

TAI B:

$$\Phi(x) = \begin{cases} N x e^{-\alpha x/3} & \text{si } 0 \leq x \leq \infty \\ 0 & \text{ailleurs} \end{cases}$$

avec $\alpha > 0$

$$N = \sqrt{\alpha^3/2}$$

1/ On a $dP(x) = |\Phi(x)|^2 dx$: proba de trouver la particule dans l'espace.

$$\text{donc } P(x) = \int_x^{x+dx} dP(x) = \int_x^{x+dx} |\Phi(x)|^2 dx.$$

$$\left(|\psi(x)|^2 = |\Phi^* \psi| = \Phi(x) = N^2 x^2 e^{-\alpha x} \right)$$

$$\langle x \rangle = \int_0^\infty x |\Phi(x)|^2 dx = \int_0^\infty x N^2 x^2 e^{-\alpha x} dx$$

$$\langle x \rangle = N^2 \int_0^\infty x^3 e^{-\alpha x} dx = N^2 \frac{3!}{\alpha^4} = \sqrt{\frac{\alpha^3}{2}} \times \frac{6}{\alpha^4} = \boxed{\frac{3}{\alpha}}$$

$$\langle x^2 \rangle = \int_0^\infty x^2 |\Phi(x)|^2 dx = \int_0^\infty x^2 N^2 x^2 e^{-\alpha x} dx$$

$$\langle x^2 \rangle = N^2 \int_0^\infty x^4 e^{-\alpha x} dx = N^2 \frac{4!}{\alpha^5} = \frac{\alpha^3}{2} \times \frac{24}{\alpha^5} = \boxed{\frac{12}{\alpha^2}}$$

2)

$$\langle p \rangle = \int_0^\infty \Phi(x) \left| \frac{\hbar}{i} \right| \frac{d\Phi}{dx} dx \quad \text{and} \quad \frac{d\Phi}{dx} = N \left(e^{-\alpha x/2} - \frac{x\alpha}{2} e^{-\alpha x/2} \right)$$

$$\langle p \rangle = \int_0^\infty N x e^{\alpha x/2} \left| \frac{\hbar}{i} \right| N \left(e^{-\alpha x/2} - \frac{x\alpha}{2} e^{-\alpha x/2} \right) dx$$

$$\langle p \rangle = N^2 \left| \frac{\hbar}{i} \right| \int_0^\infty x e^{(-\alpha x/2)} \left(e^{-\alpha x/2} - \frac{x\alpha}{2} e^{-\alpha x/2} \right) dx$$

$$\langle p \rangle = N^2 \left| \frac{\hbar}{i} \right| \int_0^\infty x e^{-\alpha x} - \frac{x^2 \alpha}{2} e^{-\alpha x} dx$$

$$\frac{d}{dx} \langle p \rangle = N^2 \left| \frac{\hbar}{i} \right| \left[\frac{1}{\alpha^2} - \frac{\alpha}{2} \frac{2}{\alpha^3} \right]$$

$$\langle p \rangle = N^2 \left| \frac{\hbar}{i} \right| \left(\frac{1}{\alpha^2} - \frac{1}{\alpha^2} \right) = 0$$

$$\langle p^2 \rangle = \int_0^\infty \Phi^* \hat{p}^2 \Phi(x) dx \quad \text{and} \quad \hat{p} = -i\hbar \frac{d}{dx}$$

$$\langle p^2 \rangle = \int_0^\infty N x e^{-\alpha x/2} \cdot \left(-i\hbar \frac{d}{dx} \right)^2 \cdot \Phi(x) dx$$

$$\langle p^2 \rangle = \int_0^\infty N x e^{-\alpha x/2} \cdot (\hbar^2) \frac{d^2}{dx^2} \Phi(x) dx \quad \text{or} \quad \frac{d^2 \Phi(x)}{dx^2} = \frac{d}{dx} \left(N \left(e^{-\alpha x/2} - \frac{x\alpha}{2} e^{-\alpha x/2} \right) \right)$$

$$\langle p^2 \rangle = \int_0^\infty N x e^{-\alpha x/2} \cdot (\hbar^2) \left(-N \frac{\alpha}{2} \left(2e^{-\alpha x/2} - \frac{\alpha x}{2} e^{-\alpha x/2} \right) \right) dx$$

$$= N \left(\frac{-\alpha}{2} e^{-\alpha x/2} - \frac{\alpha}{2} \left(e^{-\alpha x/2} - \frac{\alpha x}{2} e^{-\alpha x/2} \right) \right)$$

$$= -N \frac{\alpha}{2} \left(e^{-\alpha x/2} + e^{-\alpha x/2} - \frac{\alpha x}{2} e^{-\alpha x/2} \right)$$

$$= -N \frac{\alpha}{2} \left(2e^{-\alpha x/2} - \frac{\alpha x}{2} e^{-\alpha x/2} \right)$$

$$\langle p^2 \rangle = N^2 \hbar^2 \frac{\alpha}{2} \int_0^\infty \left(2x e^{-\alpha x} - \frac{\alpha x^2}{2} e^{-\alpha x} \right) dx$$

$$\langle p^2 \rangle = N^2 \hbar^2 \frac{\alpha}{2} \left[2 \frac{1}{\alpha^2} - \alpha \times \frac{2}{2\alpha^3} \right]$$

$$\langle p^2 \rangle = N^2 \hbar^2 \frac{\alpha}{2} \left[2 \frac{1}{\alpha^2} - \frac{1}{\alpha^2} \right] = N^2 \hbar^2 \frac{\alpha}{2} \times \frac{1}{\alpha^2} = \frac{N^2 \hbar^2}{2\alpha} = \frac{\alpha^3}{2} \times \frac{\hbar^2}{2\alpha} = \boxed{\frac{\alpha^2 \hbar^2}{4}}$$

3) $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{12}{\alpha^2} - \frac{3^2}{\alpha^2}} = \sqrt{\frac{3}{\alpha^2}} = \frac{\sqrt{3}}{\alpha}$ (14)

$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \sqrt{\frac{\hbar^2 \alpha^2}{4} - 0^2} = \frac{\hbar \alpha}{2}$

Vérifier que $\Delta x \Delta p \geq \frac{\hbar}{2}$.

$$\Delta x \Delta p = \frac{\sqrt{3}}{\alpha} \cdot \frac{\hbar \alpha}{2} = \frac{\sqrt{3}}{2} \hbar = \sqrt{3} \frac{\hbar}{2} \quad \text{donc } \underline{\underline{\geq \frac{\hbar}{2}}}$$