

TAJ EXO 6

$$\psi(x) = \begin{cases} Nxe^{-\frac{\alpha}{2}x} & \text{si } x \in \mathbb{R}^+ \\ 0 & \text{sinon} \end{cases}$$

$$\alpha > 0$$

$$N = \sqrt{\frac{\alpha^3}{2}}$$

1) Proba de trouver la particule entre x et $x+dx$

$$P(x) = \int_x^{x+dx} |\psi(x)|^2 dx$$

- Valeur attendue de la position :

$$\langle x \rangle = \int_0^{+\infty} \psi^*(x) x \psi(x) dx$$

ψ est une fonction réelle $\Rightarrow \psi^*(x)\psi(x) = \psi^2(x)$

$$\langle x \rangle = \int_0^{+\infty} N^2 x^2 e^{-\alpha x} x dx$$

$$\langle x \rangle = N^2 \int_0^{+\infty} x^3 e^{-\alpha x} dx$$

$$\langle x \rangle = \frac{\alpha^3}{2} \int_0^{+\infty} x^3 e^{-\alpha x} dx$$

$$\langle x \rangle = \frac{\alpha^3}{2} \cdot \frac{6}{\alpha^4} = \boxed{\frac{3}{\alpha}}$$

- Valeur moyenne du carré de la position :

$$\langle x^2 \rangle = \int_0^{+\infty} x^2 |\psi(x)|^2 dx$$

$$\langle x^2 \rangle = \int_0^{+\infty} N^2 x^2 e^{-\alpha x} x^2 dx$$

$$\langle x^2 \rangle = \frac{\alpha^3}{2} \int_0^{+\infty} x^4 e^{-\alpha x} dx$$

$$\langle x^2 \rangle = \frac{\alpha^3}{2} \cdot \frac{24}{\alpha^5} = \boxed{\frac{12}{\alpha^2}}$$

$$u(x) = Nx$$

$$v(x) = e^{-\frac{\alpha}{2}x}$$

$$\Rightarrow \psi(x) = Ne^{-\frac{\alpha}{2}x} - Nx \cdot \frac{\alpha}{2} e^{-\frac{\alpha}{2}x}$$

$$= Ne^{-\frac{\alpha}{2}x} \left(1 - \frac{\alpha}{2}x \right)$$

$$u'(x) = e^{-\alpha x}$$

$$u(x) = x$$

$$v(x) = \frac{1}{\alpha} e^{-\alpha x}$$

$$u'(x) = 1$$

$$\left[-\frac{x}{\alpha} e^{-\alpha x} \right]_0^{+\infty} + \frac{1}{\alpha} \int_0^{+\infty} e^{-\alpha x} dx$$

$$g(x) = -xe^{-\frac{\alpha}{2}x}$$

2) Valeur attendu de l'impulsion :

$$\langle p \rangle = \int_0^{+\infty} \psi^*(x) \left(-i\hbar \frac{\partial}{\partial x} \psi(x) \right) dx$$

$$= +i\hbar \int_0^{+\infty} Nxe^{-\frac{\alpha}{2}x} Ne^{-\frac{\alpha}{2}x} \left(1 - \frac{\alpha}{2}x \right) dx$$

$$= +i\hbar N^2 \int_0^{+\infty} xe^{-\alpha x} \left(1 - \frac{\alpha}{2}x \right) dx$$

$$= +i\hbar N^2 \left(\frac{1}{\alpha^2} - \frac{\alpha}{2\alpha^3} \right)$$

$$= 0$$

73 75

MAI

22 à 19-04

F. du TRAVAIL

S. Boris

SS. Phil., Jacq.

S. Sylvain

S. Judith

S. Prudence

S. Gisèle

VICT. 1945

S. Pacôme

S. Solange

S. Estelle

S. Achille

S. Rolande

F. Jeanne-d'

S. Denise

S. Honoré

S. Pascal

S. Eric

S. Yves

S. Bernard

S. Consta

S. Émile

S. Didier

S. Donat

ASCEN

S. Béra

S. Aug

Fête de

S. Ayn

S. Fern

Visita

$$\langle p^2 \rangle = \int_0^{+\infty} \psi^*(x) \hat{p}^2 \psi(x) dx$$

$$= \int_0^{+\infty} \psi^*(x) \cdot \left(-\hbar^2 \frac{\partial^2}{\partial x^2} \right) \psi(x) dx$$

$$\hat{p}^2 = -\hbar^2 \frac{\partial^2}{\partial x^2} \quad \frac{\partial \psi(x)}{\partial x} =$$

$$\frac{\partial^2}{\partial x^2} \psi(x) = -\frac{\alpha}{4} N e^{-\frac{\alpha}{2}x} (4 + \alpha x)$$

$$= +\hbar^2 \int_0^{+\infty} N x e^{-\frac{\alpha}{2}x} \cdot \frac{\alpha}{4} N e^{-\frac{\alpha}{2}x} (4 + \alpha x) dx$$

$$= \hbar^2 N^2 \frac{\alpha}{4} \int_0^{+\infty} x e^{-\alpha x} (4 + \alpha x) dx$$

$$\langle p^2 \rangle = \int_0^{+\infty} \psi^*(x) \cdot -\hbar^2 \frac{\partial^2}{\partial x^2} \psi(x) dx$$

$$= \int_0^{+\infty} N \psi^*(x) \cdot -\hbar^2 \frac{\partial}{\partial x} \left(e^{-\frac{\alpha}{2}x} - \frac{\alpha}{2} x e^{-\frac{\alpha}{2}x} \right) dx$$

$$= \int_0^{+\infty} \psi^*(x) (-\hbar^2) N \left(-\frac{\alpha}{2} e^{-\frac{\alpha}{2}x} - \frac{\alpha}{2} \left(e^{-\frac{\alpha}{2}x} - \frac{\alpha}{2} x e^{-\frac{\alpha}{2}x} \right) \right) dx$$

$$= \int_0^{+\infty} \psi^*(x) (-\hbar^2) N \left[-\alpha e^{-\frac{\alpha}{2}x} + \frac{\alpha^2}{4} x e^{-\frac{\alpha}{2}x} \right] dx$$

$$= \int_0^{+\infty} \psi^*(x) (-\hbar^2) N e^{-\frac{\alpha}{2}x} \left(-\alpha + \frac{\alpha^2}{4} x \right) dx$$

$$= -\hbar^2 N^2 \int_0^{+\infty} x e^{-\frac{\alpha}{2}x} \left(e^{-\frac{\alpha}{2}x} \left(-\alpha + \frac{\alpha^2}{4} x \right) \right) dx$$

$$= -\hbar^2 N^2 \int_0^{+\infty} x e^{-\alpha x} \left(-\alpha + \frac{\alpha^2}{4} x \right) dx$$

$$= -\hbar^2 N^2 \left[\int_0^{+\infty} -\alpha x e^{-\alpha x} dx + \int_0^{+\infty} x^2 e^{-\alpha x} \cdot \frac{\alpha^2}{4} dx \right]$$

$$= -\hbar^2 N^2 \left[-\alpha \cdot \frac{1}{\alpha^2} + \frac{\alpha^2}{4} \cdot \frac{2}{\alpha^3} \right]$$

$$= -\hbar^2 N^2 \left[-\frac{1}{\alpha} + \frac{1}{2\alpha} \right]$$

$$= -\hbar^2 N^2 \left(-\frac{1}{2\alpha} \right) = -\hbar^2 \left(\frac{\alpha^3}{2} \right) \left(-\frac{1}{2\alpha} \right) = -\hbar^2 \left(-\frac{\alpha^2}{4} \right) = \boxed{\frac{\hbar^2 \alpha^2}{4}}$$

$$\delta) \quad \Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{12}{\alpha^2} - \frac{9}{\alpha^2}} = \boxed{\frac{\sqrt{3}}{\alpha}}$$

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \sqrt{\frac{\hbar^2 \alpha^2}{4} - 0} = \boxed{\frac{\hbar \alpha}{2}}$$

Principe d'incertitude

$$\Delta x \Delta p = 2\pi\hbar = \hbar$$

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

$$\Delta x \Delta p = \frac{\sqrt{3}}{\alpha} \cdot \frac{\hbar \alpha}{2} = \frac{\sqrt{3}}{2} \hbar > \frac{\hbar}{2} \quad \text{COFF}$$