

✓ Ψ est solution de l'équation de Schrödinger ssi:

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x,t) = i\hbar \frac{\partial}{\partial t} \Psi(x,t).$$

• $\Psi_A(x,t) = A \cos(kx - \omega t)$.

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi_A(x,t) = -\frac{A\hbar^2}{2m} \frac{\partial}{\partial x} (-k \sin(kx - \omega t))$$

$$= \frac{A\hbar^2 k^2}{2m} \cos(kx - \omega t)$$

Nous savons que:

$$\omega = \frac{E}{\hbar}$$

$$k = \frac{p}{\hbar}$$

$$E_c = \frac{p^2}{2m}$$

$$E = E_c + V = E_c$$

↑
cor
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D'où: $\frac{A\hbar^2 k^2}{2m} \cos(kx - \omega t) = \frac{A\hbar^2 \times \frac{p^2}{\hbar^2}}{2m} \cos(kx - \omega t)$

$$= \frac{A p^2}{2m} \cos(kx - \omega t) = \boxed{A E_c \cos(kx - \omega t)} \quad (1)$$

• $i\hbar \frac{\partial}{\partial t} \Psi(x,t) = i\hbar (-\omega) A \sin(kx - \omega t) = \boxed{-i A E_c \sin(kx - \omega t)} \quad (2)$

$\Rightarrow (1) \neq (2) \Rightarrow \Psi_A(x,t)$ ne vérifie pas l'équation de Schrödinger.

• $\Psi_B(x,t) = B \sin(kx - \omega t)$.

De la même manière, on a:

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi_B(x,t) = -\frac{B\hbar^2}{2m} \frac{\partial}{\partial x} (\cos(kx - \omega t)) = +\frac{B\hbar^2 k^2}{2m} \sin(kx - \omega t)$$

$$= \frac{B p^2}{2m} \sin(kx - \omega t) = \boxed{B E_c \sin(kx - \omega t)} \quad (1)$$

• $i\hbar \frac{\partial \Psi_B(x,t)}{\partial t} = i\hbar B (-\omega) \cos(kx - \omega t) = -i\hbar B \frac{E_c}{\hbar} \cos(kx - \omega t)$

$$= \boxed{-i B E_c \cos(kx - \omega t)} \quad (2)$$

$\Rightarrow (1) \neq (2) \Rightarrow \Psi_B(x,t)$ ne vérifie pas l'équation de Schrödinger.

2/ Sur une autre planète : $\Psi(x,t) = A \exp\left(-\frac{i}{\hbar}(px - Et)\right)$ donc

il faut que $\boxed{\hbar \frac{\partial^2}{\partial x^2} \Psi(x,t) = i \hbar \frac{\partial}{\partial t} \Psi(x,t)}$

$$\bullet \frac{\partial \Psi(x,t)}{\partial t} = \frac{A i E}{\hbar} e^{-i/\hbar (px - Et)} = \frac{i E}{\hbar} \Psi(x,t).$$

$$\bullet \frac{\partial \Psi(x,t)}{\partial x} = -\frac{A i}{\hbar} e^{-i/\hbar (px - Et)} = -\frac{i p}{\hbar} \Psi(x,t)$$

$$\bullet \frac{\partial^2 \Psi(x,t)}{\partial x^2} = +\frac{i^2}{\hbar^2} A p^2 e^{-i/\hbar (px - Et)} = -\frac{p^2}{\hbar^2} \Psi(x,t).$$

$$\underline{or} : \left. \begin{aligned} E_c &= \frac{1}{2} m v^2 \\ p^2 &= m^2 v^2 \end{aligned} \right\} p^2 = 2 m E_c.$$

$$\underline{D'ail} : \frac{\partial^2 \Psi(x,t)}{\partial x^2} = -\frac{2 m E_c}{\hbar^2} \Psi(x,t) \Leftrightarrow -\frac{\hbar^2}{2 m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} = E_c \Psi(x,t)$$

$$\bullet -i \hbar \frac{\partial \Psi(x,t)}{\partial t} = E_c \Psi(x,t).$$

Avec $E = E_c$, Nous avons donc :

$$\Rightarrow -\frac{\hbar^2}{2 m} \frac{\partial^2}{\partial x^2} \Psi(x,t) = -i \hbar \frac{\partial \Psi(x,t)}{\partial t}.$$

$$\Rightarrow \boxed{i \hbar \frac{\partial}{\partial t} \Psi(x,t) = \frac{\hbar^2}{2 m} \frac{\partial^2}{\partial x^2} \Psi(x,t)}.$$

3/ Pour les particules de masse m , d'impulsion $p = \hbar k$ et $E = \hbar \omega$.

$$\boxed{\frac{\partial^2 \Psi}{\partial t^2} - c^2 \frac{\partial^2 \Psi}{\partial x^2} + \frac{m^2 c^4}{\hbar^2} \Psi = 0}$$

$\Rightarrow$ équation de Klein-Gordon.

solution : $\Psi(x,t) = A e^{i(kx - \omega t)}.$

TAI n°4. | dérivation en fonction de x et t .

$$\frac{\partial \psi}{\partial x} = ik \psi(x, t).$$

$$\frac{\partial \psi}{\partial t} = -i\omega \psi(x, t).$$

$$\frac{\partial^2 \psi}{\partial x^2} = k^2 i^2 \psi(x, t) = -k^2 \psi(x, t).$$

$$\frac{\partial^2 \psi}{\partial t^2} = +i^2 \omega^2 \psi(x, t) = -\omega^2 \psi(x, t).$$

D'ai :

$$\frac{\partial^2 \psi}{\partial t^2} - c^2 \frac{\partial^2 \psi}{\partial x^2} + \frac{m^2 c^4}{\hbar^2} \psi = 0.$$

$$-\omega^2 \psi(x, t) + c^2 k^2 \psi(x, t) + \frac{m^2 c^4}{\hbar^2} \psi(x, t) = 0.$$

$$-\frac{E^2}{\hbar^2} + c^2 \frac{p^2}{\hbar^2} + \frac{m^2 c^4}{\hbar^2} = 0$$

$$-E^2 + c^2 p^2 + m^2 c^4 = 0.$$

$$E^2 = c^2 p^2 + m^2 c^4$$

$$\Rightarrow \boxed{E = cp + mc^2.}$$

expression de l'énergie d'une
particule libre relativiste