

TAI EXO 9:

$$1) \langle E \rangle = \int_{-\infty}^{+\infty} \Psi^* \hat{H} \Psi dx$$

$$\langle E \rangle = \int_{-\infty}^{+\infty} \left(A \phi_1(x) e^{i \frac{E_1}{\hbar} t} + B \phi_2(x) e^{i \frac{E_2}{\hbar} t} \right) \left(\hat{H} A \phi_1(x) e^{-i \frac{E_1}{\hbar} t} + \hat{H} B \phi_2(x) e^{-i \frac{E_2}{\hbar} t} \right) dx$$

$$\langle E \rangle = \int_{-\infty}^{+\infty} \left(A E_1 \phi_1(x) e^{-i \frac{E_1}{\hbar} t} + B E_2 \phi_2(x) e^{-i \frac{E_2}{\hbar} t} \right) dx$$

$$\langle E \rangle = \int_{-\infty}^{+\infty} E_1 |A|^2 |\phi_1(x)|^2 + E_2 |B|^2 |\phi_2(x)|^2 dx$$

$$\langle E \rangle = |A|^2 E_1 \int |\phi_1(x)|^2 + E_2 |B|^2 \int |\phi_2(x)|^2 dx$$

$$\langle E \rangle = E_1 |A|^2 + E_2 |B|^2$$

$$\text{Or } \langle E \rangle = \frac{1}{4} E_1 + \frac{3}{4} E_2 \Rightarrow \text{par identification } |A|^2 = \frac{1}{4} \quad |B|^2 = \frac{3}{4}$$

$$\Rightarrow A = \frac{1}{2} \quad B = \frac{\sqrt{3}}{2}$$

$$\text{D'où } \Psi(x,t) = \frac{1}{2} \phi_1(x,t) + \frac{\sqrt{3}}{2} \phi_2(x,t)$$

$$2) \Delta E = \sqrt{\langle E^2 \rangle - \langle E \rangle^2}$$

$$= \sqrt{|A|^2 E_1^2 + |B|^2 E_2^2 - (|A|^2 E_1 + |B|^2 E_2)^2}$$

$$= \left(|A|^2 E_1^2 + |B|^2 E_2^2 - |A|^4 E_1^2 - 2|A|^2 |B|^2 E_1 E_2 - |B|^4 E_2^2 \right)^{1/2}$$

$$= \left(\frac{1}{4} E_1^2 + \frac{3}{4} E_2^2 - \frac{1}{16} E_1^2 - \frac{9}{16} E_2^2 - \frac{3}{8} E_1 E_2 \right)^{1/2}$$

$$= \left(\frac{3}{16} E_1^2 + \frac{3}{16} E_2^2 - \frac{6}{16} E_1 E_2 \right)^{1/2}$$

$$= \left(\frac{3}{16} [E_1^2 + E_2^2 - 2E_1 E_2] \right)^{1/2}$$

$$= \left(\frac{3}{16} (E_1 - E_2)^2 \right)^{1/2} = \boxed{\frac{\sqrt{3}}{4} (E_1 - E_2)}$$

$$\begin{aligned}
 3) |\Psi|^2 &= \Psi^*(x,t) \Psi(x,t) \\
 \Psi(x,t) &= \frac{1}{2} \phi_1(x) e^{-i \frac{E_1}{\hbar} t} + \frac{\sqrt{3}}{2} \phi_2(x) e^{-i \frac{E_2}{\hbar} t} \\
 &= \left(\frac{1}{2} \phi_1(x) e^{i \frac{E_1}{\hbar} t} + \frac{\sqrt{3}}{2} \phi_2(x) e^{i \frac{E_2}{\hbar} t} \right) \left(\frac{1}{2} \phi_1(x) e^{-i \frac{E_1}{\hbar} t} + \frac{\sqrt{3}}{2} \phi_2(x) e^{-i \frac{E_2}{\hbar} t} \right) \\
 &= \frac{1}{4} |\phi_1(x)|^2 + \frac{\sqrt{3}}{4} \phi_1(x) \phi_2(x) e^{i \frac{(E_1 - E_2)}{\hbar} t} + \frac{\sqrt{3}}{4} \phi_2(x) \phi_1(x) e^{-i \frac{(E_1 - E_2)}{\hbar} t} + \frac{3}{4} |\phi_2(x)|^2 \\
 &= \frac{1}{4} |\phi_1(x)|^2 + \frac{3}{4} |\phi_2(x)|^2 + \frac{\sqrt{3}}{4} \phi_1(x) \phi_2(x) \left[e^{i \frac{(E_1 - E_2)}{\hbar} t} + e^{-i \frac{(E_1 - E_2)}{\hbar} t} \right] \\
 &= \frac{1}{4} |\phi_1(x)|^2 + \frac{3}{4} |\phi_2(x)|^2 + \frac{\sqrt{3}}{2} \phi_1(x) \phi_2(x) \cos \left(\frac{(E_1 - E_2)}{\hbar} t \right)
 \end{aligned}$$

$|\Psi|^2$ oscille à la pulsation $\frac{(E_1 - E_2)}{\hbar}$

$$\omega = \frac{(E_1 - E_2)}{\hbar} = 2\pi \nu \Rightarrow \frac{(E_1 - E_2)}{\hbar} = \frac{2\pi}{T} \Rightarrow \frac{\Delta E}{\hbar} = \frac{2\pi}{\Delta t}$$

$$\Rightarrow \boxed{\Delta E \Delta t = 2\pi \hbar = h}$$