

particule quantique $0 \leq x \leq a$.

Fonction d'onde $\Psi(x) = Nx(a-x)$

avec $N = \text{constante}$

1/ Normer la fonction d'onde

→ condition de normalisation.

$$\int_0^a |\Psi(x)|^2 dx = 1$$

$$\begin{aligned} |\Psi(x,t)|^2 &= [Nx(a-x)]^2 = N^2 x^2 (a^2 + x^2 - 2ax) \\ &= N^2 x^2 a^2 + N^2 x^4 - 2a N^2 x^3 \end{aligned}$$

$$\begin{aligned} \int_0^a |\Psi(x,t)|^2 dx &= \int_0^a N^2 (x^2 a^2 + x^4 - 2ax^3) dx \\ &= N^2 \int_0^a (x^2 a^2 + x^4 - 2ax^3) dx \\ &= N^2 \left[\frac{x^3 a^2}{3} + \frac{x^5}{5} - \frac{2}{4} a x^4 \right]_0^a \\ &= N^2 \left(\frac{a^5}{3} + \frac{a^5}{5} - \frac{1}{2} a^5 \right) \end{aligned}$$

$$\int_0^a |\Psi(x,t)|^2 dx = N^2 a^5 \left(\frac{1}{30} \right)$$

D'où: $N^2 \frac{a^5}{30} = 1$

$$N^2 = \frac{30}{a^5} \Rightarrow N = \sqrt{\frac{30}{a^5}}$$

2/ Position moyenne.

$$\langle x \rangle = \int_0^a x |\Psi(x)|^2 dx = \int_0^a x |Nx(a-x)|^2 dx$$

$$\langle x \rangle = N^2 \int_0^a x (x^2 a^2 + x^4 - 2ax^3) dx = N^2 \int_0^a (x^3 a^2 + x^5 - 2ax^4) dx$$

$$\langle x \rangle = N^2 \left[\frac{x^4 a^2}{4} + \frac{x^6}{6} - \frac{2a}{5} x^5 \right]_0^a = N^2 \left(\frac{a^6}{4} + \frac{a^6}{6} - \frac{2}{5} a^6 \right)$$

$$\langle x \rangle = N^2 a^6 \left(\frac{15}{60} + \frac{10}{60} - \frac{24}{60} \right) = N^2 a^6 \left(\frac{1}{60} \right) = \frac{30}{a^5} a^6 \left(\frac{1}{60} \right)$$

$$\Rightarrow \langle x \rangle = \frac{a}{2}$$

Impulsion moyenne.

$$\langle p \rangle = \int_0^a \psi^*(x) \left(-i\hbar \frac{\partial}{\partial x} \right) \psi(x) dx$$

$$\langle p \rangle = -i\hbar \int_0^a (Nx a - Nx^2) (-2Nx + Na) dx.$$

$$\langle p \rangle = -i\hbar \int_0^a (-2N^2 x^2 a + Na^2 x + 2N^2 x^3 - N^2 x^2 a) dx$$

$$\langle p \rangle = -i\hbar N^2 \int_0^a (-3x^2 a + a^2 x + 2x^3) dx$$

$$\langle p \rangle = -i\hbar N^2 \left[-\frac{3a}{3} x^3 + \frac{a^2}{2} x^2 + \frac{2}{4} x^4 \right]_0^a$$

$$\langle p \rangle = -i\hbar N^2 \left(-a^4 + \frac{1}{2} a^4 + \frac{1}{2} a^4 \right)$$

$$\boxed{\langle p \rangle = 0}$$

3/ Incertainitude de position.

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

$$\langle x^2 \rangle = \int_0^a |\psi(x)|^2 x^2 dx = N^2 \int_0^a x^2 (x^2 a^2 - 2a x^3 + x^4) dx.$$

$$\langle x^2 \rangle = N^2 \int_0^a (x^4 a^2 - 2a x^3 + x^5) dx.$$

$$\langle x^2 \rangle = N^2 \left[\frac{x^5 a^2}{5} - \frac{2a x^4}{4} + \frac{x^6}{6} \right]_0^a$$

$$\langle x^2 \rangle = N^2 \left(\frac{a^7}{5} - \frac{2a^7}{6} + \frac{a^7}{6} \right)$$

$$\langle x^2 \rangle = \frac{30}{a^3} \times \frac{a^7}{105}.$$

$$\boxed{\langle x^2 \rangle = \frac{2a^2}{7}}$$

$$\bullet \underline{D'_{\text{a}}} \quad \Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

$$\Delta x = \sqrt{\frac{2a^2}{7} - \frac{1}{4} a^2}$$

$$\Delta x = \sqrt{\frac{8}{28} a^2 - \frac{7}{28} a^2} = \sqrt{\frac{a^2}{28}}$$

$$\Rightarrow \boxed{\Delta x = \frac{a}{\sqrt{28}}} = \frac{a}{2\sqrt{7}}.$$

TAI n:5 (suite).

Incertitude minimum sur l'impulsion.

$$\Delta x \cdot \Delta p \geq \frac{h}{2}$$

$$\Rightarrow \Delta p = \frac{h}{2 \Delta x}$$

A.N: $\Delta p = \frac{h \sqrt{7}}{2a}$

$$\Delta p = \frac{h \sqrt{7}}{2}$$