

$$\begin{cases} E = \omega \hbar \\ p = k \hbar \\ E_c = \frac{p^2}{2m} \end{cases}$$

1/

$$\Psi_A(x,t) = A \cos(kx - \omega t)$$

Eq. Schrödinger: $i\hbar \frac{d}{dt} \Psi(x,t) = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \Psi(x,t)$

$$\begin{aligned} \frac{d}{dt} \Psi_A(x,t) &= -A\omega \sin(kx - \omega t) \\ \frac{d^2}{dx^2} \Psi_A(x,t) &= -Ak^2 \cos(kx - \omega t) \end{aligned} \quad \begin{cases} A i \hbar \omega \sin(kx - \omega t) \stackrel{?}{=} -\frac{\hbar^2}{2m} A k^2 \cos(kx - \omega t) \\ A i E \sin(kx - \omega t) \stackrel{?}{=} -\frac{\hbar^2}{2m} A \frac{p^2}{\hbar^2} \cos(kx - \omega t) \\ A i E \sin(kx - \omega t) \stackrel{?}{=} -E_c A \cos(kx - \omega t) \end{cases}$$

$$\Psi_B(x,t) = B \sin(kx - \omega t)$$

$$\begin{aligned} \frac{d}{dt} \Psi_B(x,t) &= -B\omega \cos(kx - \omega t) \\ \frac{d^2}{dx^2} \Psi_B(x,t) &= -Bk^2 \sin(kx - \omega t) \end{aligned} \quad \begin{cases} -B i \hbar \omega \cos(kx - \omega t) \stackrel{?}{=} -\frac{\hbar^2}{2m} B k^2 \sin(kx - \omega t) \\ -B i E \cos(kx - \omega t) \stackrel{?}{=} -\frac{\hbar^2}{2m} B \frac{p^2}{\hbar^2} \sin(kx - \omega t) \\ -B i E \cos(kx - \omega t) \stackrel{?}{=} -\frac{B}{i} E_c \sin(kx - \omega t) \end{cases}$$

2/ $\Psi(x,t) = A \exp\left[\frac{-i}{\hbar} (px - Et)\right]$

$$\frac{d\Psi(x,t)}{dt} = A \frac{(-i)}{\hbar} (-E) \exp\left[\frac{-i}{\hbar} (px - Et)\right]$$

$$= \frac{Ei}{\hbar} \Psi(x,t)$$

donc $-i\hbar \frac{d}{dt} \Psi(x,t) = E \Psi(x,t)$

$$\frac{d^2 \Psi(x,t)}{dx^2} = A \frac{i^2}{\hbar^2} p^2 \exp\left[\frac{-i}{\hbar} (px - Et)\right]$$

$$= \frac{i^2 p^2}{\hbar^2} \Psi(x,t) \quad \Leftrightarrow -\frac{\hbar^2}{2m} \frac{d^2 \Psi(x,t)}{dx^2} = \Psi(x,t) \times E_c$$

$$\left(\text{car } E_c = \frac{p^2}{2m} \right)$$

$$i\hbar \frac{d}{dt} \Psi = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \Psi$$

$$\Psi(x,t) = A e^{i(kx - \omega t)}$$

$$\frac{\partial^2 \Psi(x,t)}{\partial t^2} = -(\omega)^2 \Psi(x,t)$$

$$\frac{\partial^2 \Psi(x,t)}{\partial x^2} = -(k)^2 \Psi(x,t)$$

$$\Psi = -\frac{\hbar^2 k^2}{2m} \Psi + \frac{\hbar^2 \omega^2}{2m} \Psi$$

$$\Psi = -\omega^2 + c^2 k^2 + \frac{m^2 c^4}{\hbar^2}$$

$$\frac{E^2}{\hbar^2} + \frac{c^2 p^2}{\hbar^2} + \frac{m^2 c^4}{\hbar^2}$$

$$= E^2 + c^2 p^2 + m^2 c^4$$

$$E = c \sqrt{p^2 + m^2 c^2}$$