

Exo 8 TAI MQ

1. Schrödinger : $\hat{H} \phi(x) = E \phi(x)$

$$\Leftrightarrow -\frac{\hbar^2}{2m} \phi''(x) + V(x) \phi(x) = E \phi(x) \quad (1)$$

Pour $0 \leq x < \infty$:

$$\phi'(x) = -Na x e^{-ax} + N e^{-ax}$$

$$\begin{aligned} \phi''(x) &= Na^2 x e^{-ax} - Na e^{-ax} - Na e^{-ax} \\ &= Na^2 x e^{-ax} - 2Na e^{-ax} \end{aligned}$$

$$(1) \Rightarrow -\frac{\hbar^2}{2m} (Na^2 x e^{-ax} - 2Na e^{-ax}) + V(x) N x e^{-ax} = -\frac{\hbar^2 a^2}{2m} N x e^{-ax}$$

$$\Leftrightarrow -\frac{\hbar^2}{2m} (-2Na e^{-ax}) + V(x) N x e^{-ax} = 0$$

$$\Leftrightarrow \frac{\hbar^2 a}{m} + V(x) x = 0$$

$$\Leftrightarrow \boxed{V(x) = -\frac{\hbar^2 a}{m x}}$$

2. $\psi(x, t) = \phi(x) e^{-i\omega t}$

$$\text{et } P(x) = \psi^* \times \psi$$

$$\Rightarrow P(x) = \phi(x) e^{i\omega t} \times \phi(x) e^{-i\omega t} = \phi^2(x)$$

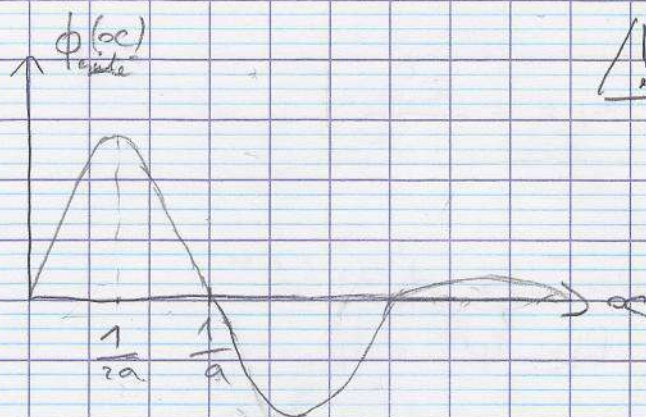
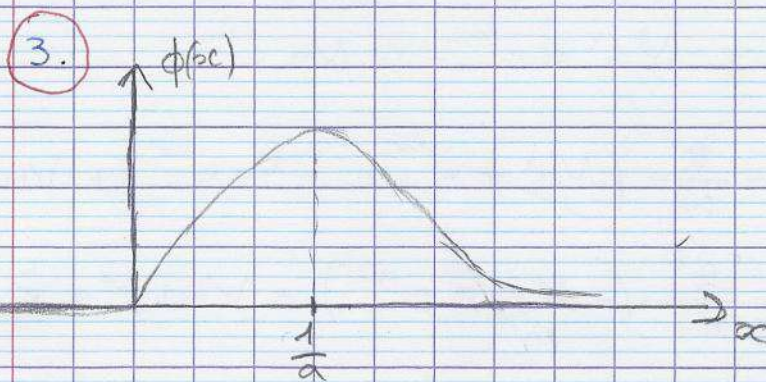
$$P(x) = \phi^2(x) = N^2 x^2 e^{-2ax}$$

$$P'(x) = N^2 (-2a x^2 e^{-2ax} + 2x e^{-2ax})$$

$$P'(x) = \underbrace{2 N^2 e^{-2ax}}_{>0} (-ax + 1) x$$

x	0	$\frac{1}{a}$	∞
$P'(x)$	0	+	-
$P(x)$			

$$\Rightarrow x_{\text{max}} = \frac{1}{a}$$



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