

TAI EXO 4:

1) $\Psi_A(x,t) = A \cos(kx - \omega t)$ $\Psi_B(x,t) = B \sin(kx - \omega t)$

Equation de Schrödinger pour une particule libre : $-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} = i\hbar \frac{\partial \Psi(x,t)}{\partial t}$

• $\Psi_A(x,t) = A \cos(kx - \omega t)$

$$\frac{\partial \Psi_A(x,t)}{\partial t} = -\omega A \sin(kx - \omega t)$$

$$\frac{\partial \Psi_A(x,t)}{\partial x} = -Ak \sin(kx - \omega t) \Rightarrow \frac{\partial^2 \Psi_A(x,t)}{\partial x^2} = -Ak^2 \cos(kx - \omega t)$$

$$-\frac{\hbar^2}{2m} Ak^2 \cos(kx - \omega t) = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi_A(x,t)}{\partial x^2}$$

$$i\hbar \frac{\partial \Psi_A(x,t)}{\partial t} = i\hbar \omega A \sin(kx - \omega t)$$

Ψ_A n'est pas solution de l'éq de Schrödinger

• $\Psi_B(x,t) = B \sin(kx - \omega t)$

$$\frac{\partial \Psi_B(x,t)}{\partial t} = -B\omega \cos(kx - \omega t)$$

$$\frac{\partial \Psi_B(x,t)}{\partial x} = Bk \cos(kx - \omega t) \Rightarrow \frac{\partial^2 \Psi_B(x,t)}{\partial x^2} = -Bk^2 \sin(kx - \omega t)$$

Ψ_B n'est pas solution de l'éq de Schrödinger

2) $\Psi(x,t) = A e^{-\frac{i}{\hbar}(px - Et)}$

On cherche à déterminer X et Y tq : $X \frac{\partial^2 \Psi(x,t)}{\partial x^2} = Y \frac{\partial \Psi(x,t)}{\partial t}$ (1)

$$\frac{\partial \Psi(x,t)}{\partial t} = \frac{E}{\hbar} i A e^{-\frac{i}{\hbar}(px - Et)}$$

$$\frac{\partial \Psi(x,t)}{\partial x} = -\frac{i p}{\hbar} A e^{-\frac{i}{\hbar}(px - Et)} \Rightarrow \frac{\partial^2 \Psi(x,t)}{\partial x^2} = -\frac{i p}{\hbar} \left(-\frac{i p}{\hbar}\right) A e^{-\frac{i}{\hbar}(px - Et)} = -\frac{p^2}{\hbar^2} A e^{-\frac{i}{\hbar}(px - Et)}$$

$$(1) \Leftrightarrow X \left(-\frac{p^2}{\hbar^2} A e^{-\frac{i}{\hbar}(px - Et)}\right) = Y \cdot \frac{E}{\hbar} i A e^{-\frac{i}{\hbar}(px - Et)}$$

$$\Leftrightarrow -X \frac{p^2}{\hbar^2} = Y \cdot \frac{E}{\hbar} i$$

$$\Leftrightarrow -X \frac{p^2}{\hbar^2} = Y \cdot \frac{E_c}{\hbar} i$$

$E = E_c$ car particule libre.

$$-X \frac{p^2}{\hbar^2} = Y \cdot \frac{E_c}{\hbar} ; (*)$$

$$\frac{p^2}{2m} \frac{1}{\hbar^2} = i\hbar \frac{E_c}{\hbar}$$

$$\text{Or } p^2 = m^2 v^2 \text{ et } E_c = \frac{1}{2} m v^2 \Rightarrow E_c = \frac{p^2}{2m}$$

$$\text{si } X = -\frac{\hbar^2}{2m} \text{ et } Y = i\hbar \Rightarrow \frac{p^2}{2m} = -E_c \text{ FAUX}$$

$$\text{Donc il faut } \boxed{X = \frac{\hbar^2}{2m} \text{ et } Y = i\hbar}$$

$$-\frac{\hbar^2}{2m} \frac{p^2}{\hbar^2} = i\hbar \frac{E_c}{\hbar} ; \Leftrightarrow -\frac{p^2}{2m} = -E_c \Leftrightarrow \frac{p^2}{2m} = E_c \checkmark$$

$$\text{D'où } \boxed{\frac{\hbar^2}{2m} \frac{\partial^2 \psi(x,t)}{\partial x^2} = i\hbar \frac{\partial \psi(x,t)}{\partial t}}$$

$$3) \text{ Equat° de Klein-Gordon : } \frac{\partial^2 \psi}{\partial t^2} - c^2 \frac{\partial^2 \psi}{\partial x^2} + \frac{m^2 c^4}{\hbar^2} \psi = 0$$

$$\psi(x,t) = A e^{i(kx - \omega t)}$$

$$\frac{\partial \psi(x,t)}{\partial t} = -i\omega A e^{i(kx - \omega t)} \Rightarrow \frac{\partial^2 \psi(x,t)}{\partial t^2} = -i\omega(-i\omega) A e^{i(kx - \omega t)} = -\omega^2 A e^{i(kx - \omega t)}$$

$$\frac{\partial \psi(x,t)}{\partial x} = ik A e^{i(kx - \omega t)} \Rightarrow \frac{\partial^2 \psi(x,t)}{\partial x^2} = -k^2 A e^{i(kx - \omega t)}$$

$$\frac{\partial^2 \psi}{\partial t^2} - c^2 \frac{\partial^2 \psi}{\partial x^2} + \frac{m^2 c^4}{\hbar^2} \psi = -\omega^2 A e^{i(kx - \omega t)} + c^2 k^2 A e^{i(kx - \omega t)} + \frac{m^2 c^4}{\hbar^2} A e^{i(kx - \omega t)}$$

$$= A e^{i(kx - \omega t)} \left[-\omega^2 + c^2 k^2 + \frac{m^2 c^4}{\hbar^2} \right]$$

$$= A e^{i(kx - \omega t)} \left[-\frac{E^2}{\hbar^2} + c^2 \frac{p^2}{\hbar^2} + \frac{m^2 c^4}{\hbar^2} \right]$$

$$= \frac{A e^{i(kx - \omega t)}}{\hbar^2} \left[-E^2 + c^2 p^2 + m^2 c^4 \right] \quad (2)$$

$$\underline{\text{Or } E^2 = p^2 c^2 + m^2 c^4} \Rightarrow (2) \Rightarrow \frac{A e^{i(kx - \omega t)}}{\hbar^2} \left[-p^2 c^2 - m^2 c^4 + c^2 p^2 + m^2 c^4 \right] = 0$$

Donc l'équat° de Klein-Gordon admet des solut° de la forme $\psi(x,t) = A e^{i(kx - \omega t)}$

$$-E^2 + c^2 p^2 + m^2 c^4 = 0 \Rightarrow E^2 = c^2 p^2 + m^2 c^4$$

$$\text{E) } E^2 = c^2 (p^2 + m^2 c^2)$$

$$\text{E) } E = c \sqrt{p^2 + m^2 c^2}$$