

$$\text{Soit } \phi(x) = \begin{cases} N x e^{-\alpha x/2} & \text{si } 0 \leq x < \infty \\ 0 & \text{ailleurs.} \end{cases}$$

avec $\alpha \in \mathbb{R}^+$

$$N = \sqrt{\frac{\alpha^3}{2}}$$

1) Probabilité de trouver la particule dans l'espace dx .

$$dP(x) = |\phi(x)|^2 dx$$

• Probabilité de trouver la particule entre x et $x+dx$.

$$P(x) = \int_x^{x+dx} dP(x)$$

$$\begin{aligned} \text{• Soit, } dP(x) &= |\phi(x)|^2 \\ &= \phi^*(x) \cdot \phi(x) \\ &= \phi^2(x) \\ &= N^2 x^2 e^{-\alpha x} \end{aligned}$$

$$\begin{aligned} \text{avec: } \phi^*(x) &= \phi(x) \\ \Rightarrow \phi^*(x) \wedge \phi(x) &= \phi^2(x) \end{aligned}$$

$$\Rightarrow dP(x) = \frac{\alpha^3}{2} x^2 e^{-\alpha x}$$

• position $\langle x \rangle$ et carré de la position $\langle x^2 \rangle$. $\int_0^\infty x^n e^{-\alpha x} dx = \frac{n!}{\alpha^{n+1}}$

$$\langle x \rangle = \int_0^\infty x |\phi(x)|^2 dx$$

$$\langle x \rangle = \frac{\alpha^3}{2} \int_0^\infty x^3 e^{-\alpha x} dx$$

$$\langle x \rangle = \frac{\alpha^3}{2} \frac{3!}{\alpha^4}$$

$$\langle x \rangle = \frac{3}{\alpha}$$

$$\langle x^2 \rangle = \int_0^\infty x^2 |\phi(x)|^2 dx$$

$$\langle x^2 \rangle = \frac{\alpha^3}{2} \int_0^\infty x^4 e^{-\alpha x} dx$$

$$\langle x^2 \rangle = \frac{\alpha^3}{2} \frac{4!}{\alpha^5}$$

$$\langle x^2 \rangle = \frac{12}{\alpha^2}$$

2/ impulsion $\langle p \rangle$ et du carré de l'impulsion $\langle p^2 \rangle$.

$$\langle p \rangle = i\hbar \int_0^\infty \left(\phi^* \frac{d\phi}{dx} \right) dx = i\hbar \int_0^\infty \phi^* \frac{d}{dx} (N x e^{-\alpha x/2}) dx.$$

$$\langle p \rangle = i\hbar \int_0^\infty \left(\phi^* \left[N x \left(e^{-\alpha x/2} - \frac{\alpha}{2} x e^{-\alpha x/2} \right) \right] \right) dx$$

$$\langle p \rangle = i\hbar \int_0^\infty \left(\phi^* \left[N \cdot e^{-\alpha x/2} \left(1 - \frac{\alpha x}{2} \right) \right] \right) dx$$

$$\langle p \rangle = i\hbar \int_0^\infty \left(N^2 x e^{-\alpha x} \left(1 - \frac{\alpha x}{2} \right) \right) dx$$

$$\langle p \rangle = i\hbar N^2 \int_0^\infty \left(x e^{-\alpha x} - \frac{\alpha}{2} x^2 e^{-\alpha x} \right) dx$$

$$\langle p \rangle = i\hbar N^2 \left(\int_0^\infty x e^{-\alpha x} - \frac{\alpha}{2} \int_0^\infty x^2 e^{-\alpha x} \right) dx$$

$$\langle p \rangle = i\hbar N^2 \left(\frac{1}{\alpha^2} - \frac{\alpha}{2} \times \frac{2}{\alpha^3} \right)$$

$$\boxed{\langle p \rangle = 0}$$

$$\bullet \langle p^2 \rangle = \int_0^\infty \phi^*(x) \hat{p}^2 \phi(x) dx = \int_0^\infty \left[N x e^{-\alpha x/2} (-\hbar)^2 \frac{d}{dx} (N x e^{-\alpha x/2}) \right] dx.$$

$$\langle p^2 \rangle = \int_0^\infty \left[\phi^* (-\hbar^2) N x \frac{d}{dx} (e^{-\alpha x/2} (1 - \frac{\alpha}{2} x)) \right] dx.$$

$$\langle p^2 \rangle = \int_0^\infty \left[\phi^* (-\hbar^2) N \frac{d}{dx} (e^{-\alpha x/2} - \frac{\alpha}{2} x e^{-\alpha x/2}) \right] dx$$

$$\langle p^2 \rangle = \int_0^\infty \left[\phi^* (-\hbar^2) N \left[-\frac{\alpha}{2} e^{-\alpha x/2} - \frac{\alpha}{2} (e^{-\alpha x/2} - \frac{\alpha}{2} x e^{-\alpha x/2}) \right] \right] dx.$$

$$\langle p^2 \rangle = \int_0^\infty \left[\phi^* (-\hbar^2) N \left[-\frac{\alpha}{2} e^{-\alpha x/2} + \frac{\alpha^2}{4} x e^{-\alpha x/2} \right] \right] dx.$$

$$\langle p^2 \rangle = \int_0^\infty \left[\phi^* (-\hbar^2) N \left[e^{-\alpha x/2} \left(-\alpha + \frac{\alpha^2}{4} x \right) \right] \right] dx$$

$$\langle p^2 \rangle = (-\hbar)^2 N^2 \int_0^\infty x e^{-\alpha x} \left(-\alpha + \frac{\alpha^2}{4} x \right) dx$$

TA in: 6.

$$\langle p^2 \rangle = -\hbar^2 N^2 \left(-\alpha \int_0^\infty x e^{-\alpha x} dx + \frac{\alpha^2}{4} \int_0^\infty x^2 e^{-\alpha x} dx \right)$$

$$\langle p^2 \rangle = -\hbar^2 N^2 \left(-\frac{\alpha}{\alpha^2} + \frac{\alpha^2}{4} \frac{2!}{\alpha^3} \right)$$

$$\langle p^2 \rangle = -\hbar^2 N^2 \left(-\frac{1}{\alpha} + \frac{1}{2\alpha} \right)$$

$$\langle p^2 \rangle = -\hbar^2 N^2 \left(-\frac{1}{2\alpha} \right)$$

$$\langle p^2 \rangle = -\hbar^2 \frac{\alpha^3}{2} \left(-\frac{1}{2\alpha} \right)$$

$$\Rightarrow \boxed{\langle p^2 \rangle = \hbar^2 \frac{\alpha^2}{4}}$$

3/. Incertitudes Δx et Δp

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$$

$$\Delta x = \sqrt{\frac{12}{\alpha^2} - \frac{9}{\alpha^2}}$$

$$\Delta x = \sqrt{\frac{3}{\alpha^2}}$$

$$\boxed{\Delta x = \frac{\sqrt{3}}{\alpha}}$$

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$$

$$\Delta p = \sqrt{\hbar^2 \frac{\alpha^2}{4}}$$

$$\boxed{\Delta p = \frac{\hbar \alpha}{2}}$$

$$\boxed{\Delta x \Delta p = \frac{\hbar \sqrt{3}}{2} \geq \frac{1}{2} \hbar} \Rightarrow \text{principe d'Heisenberg respecté.}$$