

exo 10 | Q2

$$1) \Psi(x, t) = \frac{1}{\sqrt{2}} \left(\Phi_0(x) e^{-\frac{iE_0 t}{\hbar}} + \Phi_1(x) e^{-\frac{iE_1 t}{\hbar}} \right)$$

Or $E_n = \left(n + \frac{1}{2}\right) \hbar \omega$

D'où $E_0 = \frac{1}{2} \hbar \omega$ et $E_1 = \frac{3}{2} \hbar \omega$

Ce qui donne : $\Psi(x, t) = \frac{1}{\sqrt{2}} \left(\Phi_0(x) e^{-\frac{i\omega t}{2}} + \Phi_1(x) e^{-\frac{3i\omega t}{2}} \right)$

Montrons que $\Psi(x, t)$ normée :

$$\int_{-\infty}^{+\infty} \Psi(x, t) \cdot \Psi^*(x, t) dx = \frac{1}{2} \int_{-\infty}^{+\infty} \left(\Phi_0(x) e^{-\frac{i\omega t}{2}} + \Phi_1(x) e^{-\frac{3i\omega t}{2}} \right) \left(\Phi_0(x) e^{\frac{i\omega t}{2}} + \Phi_1(x) e^{\frac{3i\omega t}{2}} \right) dx$$

$$= \frac{1}{2} \int_{-\infty}^{+\infty} \left(\Phi_0^2(x) + \Phi_0(x) \cdot \Phi_1(x) e^{i\omega t} + \Phi_0(x) \Phi_1(x) e^{-i\omega t} + \Phi_1^2(x) \right) dx$$

$$= \frac{1}{2} \underbrace{\int_{-\infty}^{+\infty} \Phi_0^2(x) dx}_1 + \frac{1}{2} \underbrace{\int_{-\infty}^{+\infty} \Phi_1^2(x) dx}_1 + \frac{1}{2} \int_{-\infty}^{+\infty} \Phi_0(x) \Phi_1(x) (e^{i\omega t} + e^{-i\omega t}) dx$$

Or Φ_0 et Φ_1 sont orthogonaux et normés

D'où $\begin{cases} \int \Phi_0 \cdot \Phi_1 = 0 \\ \int \Phi_0^2(x) dx = 1 \text{ de même pour } \Phi_1(x) \end{cases}$

$$\Rightarrow \int_{-\infty}^{+\infty} \Psi(x, t) \cdot \Psi^*(x, t) dx = \frac{1}{2} + \frac{1}{2} + 0$$

$\int_{-\infty}^{+\infty} \Psi(x, t) \cdot \Psi^*(x, t) dx = 1$

 ✓

$$2) \langle x \rangle = \int_{-\infty}^{+\infty} \Psi(x,t)^* \cdot x \cdot \Psi(x,t) dx$$

$$= \frac{1}{2} \int_{-\infty}^{+\infty} (\Phi_0^2(x) + \Phi_1^2(x) + \Phi_0(x) \Phi_1(x) (e^{i\omega t} + e^{-i\omega t})) x dx$$

$$= \frac{1}{2} \int_{-\infty}^{+\infty} \Phi_0^2(x) x dx + \frac{1}{2} \int_{-\infty}^{+\infty} \Phi_1^2(x) x dx + \int_{-\infty}^{+\infty} \Phi_0(x) \Phi_1(x) \cos(\omega t) x dx$$

$$= \frac{1}{2} \langle x \rangle_{\Phi_0(x)} + \frac{1}{2} \langle x \rangle_{\Phi_1(x)} + \int_{-\infty}^{+\infty} \Phi_0(x) \Phi_1(x) \cos(\omega t) x dx$$

D'après l'énoncé : $\langle x \rangle_{\Phi_n} = 0$

$$\text{D'où } \langle x \rangle = \int_{-\infty}^{+\infty} \cos(\omega t) \Phi_0(x) \Phi_1(x) x dx$$

$$\Leftrightarrow \boxed{\langle x \rangle = A \cos(\omega t)}$$

$$E_m = (n + \frac{1}{2}) \hbar \omega \Leftrightarrow \boxed{\omega = \frac{E_m}{(n + \frac{1}{2}) \hbar}}$$

3) Recherche de k $\omega = \sqrt{\frac{k}{\mu}}$

$$\lambda = \frac{c}{\nu} = \frac{2\pi c}{\omega} = 2\pi c \sqrt{\frac{\mu}{k}} \Rightarrow k = \frac{(2\pi c)^2 \mu}{\lambda^2}$$

$$\mu = \frac{m_N \cdot m_\alpha}{m_N + m_\alpha} = \frac{\frac{M_o}{N_A} \cdot \frac{M_N}{N_A}}{\frac{M_o + M_N}{N_A}} = \frac{M_o \cdot M_N}{M_o + M_N} \times \frac{1}{N_A}$$

$$m = nM = \frac{N}{N_A} M = \frac{M}{N_A} \text{ car } N=1$$

$$\Rightarrow \mu = \frac{14 \times 16}{14 + 16} \cdot \frac{1}{6,02 \times 10^{23}} = 1,24 \cdot 10^{-23} \text{ kg}$$

$$\text{D'où } k = \frac{(2\pi \cdot 3 \cdot 10^8)^2 \times 1,24 \cdot 10^{-23}}{(15330 \times 10^{-9})^2}$$

$$\boxed{k = 1,65 \cdot 10^6 \text{ N} \cdot \text{m}^{-1}}$$