \left(\frac{\hbar^2}{m} \alpha + V\alpha \right) = 0$$

$$V(x) = \frac{-\hbar^2 \alpha}{m a}$$

$$2) \quad \frac{dP(x)}{dx} = |\phi|^2 \cdot dx = N^2 \alpha^2 e^{-2\alpha x} \cdot dx \\ = N^2 2 e^{-2\alpha x} (\alpha - \alpha x^2)$$

$$\frac{dP(x)}{dx} = 0 \Leftrightarrow \begin{cases} 2N^2 e^{-2\alpha x} = 0 \\ \alpha - \alpha x^2 = 0 \end{cases} \Leftrightarrow \begin{cases} \alpha = 0 \\ \alpha - \alpha x = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} \alpha = 0 \\ \alpha = \frac{1}{a} \end{cases} \Rightarrow \boxed{\alpha_{\max} = \frac{1}{a}}$$



Ex 9: 1) $\langle E \rangle = \int \Psi^* \hat{H} \Psi \cdot dx$

$$= \int (A\Psi_1^* + B\Psi_2^*) \hat{H} (A\Psi_1 + B\Psi_2) \cdot dx$$

Or $\hat{H}\Psi = E\Psi$ donc $\hat{H}(A\Psi_1 + B\Psi_2)$

$$= A\hat{H}\Psi_1 + B\hat{H}\Psi_2 = AE_1\Psi_1 + BE_2\Psi_2$$

$$\langle E \rangle = \int (A\Psi_1^* + B\Psi_2^*) (AE_1\Psi_1 + BE_2\Psi_2) \cdot dx$$

$$= \int \underbrace{A^2 E_1 \Psi_1^* \Psi_1}_{|\Psi_1|^2} + \underbrace{B^2 E_2 \Psi_2^* \Psi_2}_{|\Psi_2|^2} + ABE_2 \Psi_1^* \Psi_2 + BAE_1 \Psi_1 \Psi_2^* \cdot dx$$

$$= A^2 E_1 \underbrace{\int |\Psi_1|^2 dx}_1 + B^2 E_2 \underbrace{\int |\Psi_2|^2 dx}_1 + AB E_2 \underbrace{\int \Psi_1^* \Psi_2 dx}_0 + AB E_1 \underbrace{\int \Psi_2^* \Psi_1 dx}_0$$

car normées car normées car $\perp$ car $\perp$

$$= A^2 E_1 + B^2 E_2$$

Par identification : $\boxed{\langle E \rangle = \frac{1}{4} E_1 + \frac{3}{4} E_2}$

$$\Rightarrow \boxed{\Psi(x,t) = \frac{1}{2} \Psi_1 + \sqrt{\frac{3}{2}} \Psi_2} \quad A = \frac{1}{2} \text{ et } B = \sqrt{\frac{3}{2}}$$

2) $\Delta E = \sqrt{\langle E^2 \rangle - \langle E \rangle^2}$

Calcul de $\langle E^2 \rangle = \int (A \Psi_1^* + B \Psi_2^*) H^2 \Psi dx$

or $H^2 \Psi = \hat{H}(\hat{H} \Psi) = \hat{H}(A E_1 \Psi_1 + B E_2 \Psi_2) = A E_1 \underbrace{\hat{H} \Psi_1}_{E_1 \Psi_1} + B E_2 \underbrace{\hat{H} \Psi_2}_{E_2 \Psi_2}$
 $= A E_1^2 \Psi_1 + B E_2^2 \Psi_2$

$$\langle E^2 \rangle = \int (A \Psi_1^* + B \Psi_2^*) (A E_1^2 \Psi_1 + B E_2^2 \Psi_2) dx$$

$$\boxed{\langle E^2 \rangle = A^2 E_1^2 + B^2 E_2^2}$$

Donc $\Delta E = \sqrt{A^2 E_1^2 + B^2 E_2^2 - \left(\frac{1}{4} E_1 + \frac{3}{4} E_2 \right)^2}$

$$= \sqrt{\frac{1}{4} E_1^2 + \frac{3}{4} E_2^2 - \frac{1}{16} E_1^2 - \frac{3}{8} E_1 E_2 - \frac{9}{16} E_2^2}$$

$$= \frac{3}{16} E_1^2 + \frac{3}{16} E_2^2 - \frac{6}{16} E_1 E_2$$

$$= \sqrt{\frac{\sqrt{3}}{4}} (\epsilon_1^2 - \epsilon_2^2)^{1/2}$$

$$\Delta E = \frac{\sqrt{3}}{4} (\epsilon_1^2 - \epsilon_2^2)$$

$$\begin{aligned} 3) \quad \mathcal{P} &= |\Psi|^2 \cdot dx = \Psi^* \Psi \cdot dx \\ &= (A\Psi_1^* + B\Psi_2^*)(A\Psi_1 + B\Psi_2) \cdot dx \\ &= A^2 |\Psi_1|^2 + B^2 |\Psi_2|^2 + AB(\Psi_1^* \Psi_2 + \Psi_1 \Psi_2^*) \\ &= A^2 \phi_1^2 + B^2 \phi_2^2 + 2AB \phi_1 \phi_2 \cos((\epsilon_1 - \epsilon_2)t/\hbar) \end{aligned}$$

$$\text{Or } \omega = \frac{\epsilon_1 - \epsilon_2}{\hbar} = \frac{4}{\sqrt{3}} \frac{\Delta E}{\hbar}$$

$$\mathcal{P}(t) = A^2 \phi_1^2 + B^2 \phi_2^2 + 2AB \phi_1 \phi_2 \cos(\omega t)$$

$$\begin{aligned} &= e^{i\epsilon_1 t/\hbar} e^{-i\epsilon_2 t/\hbar} + e^{i\epsilon_2 t/\hbar} e^{-i\epsilon_1 t/\hbar} \\ &= e^{i(\epsilon_1 - \epsilon_2)t/\hbar} + e^{-i(\epsilon_1 - \epsilon_2)t/\hbar} \\ &= 2 \cos[(\epsilon_1 - \epsilon_2)t/\hbar] \end{aligned}$$

Donc $|\Psi|^2$ oscille dans le temps.

• Relation entre période et incertitude sur E :

Ex 10: 1) $E_0 = \frac{\hbar}{2} \omega$; $E_1 = \frac{3\hbar}{2} \omega$

$\omega = \frac{E}{\hbar}$; $\Psi(x, t) = \Psi(x, 0) e^{-iEt/\hbar}$

$\Psi_0(x, t) = \phi_0 e^{-iE_0 t/\hbar}$; $\Psi_1(x, t) = \phi_1 e^{-iE_1 t/\hbar}$

$\Rightarrow \Psi(x, t) = \frac{1}{\sqrt{2}} (\Psi_0 + \Psi_1)$

$\Psi(x, t) = \frac{1}{\sqrt{2}} \left(\phi_0 e^{-iE_0 t/\hbar} + \phi_1 e^{-iE_1 t/\hbar} \right)$

$\Psi(x, t)$ normé $\Leftrightarrow \int_{-\infty}^{\infty} |\Psi(x, t)|^2 dx = 1$

$$\begin{aligned} \int_{-\infty}^{\infty} |\Psi|^2 dx &= \frac{1}{2} \left(\underbrace{\Psi_0^* \Psi_0}_1 + \underbrace{\Psi_1^* \Psi_1}_1 + \underbrace{\Psi_1^* \Psi_0}_0 + \underbrace{\Psi_0^* \Psi_1}_0 \right) dx \\ &= \frac{1}{2} \times (1 + 1) = \frac{1}{2} \times 2 \end{aligned}$$

$\int_{-\infty}^{\infty} |\Psi|^2 dx = 1 \Rightarrow \underline{\Psi(x, t) \text{ normé}}$

2) $\langle x \rangle = \int_{-\infty}^{\infty} x |\Psi|^2 dx$; $\langle x \rangle_0 = \int \Psi^* x \Psi dx = 0$

$\langle x \rangle = \frac{1}{2} \int_{-\infty}^{\infty} x \left(\underbrace{|\Psi_1|^2}_0 + \underbrace{|\Psi_0|^2}_0 + \Psi_0^* \Psi_1 + \Psi_1^* \Psi_0 \right) dx$

$= \frac{1}{2} \int_{-\infty}^{\infty} \underbrace{\Psi_0^* \Psi_1}_0 + \underbrace{\Psi_1^* \Psi_0}_0 dx$

$= \frac{1}{2} \int_{-\infty}^{\infty} x \left(\phi_0^* \phi_1 e^{iE_0 t/\hbar} e^{-iE_1 t/\hbar} + \phi_1^* \phi_0 e^{iE_1 t/\hbar} e^{-iE_0 t/\hbar} \right) dx$

$$\text{Or } \cos \theta = (e^{i\theta} + e^{-i\theta})/2$$

$$\langle x \rangle = \frac{1}{2} \int_{-\infty}^{\infty} \phi_0 \phi_1 x 2 \cos((E_0 - E_1) \frac{t}{\hbar})$$

$$= \int_{-\infty}^{\infty} \phi_0 \phi_1 x \underbrace{\cos(\omega t)}_{\text{dépend pas de } x} \cdot dx = \cos(\omega t) \underbrace{\int_{-\infty}^{\infty} \phi_0 \phi_1 x dx}_A$$

$$\text{avec } \omega = \frac{E_0 - E_1}{\hbar}$$

$$\text{Donc } \boxed{\langle x \rangle = A \cos \omega t}$$

Signification physique : oscillateur harmonique

3) $F = -kx$; $x = l - l_0$
 $ma = -kx$
 $m\ddot{x} = -kx$
 $x = A \cos(\omega t)$
 $-m A \omega^2 \cos(\omega t) = -k A \cos(\omega t)$
 $\omega = \sqrt{\frac{k}{m}}$

$$m = \text{masse réduite} = \frac{m_1 m_2}{m_1 + m_2} ; m = \frac{n}{N}$$

$$\text{On a } 1 \text{ mole} = 6,02 \cdot 10^{23} \text{ N} \quad \begin{matrix} 14 \\ \downarrow \\ N_N \end{matrix} \Rightarrow m_N = \frac{14 \cdot 10^{-3}}{6,02 \cdot 10^{23}}$$

$$\quad \quad \quad \begin{matrix} 16 \\ \downarrow \\ N_O \end{matrix} \Rightarrow m_O = \frac{16 \cdot 10^{-3}}{6,02 \cdot 10^{23}}$$

$$\text{On a } k = \omega^2 m ; m = \frac{m_1 m_2}{m_1 + m_2} \text{ et } \omega = \frac{2\pi c}{\lambda}$$

$$E_1 - E_0 = \hbar \omega = \frac{\hbar c}{\lambda}$$

$$\textcircled{M}: k = \frac{4\pi^2 c^2}{\lambda^2} \cdot \frac{m_N m_O}{m_N + m_O} = \omega^2 m = 1,25 \times 10^{29} \times 1,24 \times 10^{-26} = 1550 \text{ N.m}^{-1}$$

Ex 11 : 1) Equation de Schrödinger:

$$\text{Milieu 1: } \frac{-\hbar^2}{2m} \frac{\partial^2 \phi}{\partial x^2} + \cancel{U_0 \phi} = E\phi ; E < U_0.$$

$$\text{Milieu 2: } \frac{-\hbar^2}{2m} \frac{\partial^2 \phi}{\partial x^2} = (E - U_0)\phi.$$

• Fonction d'onde:

$$\text{Milieu 1: } \phi_1 = \underbrace{A e^{ikx}}_{\text{progressive} \rightarrow} + \underbrace{B e^{-ikx}}_{\text{rétrograde} \leftarrow}$$

$$\text{Milieu 2: } \phi_2 = \underbrace{C e^{\beta x}}_{\leftarrow} + \underbrace{D e^{-\beta x}}_{\text{évanescence} \rightarrow}$$

• Vecteur d'onde:

$$\text{Milieu 1: } E_1 = \frac{\hbar^2 k^2}{2m} \Rightarrow k^2 = \frac{2m E_1}{\hbar^2}$$

$$k = \frac{\sqrt{2m(E - U_0)}}{\hbar}$$

$$\text{Milieu 2: } E_2 = \frac{-\hbar^2 \beta^2}{2m} \Rightarrow \beta^2 = \frac{-2m E_2}{\hbar^2}$$

$$\beta = \frac{\sqrt{2m(U_0 - E)}}{\hbar}$$

2) Conditions de continuité en $x=0$:

$$\begin{cases} \phi_1(x=0) = \phi_2(x=0) \\ \frac{d\phi_1}{dx}(x=0) = \frac{d\phi_2}{dx}(x=0) \end{cases}$$

$$\text{On veut } \begin{cases} A+B = C+D \\ ikA - ikB = \beta C - \beta D \end{cases}$$

$$C=B$$

$$\text{Soit } \begin{cases} A+B=D \\ ik(A-B)=-\beta D \end{cases} \Leftrightarrow \begin{cases} B=C=\frac{ik+\beta}{ik-\beta} \\ D=\frac{-ik(A-B)}{\beta} \end{cases}$$

• Coeff de réflexion R :

$$R = \left| \frac{B}{A} \right|^2 = \left| \frac{(ik+\beta)^2}{(ik-\beta)^2} \right| \quad \text{avec } k > 0 \text{ et } \beta > 0$$

$R = 1 \Rightarrow$ Tout est réfléchi = réflexion totale, comme en physique classique.

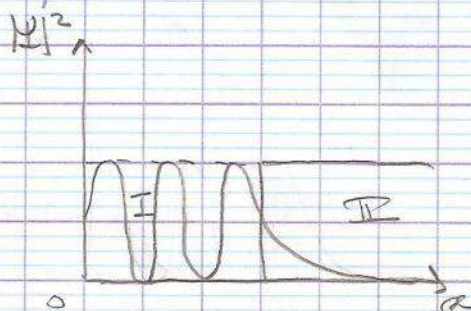
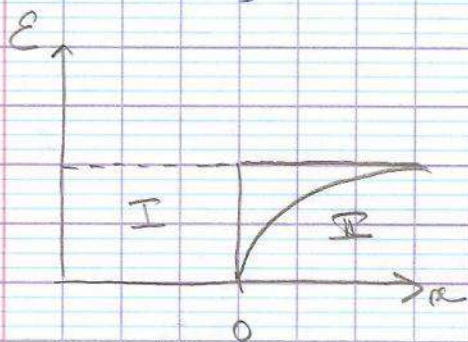
3) $\exists$ l'onde évanescente $e^{-\beta x}$, la particule a une proba $\neq 0$ d'être présente dans le milieu II, qui classiquement lui est interdit.

Cette proba $\rightarrow$ exponentiellement en x et devient négligeable à une épaisseur caractéristique

$$\frac{1}{\beta} = \frac{\hbar^2}{2m(V_0 - E)} \rightarrow 0 \quad \text{si } \hbar = 0$$

$$\rightarrow \infty \quad \text{si } E \rightarrow V_0$$

$\Rightarrow$ Comme en physique classique



$$\frac{\lambda}{2} = \frac{\hbar}{p}$$

EX 2: 1) Conservation E avant et après le choc:

$$E_i = E_f + E_e$$

$$E_i + \sqrt{\underbrace{p_i^2 c^2}_{\text{initial}} + m_e^2 c^4} = E_f + \sqrt{\underbrace{p_e^2 c^2}_{\text{final}} + m_e^2 c^4}$$

$$E = pc$$

$$p_i c + m_e c^2 = p_f c + c \sqrt{p_e^2 + m_e^2 c^2}$$

Sur Ox: $p_i = p_f \cos \theta + p_e \cos \phi$

Oy: $0 = p_f \sin \theta + p_e \sin \phi$ ou $p_e^2 = p_i^2 + p_f^2 - 2 p_i p_f \cos \theta$

2) $p_e^2 c^2 = p_i^2 c^2 + p_f^2 c^2 - 2 p_i p_f c^2 \cos \theta$
 $= h^2 v_i^2 + h^2 v_f^2 - 2 \frac{h v_i}{c} \times \frac{h v_f}{c} \cos \theta$

$$p = \frac{h\nu}{c}$$

$$p_e^2 c^2 = h^2 (v_i^2 + v_f^2) - 2 h^2 v_i v_f \cos \theta$$

3) $p_e = m_e v$ et $v_i \approx v_f$ et $E_i = E_f + E_e$.

L'éq préc devient: $m_e^2 v^2 c^2 = 2 h^2 v_i^2 - 2 h^2 v_i^2 \cos \theta$
 $= 2 h^2 v_i^2 (1 - \cos \theta)$

Or $1 - \cos \theta = 2 \sin^2 \left(\frac{\theta}{2} \right)$. On a donc

$$m_e v = \frac{2 h v_i \sin \frac{\theta}{2}}{c}$$

On reporte v dans l'équation $h\nu_i = h\nu_f + \frac{1}{2} m_e v^2$
 $E_i = E_f + E_e$

$$\Delta \nu = \nu_i - \nu_f = \frac{h \nu_i^2}{m_e c^2} (1 - \cos \theta)$$

4) $\theta = 90^\circ$.

$$\Delta v = \frac{h v_1^2}{m_e c^2} (1 - \cos 90^\circ) = \frac{6,62 \times 10^{-34} \times v_1^2}{9,1 \times 10^{-31} \times (3 \times 10^8)^2} (1 - \cos 90^\circ)$$

$$\Delta v = 8,08 \times 10^{-21} \cdot v_1^2$$

$$\frac{\Delta v}{v_1} = \frac{h v_1}{m_e c^2} (1 - 0) = \frac{h v_1}{m_e c^2} =$$

Valeur infinitésimale $\Rightarrow$ approximation 90° proc justifiée.

EX 1: 1) $E_c = h\nu - W$

$$\text{or } \nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{450 \times 10^{-9}} = 6,67 \cdot 10^{14} \text{ Hz}$$

$$h\nu = 6,62 \times 10^{-34} \times 6,67 \cdot 10^{14} = 4,41 \cdot 10^{-19} \text{ J} = 2,76 \text{ eV}$$

$\swarrow \times 6,24 \times 10^{18}$

Pour le magnésium: $E_c = 2,76 - W_{Mg} = 2,76 - 3,68$
 $= -0,92 \text{ eV}$

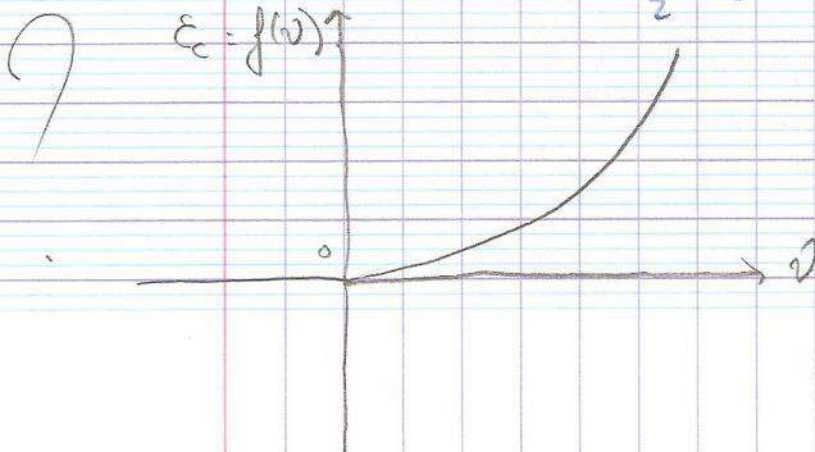
$\Rightarrow E_c$ insuffisante

potassium: $E_c = 2,76 - 2 = 0,76 \text{ eV}$

$\Rightarrow E_c$ suffisante.

Pour K: $E_c = \frac{1}{2} m_e v^2 \Leftrightarrow v = \sqrt{\frac{2 \cdot E_c}{m_e}} = \frac{2 \times 0,76 \times 1,6 \times 10^{-19}}{9,1 \times 10^{-31}}$

$$v = 5,17 \times 10^5 \text{ m.s}^{-1}$$



2) On voit que $\tan \theta = \frac{\Delta/2}{b} = \frac{\Delta}{2b} \Rightarrow \Delta = 2b \tan \theta$

On cherche Δ . Pour cela calculons v :

$$E_c = 200 \text{ eV} \quad v = \sqrt{\frac{2E_c}{m}} = \sqrt{\frac{2 \times 200 \times 1,6 \times 10^{-19}}{9,1 \times 10^{-31}}} = 8,39 \cdot 10^6 \text{ m.s}^{-1}$$

$$\text{Or } \lambda = \frac{h}{mv} = \frac{6,62 \times 10^{-34}}{9,1 \times 10^{-31} \times 8,39 \times 10^6} = 8,67 \cdot 10^{-11} \text{ m.}$$

$$\text{De plus } \sin \theta = \frac{\lambda}{D} = \frac{8,67 \times 10^{-11}}{2 \times 10^{-2}} = 4,34 \times 10^{-9}$$

$$\text{Soit } \theta = \arcsin \theta = (2,48 \times 10^{-7})^\circ$$

$$\text{On a donc } \Delta = 2b \tan \theta \\ = 2 \times 20 \times 10^{-2} \times 4,33 \times 10^{-9}$$

$$\boxed{\Delta = 1,73 \times 10^{-9} \text{ m.}}$$

J.J. Thomson n'observa aucun comportement ondulatoire du faisceau car le diamètre de la tache de diffraction Δ est trop petit pour être observé à l'œil nu.

3) Longueur d'onde de De Broglie :

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

$$v = \sqrt{\frac{2E_c}{m}} = \frac{2 \times 7 \times 1,6 \times 10^{-19}}{9,1 \times 10^{-31}} = 1,57 \times 10^6 \text{ m.s}^{-1}$$

$$\Rightarrow \lambda = \frac{6,62 \times 10^{-34}}{9,1 \times 10^{-31} \times 1,57 \times 10^6} = 4,63 \times 10^{-10} \text{ m.}$$

$$\boxed{\lambda = 4,63 \text{ \AA}}$$

λ de l'ordre de grandeur de l' $\text{\AA}$ donc de l'atome.
Donc il y aura diffraction.

Ex 12: 1) Moment cinétique $\vec{L} = r \vec{p}$ ($L = rp \sin \alpha$)

$$|\vec{L}| = mvr \sin \theta = 9,1 \cdot 10^{-31} \times 10 \times 10^{-3} \sin(90^\circ) \\ = 9,1 \times 10^{-33} \\ = \text{moment cinétique classique } e^-.$$

En quantique: $L = \sqrt{l(l+1)} \hbar = \sqrt{l(l+1)} \times 6,62 \times 10^{-34}$

$l = 10$: $L = \sqrt{10 \times 11} \times 6,62 \times 10^{-34} = 1,105 \times 10^{-33}$

$l = 85$: $L = \sqrt{85 \times 86} \times 6,62 \times 10^{-34} = 9,01 \times 10^{-33}$

$l = 86$: $L = \sqrt{86 \times 87} \times 6,62 \times 10^{-34} = 9,11 \times 10^{-33}$

$\downarrow$
le + près $l = 86$: pour l fixé, $(2l+1)$ valeurs possibles.
173 valeurs possibles pour l_y .

2) $E_R = \frac{L^2}{m a^2} = \frac{l(l+1) \hbar^2}{m a^2}$

1^{er} niveau: $l = 1$

$$E_R = \frac{1 \times 2 \times \hbar^2}{m a^2} = \frac{2 \times (6,62 \times 10^{-34})^2}{40^2 \times 1,7 \times 10^{-29} (7,5 \times 10^{-11})^2} \\ = 2,32 \cdot 10^{-23} \text{ J} = 1,45 \cdot 10^{-2} \text{ eV}$$

$$E_{1h} = \frac{1}{40} \text{ eV} = 2,5 \times 10^{-2} \text{ eV} > E_R$$

Suffisante pour induire transitions au 1^{er} niveau d'excitation.