

Exercice 5.

1) On cherche à vérifier que: $\int_0^a |\Psi(x, A)|^2 dx = 1$

$$|\Psi(x, A)|^2 = [N x (a-x)]^2 = N^2 x^2 (a^2 - 2ax + x^2) \\ = N^2 x^2 a^2 - 2N^2 a x^3 + N^2 x^4$$

$$\Rightarrow \int_0^a N^2 (x^2 a^2 - 2ax^3 + x^4) dx = N^2 \left[\frac{1}{3} x^3 a^2 - \frac{2}{4} a x^4 + \frac{x^5}{5} \right]_0^a \\ = N^2 \left(\frac{a^5}{3} - \frac{a^5}{2} + \frac{a^5}{5} \right) \\ = N^2 a^5 \times \left(\frac{1}{30} \right)$$

Nous voulons: $N^2 a^5 \times \frac{1}{30} = 1 \Leftrightarrow \boxed{N = \sqrt{\frac{30}{a^5}}}$

2) $\langle x \rangle = \int_0^a x |\Psi(x)|^2 dx = N^2 \int_0^a x (x^2 a^2 - 2ax^3 + x^4) dx$

$$\Rightarrow \langle x \rangle = N^2 \int_0^a (x^3 a^2 - 2ax^4 + x^5) dx = N^2 \left[\frac{a^2 x^4}{4} - \frac{2}{5} a x^5 + \frac{x^6}{6} \right]_0^a$$

$$= N^2 \left(\frac{a^6}{4} - \frac{2a^6}{5} + \frac{a^6}{6} \right) = N^2 a^6 \frac{1}{60} = \frac{30}{a^5} \frac{a^6}{60}$$

$$\Rightarrow \boxed{\langle x \rangle = \frac{1}{2} a}$$

complexe conjuguée
ici $\Psi^* = \Psi$ car pas complexe

$$\langle p \rangle = \int_0^a \Psi^*(x) \cdot \frac{\hbar}{i} \frac{\partial \Psi}{\partial x} dx = -i\hbar \int_0^a (Nxa - Nx^2)(-2Nx + Na) dx$$

$$= -i\hbar \int_0^a (-2N^2 x^2 a + N^2 a^2 x + 2N^2 x^3 - N^2 x^2 a) dx = -i\hbar N^2 \int_0^a (-3x^2 a + dx + 2x^3) dx$$

$$= -i\hbar N^2 \left[-\frac{3ax^3}{3} + \frac{a^2 x^2}{2} + \frac{2x^4}{4} \right]_0^a = -i\hbar N^2 \left(-a^4 + \frac{1}{2} a^4 + \frac{1}{2} a^4 \right)$$

$$\boxed{\langle p \rangle = 0}$$

3) $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$

$$\langle x^2 \rangle = \int_0^a x^2 |\Psi(x)|^2 dx$$

$$= N^2 \int_0^a x^2 (x^2 a^2 - 2ax^3 + x^4) dx$$

$$= N^2 \int_0^a (x^4 a^2 - 2ax^5 + x^6) dx$$

$$= N^2 \left[\frac{x^5 a^2}{5} - \frac{2ax^6}{6} + \frac{x^7}{7} \right]$$

$$= N^2 \left(\frac{a^7}{5} - \frac{2a^7}{6} + \frac{a^7}{7} \right) = \frac{30}{a^5} \frac{a^7}{42} = \boxed{\frac{2a^2}{7}}$$

D'où $\Delta x = \sqrt{\frac{2a^2}{7} - \frac{1}{4} a^2}$

$$\Delta x = \sqrt{\frac{a^2}{28}} = \boxed{\frac{a}{\sqrt{28}}}$$

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Leftrightarrow \Delta p = \frac{\hbar}{2\Delta x} = \boxed{\frac{\sqrt{28} \hbar}{a}}$$

exercice 5

1) Les fonctions d'ondes peuvent s'additionner donc des sommes de fonctions d'ondes forment un espace vectoriel
La condition de normalisation permet donc de formaliser ce fait en posant que l'onde se situe dans notre espace à coup sur. Donc l'intégrale de 0 à $a = 1$ nous donne une proba de 1 pour que notre onde soit dans notre espace

$$\int_0^a |\Psi(x)|^2 dx \quad \text{or} \quad \Psi(x) = Nx(a-x) \quad \text{donc}$$

$$\int_0^a |Nx(a-x)|^2 dx = |N|^2 \int_0^a |x(a-x)|^2 dx = 1 = |N|^2 \int_0^a (xa - x^2)^2 dx$$

~~$$|N|^2 \int_0^a (xa - x^2)^2 dx = 1$$~~

$$|N|^2 \int_0^a x^2 a^2 - 2x^3 a + x^4 dx$$

$$= |N|^2 \left[\frac{a^2 x^3}{3} - \frac{a x^4}{2} + \frac{x^5}{5} \right]_0^a = |N|^2 \left[\frac{a^5}{3} - \frac{a^5}{2} + \frac{a^5}{5} \right]$$

$$= \frac{|N|^2 a^5}{30} = 1$$

$$N^2 = \frac{30}{a^5} \Rightarrow \boxed{N = \sqrt{\frac{30}{a^5}}}$$

Exo 6 $\frac{3}{4}$ 3) incertitude de position :

$$\langle x^2 \rangle = \int_0^a \psi^*(x) \hat{x}^2 \psi(x) dx \quad \text{avec } \hat{x}^2 = x^2$$

$$= \int_0^a \psi(x^2) x^2 dx$$

$$= N^2 \int_0^a x^2 (a-x)^2 x^2 dx = N^2 \int_0^a x^4 (a-x)^2 dx$$

$$= N^2 \int_0^a x^4 a^2 - 2ax^5 + x^6 dx$$

$$= N^2 \left[\frac{x^5 a^2}{5} - \frac{2ax^6}{6} + \frac{x^7}{7} \right]_0^a = N^2 \left(\frac{a^7}{5} - \frac{2a^7}{6} + \frac{a^7}{7} \right)$$

$$= \frac{30a^7}{a^5 \times 105} = \frac{2a^2}{7}$$

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{2a^2}{7} - \frac{a^2}{4}} = \sqrt{\frac{8a^2 - 7a^2}{28}}$$

$$\Delta x = \sqrt{\frac{a^2}{28}} = \frac{a}{\sqrt{28}}$$