

TAI EXO 10:

$$\text{à } t=0 \quad \psi(x,0) = \frac{1}{\sqrt{2}} (\Phi_0(x) + \Phi_1(x)) \quad \Phi_0(x) \in \mathbb{R} \quad \Phi_1(x) \in \mathbb{R} \quad E_n = (n + \frac{1}{2}) \hbar \omega$$

$$1) \quad \forall t \quad \psi(x,t) = \frac{1}{\sqrt{2}} \left(\Phi_0(x) e^{-i \frac{E_0}{\hbar} t} + \Phi_1(x) e^{-i \frac{E_1}{\hbar} t} \right)$$

$$\text{Or } E_n = (n + \frac{1}{2}) \hbar \omega \Rightarrow E_0 = \frac{1}{2} \hbar \omega \text{ et } E_1 = \frac{3}{2} \hbar \omega$$

$$\text{Donc } \psi(x,t) = \frac{1}{\sqrt{2}} \left(\Phi_0(x) e^{-i \frac{\omega t}{2}} + \Phi_1(x) e^{-i \frac{3\omega t}{2}} \right)$$

→ Montrons que $\psi(x,t)$ est normée

$$\begin{aligned} \int_{-\infty}^{+\infty} \psi(x,t) \psi^*(x,t) dx &= \frac{1}{2} \int_{-\infty}^{+\infty} \left(\Phi_0(x) e^{-i \frac{\omega t}{2}} + \Phi_1(x) e^{-i \frac{3\omega t}{2}} \right) \left(\Phi_0(x) e^{i \frac{\omega t}{2}} + \Phi_1(x) e^{i \frac{3\omega t}{2}} \right) dx \\ &= \frac{1}{2} \int_{-\infty}^{+\infty} \Phi_0^2(x) + \Phi_0(x) \Phi_1(x) e^{i\omega t} + \Phi_0(x) \Phi_1(x) e^{-i\omega t} + \Phi_1^2(x) dx \\ &= \frac{1}{2} \int_{-\infty}^{+\infty} \Phi_0^2(x) dx + \frac{1}{2} \int_{-\infty}^{+\infty} \Phi_1^2(x) dx + \underbrace{\int_{-\infty}^{+\infty} \Phi_0(x) \Phi_1(x) (e^{i\omega t} + e^{-i\omega t}) dx}_{=0} \\ &= 1 \end{aligned}$$

Donc $\psi(x,t)$ est normée.

$$\begin{aligned} 2) \quad \langle x \rangle &= \int_{-\infty}^{+\infty} \psi^* x \psi dx \\ &= \frac{1}{2} \int_{-\infty}^{+\infty} \left(\Phi_0(x) e^{i \frac{\omega t}{2}} + \Phi_1(x) e^{i \frac{3\omega t}{2}} \right) \left(\Phi_0(x) x e^{-i \frac{\omega t}{2}} + \Phi_1(x) x e^{-i \frac{3\omega t}{2}} \right) dx \\ &= \frac{1}{2} \int_{-\infty}^{+\infty} \Phi_0^2(x) x + \Phi_0(x) \Phi_1(x) x e^{-i\omega t} + x \Phi_1(x) \Phi_0(x) e^{i\omega t} + \Phi_1^2(x) x dx \\ &= \frac{1}{2} \int_{-\infty}^{+\infty} \Phi_0^2(x) x dx + \frac{1}{2} \int_{-\infty}^{+\infty} \Phi_0(x) \Phi_1(x) x (e^{-i\omega t} + e^{i\omega t}) dx + \frac{1}{2} \int_{-\infty}^{+\infty} \Phi_1^2(x) x dx \\ &= \int_{-\infty}^{+\infty} \Phi_0(x) \Phi_1(x) x \cos(\omega t) dx \\ &= \cos(\omega t) \int_{-\infty}^{+\infty} \Phi_0(x) \Phi_1(x) x dx \\ &= A \cos(\omega t) \end{aligned}$$

$$E_n = (n + \frac{1}{2}) \hbar \omega \Rightarrow \omega = \frac{E_n}{\hbar} - n - \frac{1}{2}$$

On a bien $\langle x \rangle_{\Phi_n} = 0$

3) Molécule de NO : $M_N = 14 \text{ g.mol}^{-1}$ $M_O = 16 \text{ g.mol}^{-1}$

Longueurs d'ondes autorisées : $\lambda = \frac{hc}{E_r} = 2\pi c \sqrt{\frac{\mu}{k}}$

Soit μ la masse réduite : $\mu = \frac{M_N M_O}{M_N + M_O} = 7,47 \text{ g.mol}^{-1} \downarrow \frac{\mu}{\text{g.mol}^{-1}}$
 $= 1,24 \cdot 10^{-23} \text{ g}$

$\lambda = 5330 \text{ nm} \Rightarrow k = \frac{4\pi^2 c^2 \mu}{\lambda^2} = \frac{4\pi^2 (3 \cdot 10^8)^2 (1,24 \cdot 10^{-23})}{(5330 \cdot 10^{-9})^2}$

$= \underline{\underline{1,65 \cdot 10^6 \text{ N.m}^{-1}}}$