exo 4 | Q2

1) $\Psi_A = A\cos(kx-\omega t)$

$\begin{cases} \frac{\partial \Psi_A(x,t)}{\partial x} = -Ak\sin(kx-\omega t) \\ \frac{\partial^2 \Psi(x,t)}{\partial x^2} = -Ak^2\cos(kx-\omega t) \end{cases}$ | $\frac{\partial \Psi(x,t)}{\partial t} = A\omega\sin(kx-\omega t)$

Membre de gauche: ($k = p/\hbar$, $E_c = \frac{p^2}{2m}$)

$-\frac{\hbar^2}{2m}(-A)k^2\cos(kx-\omega t) = \frac{\hbar^2}{2m}\frac{p^2}{\hbar^2}A\cos(-) = A\cdot\frac{p^2}{2m}\cos(-) = \underbrace{AE_c\cos(kx-\omega t)}_{(1)}$

Mb de droite: $i\hbar(A)\omega\sin(kx-\omega t) = \underbrace{iAE\sin(kx-\omega t)}_{(2)}$

D'où (1) ≠ (2), $\Psi_A$ ∅ vérifie

$\Psi_B(x,t) = B\sin(kx-\omega t)$

Idem

$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} = -\frac{\hbar^2}{2m}(-B)k^2\sin(kx-\omega t) = \frac{B\hbar^2}{2m}\frac{p^2}{\hbar^2}\sin(-) = \underbrace{BE_c\sin(kx-\omega t)}_{(1)}$

ET $i\hbar B(-\omega)\cos(kx-\omega t) = \underbrace{-iBE\cos(kx-\omega t)}_{(2)}$

(1) ≠ (2)

3) Mq $\Psi(x,t) = Ae^{i(kx-\omega t)}$ vérifie l'éq de l'énoncé

$\frac{\partial \Psi(x,t)}{\partial x} = ki\Psi(x,t)$ | $\frac{\partial \Psi(x,t)}{\partial t} = -\omega i\Psi(x,t)$

$\frac{\partial^2 \Psi(x,t)}{\partial x^2} = k^2 i^2 \Psi(x,t) = -k^2\Psi(x,t)$ | $\frac{\partial^2 \Psi(x,t)}{\partial t^2} = +\omega^2 i^2 \Psi(x,t) = -\omega^2\Psi(x,t)$

D'où : $-\omega^2\Psi(x,t) + c^2k^2\Psi(x,t) + \frac{m^2c^4}{\hbar^2}\Psi(x,t) = 0$

$\Leftrightarrow -\hbar^2\omega^2\Psi(x,t) + c^2k^2\hbar^2\Psi(x,t) + m^2c^4\Psi(x,t) = 0$

$\Leftrightarrow -E^2\Psi(x,t) + c^2p^2\Psi(x,t) + c^4m^2\Psi(x,t) = 0$

$\Leftrightarrow \Psi(x,t)(-E^2 + c^2p^2 + c^4m^2) = 0$

$\Rightarrow \boxed{E^2 = p^2c^2 + m^2c^4}$

2) Enoncé : $\Psi(x,t) = A e^{-\frac{i}{\hbar}(px - Et)}$

① $\begin{cases} \dfrac{\partial \Psi(x,t)}{\partial x} = \dfrac{-Ai}{\hbar} p e^{-\frac{i}{\hbar}(px-Et)} = -\dfrac{ip}{\hbar}\Psi(x,t) \\ \dfrac{\partial^2 \Psi(x,t)}{\partial x^2} = \dfrac{-i^2}{\hbar^2} A p^2 e^{\sim} = -\dfrac{p^2}{\hbar^2}\Psi(x,t) \end{cases}$

et $\dfrac{\partial \Psi(x,t)}{\partial t} = +\dfrac{Ai}{\hbar} E e^{-\frac{i}{\hbar}(px-Et)} = i\dfrac{E}{\hbar}\Psi(x,t)$

② Or $p^2 = m^2 v^2$ et $E_c = \frac{1}{2} m v^2$

D'où $p^2 = 2m E_c$

③ D'où $\dfrac{\partial^2 \Psi(x,t)}{\partial x^2} = -\dfrac{2m E_c}{\hbar}\Psi(x,t) \iff -\dfrac{\hbar}{2m}\dfrac{\partial^2 \Psi(x,t)}{\partial x^2} = E_c \Psi(x,t)$

et : $-i\hbar \dfrac{\partial \Psi(x,t)}{\partial t} = E \Psi(x,t)$

Avec $E = E_c$, nous avons :

$$\boxed{\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} = i\hbar \frac{\partial \Psi(x,t)}{\partial t}}$$