

exo 7

$$1) \forall t \in [-\frac{\tau}{2}; \frac{\tau}{2}]$$

$$|\varphi(t)|^2 = \varphi^*(t) \cdot \varphi(t)$$

$$\Rightarrow |\varphi(t)|^2 = \frac{1}{\sqrt{\tau}} \cdot e^{i\omega_0 t} \times \frac{1}{\sqrt{\tau}} e^{-i\omega_0 t}$$

$$\Rightarrow \boxed{|\varphi(t)|^2 = \frac{1}{\tau}}$$

$$2) A(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\tau/2}^{\tau/2} \varphi(t) e^{i\omega t} dt = \frac{1}{\sqrt{2\pi\tau}} \int_{-\tau/2}^{\tau/2} e^{-i\omega_0 t} e^{i\omega t} dt$$

$$\Rightarrow A(\omega) = \frac{1}{\sqrt{2\pi\tau}} \int_{-\tau/2}^{\tau/2} e^{i(\omega - \omega_0)t} dt = \frac{1}{\sqrt{2\pi\tau}} \left[\frac{1}{i(\omega - \omega_0)} e^{i(\omega - \omega_0)t} \right]_{-\tau/2}^{\tau/2}$$

$$\Rightarrow A(\omega) = \frac{1}{\sqrt{2\pi\tau}} \times \frac{-i}{\omega - \omega_0} \left(e^{i(\omega - \omega_0)\tau/2} - e^{-i(\omega - \omega_0)\tau/2} \right)$$

$$\Rightarrow A(\omega) = \frac{2}{\sqrt{2\pi\tau}(\omega - \omega_0)} \sin((\omega - \omega_0)\tau/2)$$

$$\Rightarrow A(\omega) = \frac{2 \sin((\omega - \omega_0)\tau/2)}{\sqrt{2\pi\tau}(\omega - \omega_0)} \times \frac{(\omega - \omega_0)\tau/2}{(\omega - \omega_0)\tau/2}$$

$$\Rightarrow A(\omega) = \frac{\tau}{\sqrt{2\pi\tau}} \times \frac{\sin((\omega - \omega_0)\tau/2)}{(\omega - \omega_0)\tau/2}$$

$$\Rightarrow \boxed{A(\omega) = \sqrt{\frac{\tau}{2\pi}} \cdot \frac{\sin((\omega - \omega_0)\tau/2)}{(\omega - \omega_0)\tau/2}} \quad C = \sqrt{\frac{\tau}{2\pi}}$$

$$3) |A(\omega)|^2 = \frac{\gamma}{2\pi} \sin^2((\omega - \omega_0)\gamma/2)$$

$$|A(\omega)|^2 = 0 \Leftrightarrow \sin^2((\omega - \omega_0)\gamma/2) = 0$$

$$\Leftrightarrow (\omega - \omega_0)\gamma/2 = n\pi$$

$$\Leftrightarrow \omega - \omega_0 = \frac{n\pi \cdot 2}{\gamma}$$

$$\Leftrightarrow \boxed{\omega_0 = \frac{-n\pi \cdot 2}{\gamma} + \omega}$$