

Exercice n°3 =

TA1 : Mécanique Quantique

$$1) \frac{mv^2}{r} = \frac{e^2}{4\pi\epsilon_0 r^2} \Rightarrow \frac{4\pi\epsilon_0}{e^2} = \frac{r}{r^2 m v^2}$$

$$\Rightarrow \frac{4\pi\epsilon_0}{e^2} = \frac{1}{r m v^2}$$

$$r \gg a_0 \Rightarrow r \gg \frac{4\pi\epsilon_0 \hbar^2}{e^2 m_e}$$

$$\Rightarrow r \gg \frac{\hbar^2}{r m v^2 m_e}$$

$$\Rightarrow r^2 m^2 v^2 \gg \hbar^2 \quad (\text{car } m_e = m)$$



$$r m v \gg \hbar$$

Or $L = mvr$ d'où $L \gg \hbar$

2) Incertitude minimum :

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

ou $\Delta x \overset{\text{après interaction}}{\approx} a \Rightarrow$

$$\Delta p \geq \frac{\hbar}{2a}$$

De plus : $E_c = \frac{p^2}{2m_e} = \frac{\hbar^2}{8a^2 m_e} = E_c$

(car $p = \sqrt{2E_c m}$)

impulsion : $p = mv \Rightarrow$

$$v = \frac{p}{m} \Rightarrow$$

$$v = \frac{\hbar}{2am}$$

3) $\lambda = 97,24 \text{ nm}$

$\begin{array}{c} \text{--- } E_n \\ \uparrow \Delta E_m \\ \text{--- } E_0 \end{array}$

Transition électrique de H :

$$E_m = -\frac{E_0}{n^2}$$

$$\Delta E = \frac{hc}{\lambda}$$

$$\Delta E = E_m - E_0 = -\frac{E_0}{n^2} + \frac{E_0}{1^2} = E_0 \left(1 - \frac{1}{n^2}\right)$$

$$\Delta E = 12,76 \text{ eV}$$
$$E_0 = 13,6 \text{ eV}$$



$$\frac{\Delta E}{E_0} = 1 - \frac{1}{n^2}$$

$$\frac{\Delta E}{E_0} - 1 = -\frac{1}{n^2} \Rightarrow -\frac{1}{\frac{\Delta E}{E_0} - 1} = n^2 \Rightarrow n^2 = 16$$
$$\Rightarrow \boxed{n=4}$$

Exercice n°4

TAF de Mécanique Quantique

1. ψ est solution de l'équation de Schrödinger ssi $-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x,t) = i\hbar \frac{\partial}{\partial t} \psi(x,t)$

$$\psi_A(x,t) = A \cos(kx - \omega t)$$

$$\begin{aligned} \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi_A(x,t) &= -\frac{A\hbar^2}{2m} \frac{\partial}{\partial x} (-k \sin(kx - \omega t)) \\ &= \frac{A\hbar^2 k^2}{2m} \frac{\partial}{\partial x^2} \cos(kx - \omega t) \end{aligned}$$

$$\left(= \frac{\hbar^2 k^2}{2m} \frac{\partial}{\partial x^2} \psi_A(x,t) \right)$$

On sait que $\boxed{\omega = \frac{E}{\hbar}}$ et $\boxed{k = \frac{p}{\hbar}}$ et $\boxed{E_c = \frac{p^2}{2m}}$ et $\boxed{E = E_c + V = E_c}$

$$\Rightarrow \frac{A\hbar^2 \frac{p^2}{\hbar^2}}{2m} \cos(kx - \omega t) = \frac{A p^2}{2m} \cos(kx - \omega t)$$

↑ cas particule libre

$$= \underline{A E_c \cos(kx - \omega t)} \quad (1)$$

$$i\hbar \frac{\partial}{\partial t} \psi_A(x,t) = +i\hbar A \omega \sin(kx - \omega t) = \underline{i A E_c \sin(kx - \omega t)} \quad (2)$$

$(1) \neq (2) \Rightarrow \psi_A(x,t)$ ne vérifie pas l'équation de Schrödinger

$$\psi_B(x,t) = B \sin(kx - \omega t)$$

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi_B(x,t) = -\frac{B\hbar^2 k^2}{2m} \frac{\partial}{\partial x} (\cos(kx - \omega t)) = \underline{\frac{B\hbar^2 k^2}{2m} \frac{\partial}{\partial x^2} (\sin(kx - \omega t))}$$

$$= \frac{B p^2}{2m} \sin(kx - \omega t) = B E_c \sin(kx - \omega t) \quad (1)$$

$$i\hbar \frac{\partial}{\partial t} \psi_B(x,t) = -i\hbar B \omega \cos(kx - \omega t) = -i B E_c \cos(kx - \omega t) \quad (2)$$

$(1) \neq (2) \Rightarrow \psi_B(x,t)$ ne vérifie pas l'équation de Schrödinger

2) Soit une autre planète $\psi(x,t) = A \exp\left(-\frac{i}{\hbar}(px - Et)\right)$ donc il faut que

$$X \frac{\partial^2}{\partial x^2} \psi(x,t) = Y \frac{\partial}{\partial t} \psi(x,t)$$

$$\Leftrightarrow A X \frac{\partial}{\partial x} \left(-\frac{i}{\hbar} \exp\left(\frac{i}{\hbar}(px - Et)\right) \right) = Y A \times \left(\frac{iE}{\hbar} \exp(px - Et) \right)$$

$$\Leftrightarrow -A X \frac{p^2}{\hbar^2} \exp(px - Et) = Y A \frac{iE}{\hbar} \exp(px - Et)$$

$$\Leftrightarrow -X \frac{p^2}{\hbar^2} = Y \frac{E i}{\hbar} \quad (E = E_c \text{ car particule libre})$$

Si on remplace X par $-\frac{\hbar^2}{2m}$ et Y par $i\hbar$ on obtient:

$$\frac{p^2}{2m} = -E \Rightarrow \text{il faut } \boxed{X = -\frac{\hbar^2}{2m} \text{ et } Y = -i\hbar}$$

$$\Rightarrow -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x,t) = -i\hbar \frac{\partial}{\partial t} \psi(x,t)$$

$$\Rightarrow \boxed{\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x,t) = i\hbar \frac{\partial}{\partial t} (\psi(x,t))}$$

Exercice n°4 (suite)TAI de Mécanique Quantique

$$3. \psi(x,t) = A e^{i(kx - \omega t)} \Rightarrow \frac{\partial}{\partial x} \psi(x,t) = A i k e^{i(kx - \omega t)}$$

$$\Downarrow$$
$$\frac{\partial}{\partial x^2} \psi(x,t) = A i^2 k^2 e^{i(kx - \omega t)}$$
$$= \underline{-A k^2 e^{i(kx - \omega t)}}$$

$$\Rightarrow \frac{\partial}{\partial t} \psi(x,t) = -A i \omega e^{i(kx - \omega t)}$$

$$\Downarrow$$
$$\frac{\partial}{\partial t^2} \psi(x,t) = +A i^2 \omega^2 e^{i(kx - \omega t)}$$
$$= \underline{-A \omega^2 e^{i(kx - \omega t)}}$$

$$\Rightarrow -A \omega^2 e^{i(kx - \omega t)} + c^2 A k^2 e^{i(kx - \omega t)} + \frac{m^2 c^4}{\hbar^2} A e^{i(kx - \omega t)}$$

$$= A e^{i(kx - \omega t)} \left(-\omega^2 + c^2 k^2 + \frac{m^2 c^4}{\hbar^2} \right)$$

$$= A e^{i(kx - \omega t)} \left(-\frac{E^2}{\hbar^2} + c^2 \frac{p^2}{\hbar^2} + \frac{m^2 c^4}{\hbar^2} \right)$$

$$= \frac{1}{\hbar^2} A e^{i(kx - \omega t)} \left(c^2 p^2 + m^2 c^4 - E^2 \right) = 0$$

$$\Rightarrow E^2 = c^2 p^2 + m^2 c^4 \Rightarrow \boxed{E = c p + m c^2}$$

finir la 1^{ère}
partie de la
question

TAI : Mécanique Quantique

Exercice n°5

$$1. \int_0^a |\psi(x)|^2 dx = 1 \Rightarrow \int_0^a |Nx(a-x)|^2 dx = 1$$

$$\Rightarrow |N|^2 \int_0^a |xa - x^2|^2 dx = 1$$

$$\Rightarrow |N|^2 \int_0^a (xa - x^2)^2 dx = 1$$

$$\Rightarrow |N|^2 \int_0^a (x^2 a^2 - 2ax^3 + x^4) dx = 1$$

$$\Rightarrow |N|^2 \left[\frac{x^3 a^2}{3} - \frac{2ax^4}{4} + \frac{x^5}{5} \right]_0^a = 1$$

$$\Rightarrow |N|^2 \left(\frac{a^5}{3} - \frac{a^5}{2} + \frac{a^5}{5} \right) = 1$$

$$\Rightarrow |N|^2 \frac{a^5}{30} = 1$$

$$\Rightarrow N^2 = \frac{30}{a^5} \Rightarrow \boxed{N = \sqrt{\frac{30}{a^5}}}$$

2. Position moyenne : $\langle x \rangle = \int_0^a x |\psi(x)|^2 dx = \int_0^a x |Nx(a-x)|^2 dx$

$$\langle x \rangle = |N|^2 \int_0^a x^3 (a^2 - 2ax - x^2) dx = |N|^2 \left[\frac{x^4 a^2}{4} - \frac{2ax^5}{5} - \frac{x^6}{6} \right]_0^a$$

$$\langle x \rangle = |N|^2 \left(\frac{a^6}{4} - \frac{2a^6}{5} - \frac{a^6}{6} \right) \Rightarrow \langle x \rangle = |N|^2 \frac{a^6}{60} \Rightarrow \boxed{\langle x \rangle = \frac{a}{2}}$$

impulsion moyenne :

$$\langle p \rangle = \int_{-\infty}^{\infty} \psi^*(x) \left| -\frac{\hbar}{i} \right| \frac{\partial \psi}{\partial x} dx$$

$$= \int_0^a (N x(a-x)) \left| -\frac{\hbar}{i} \right| (-N(x-a) - N x) dx$$

$$= -\frac{\hbar}{i} \int_0^a N^2 x(a-x)(lx-a) dx$$

$$\boxed{\langle p \rangle = N^2 \frac{\hbar}{i} [lx^2 a - lx^3]_0^a = 0}$$

3. Incertitude de position

$$\langle x^2 \rangle = \int_0^a \psi^*(x) \hat{x}^2 \psi(x) dx \quad \text{avec} \quad \boxed{\hat{x}^2 = x^2}$$

$$= N^2 \int_0^a x^2 (a-x)^2 x^2 dx = N^2 \int_0^a x^4 (a-x)^2 dx$$

$$= N^2 \int_0^a x^4 a^2 - 2ax^5 + x^6 dx$$

$$= N^2 \left[\frac{x^5 a^2}{5} - \frac{2ax^6}{6} + \frac{x^7}{7} \right]_0^a$$

$$= N^2 \left(\frac{a^7}{5} - \frac{2a^7}{6} + \frac{a^7}{7} \right)$$

$$= \frac{30}{a^5} \times \frac{a^7}{105} = \boxed{\frac{2a^2}{7} = \langle x^2 \rangle}$$

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{2a^2}{7} - \frac{a^2}{4}} = \sqrt{\frac{8a^2 - 7a^2}{28}}$$

$$\Delta x = \sqrt{\frac{a^2}{28}}$$

$$\boxed{\Delta x = \frac{a}{2\sqrt{7}}}$$

Exercice n°5 (suite)

TA1 : Mécanique Quantique

Incertitude d'impulsion: $\langle p^2 \rangle = \int_0^a \psi^*(x,t) \hat{p}^2 \psi(x,t) dx$

$$= \int_0^a (Nx(a-x)) \hat{p}^2 \psi(x,t) dx$$

$$\hat{p}^2 \psi(x,t) = \left(-i\hbar \frac{\partial}{\partial x}\right)^2 \psi(x,t) = -\hbar^2 \frac{\partial^2}{\partial x^2} \psi(x,t)$$

$$\text{or } \frac{\partial}{\partial x} \psi(x,t) = -N(a-x)$$

$$\Rightarrow \hat{p}^2 \psi(x,t) = \hbar^2 x(-N)$$

$$\hookrightarrow \langle p^2 \rangle = \hbar^2 N^2 \int_0^a x(a-x) dx = \hbar^2 N^2 \int_0^a -x^2 + ax dx$$

$$= \hbar^2 N^2 \left[-\frac{x^3}{3} + \frac{ax^2}{2} \right]_0^a$$

$$= \hbar^2 N^2 \left(-\frac{a^3}{3} + \frac{a^3}{2} \right) = \hbar^2 N^2 \frac{a^3}{6}$$

$$= \hbar^2 \frac{30}{a^5} \cdot \frac{a^3}{6} = \hbar^2 \frac{10}{a^2}$$

$$\Rightarrow \Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$$

$$\Delta p = \sqrt{\hbar^2 \frac{10}{a^2}}$$

$$\boxed{\Delta p = \frac{\hbar}{a} \sqrt{10}}$$

TAI : Mécanique Quantique

Exercice n°6

Soit $\psi(x) = \begin{cases} Nxe^{-\alpha x/2} & \text{si } 0 \leq x \leq \infty \\ 0 & \text{sinon} \end{cases}$

$\left(\begin{array}{l} \alpha \in \mathbb{R}^+ \\ N = \sqrt{\alpha^3/2} \end{array} \right)$

1) Proba de trouver la particule dans l'espace $dx =$

$$dP(x) = |\psi(x)|^2 dx$$

Proba entre x et $x+dx$ =

$$P(x) = \int_x^{x+dx} dP(x)$$

Soit $dP(x) = |\psi(x)|^2$

$$= \psi^*(x) \wedge \psi(x)$$

$$= N^2 x^2 e^{-\alpha x}$$

$$= \frac{\alpha^3}{2} (x^2 e^{-\alpha x})$$

ici $\psi^*(x) = \psi(x)$

$\rightarrow \psi^*(x) \wedge \psi(x) = \psi^2(x)$

• $\langle x \rangle = \int_0^\infty x |\psi(x)|^2 dx$

$$= \frac{\alpha^3}{2} \int_0^\infty x^3 e^{-\alpha x} dx$$

$$= \frac{\alpha^3}{2} \frac{3!}{\alpha^4}$$

$\langle x \rangle = \frac{3}{\alpha}$

$\langle x^2 \rangle = \int_0^\infty x^2 |\psi(x)|^2 dx$

$$= \frac{\alpha^3}{2} \int_0^\infty x^4 e^{-\alpha x} dx$$

$$= \frac{\alpha^3}{2} \frac{4!}{\alpha^5}$$

$\langle x^2 \rangle = \frac{12}{\alpha^2}$

$$\begin{aligned}
2. \langle p \rangle &= i\hbar \int_0^{\infty} \left(\psi^* \frac{d\psi}{dx} \right) dx = i\hbar \int_0^{\infty} \psi^* \frac{d}{dx} \left(N x e^{-\frac{\alpha x}{2}} \right) dx \\
&= i\hbar \int_0^{\infty} \left(\psi^* x \left[N x e^{-\frac{\alpha x}{2}} - \frac{\alpha x}{2} e^{-\frac{\alpha x}{2}} \right] \right) dx \\
&= i\hbar \int_0^{\infty} \left(\psi^* x \left[N x e^{-\frac{\alpha x}{2}} \left(1 - \frac{\alpha x}{2} \right) \right] \right) dx \\
&= i\hbar \int_0^{\infty} \left(N^2 x x e^{-\alpha x} \left(1 - \frac{\alpha}{2} x \right) \right) dx \\
&= i\hbar N^2 \int_0^{\infty} \left(x e^{-\alpha x} - \frac{\alpha}{2} x^2 e^{-\alpha x} \right) dx \\
&= i\hbar N^2 \left(\int_0^{\infty} x e^{-\alpha x} - \frac{\alpha}{2} \int_0^{\infty} x^2 e^{-\alpha x} \right) dx \\
&= i\hbar N^2 \left(\frac{1}{\alpha^2} - \frac{2\alpha}{2\alpha^3} \right) = 0
\end{aligned}$$

$$\begin{aligned}
\langle p^2 \rangle &= \int_0^{\infty} \psi^* \hat{p}^2 \psi(x) dx = \int_0^{\infty} \left[N x e^{-\frac{\alpha x}{2}} \left(-\hbar^2 \frac{\partial^2}{\partial x^2} \right) \left(N x e^{-\frac{\alpha x}{2}} \right) \right] dx \\
&= \int_0^{\infty} \left[\psi^* \left(-\hbar^2 \right) N x \frac{d}{dx} \left(e^{-\frac{\alpha x}{2}} \left(1 - \frac{\alpha}{2} x \right) \right) \right] dx \\
&= \int_0^{\infty} \left[\psi^* \left(-\hbar^2 \right) N \frac{d}{dx} \left(e^{-\frac{\alpha x}{2}} - \frac{\alpha}{2} x e^{-\frac{\alpha x}{2}} \right) \right] dx \\
&= \int_0^{\infty} \left[\psi^* \left(-\hbar^2 \right) N \left[-\frac{\alpha}{2} e^{-\frac{\alpha x}{2}} - \frac{\alpha}{2} \left(e^{-\frac{\alpha x}{2}} - \frac{\alpha}{2} x e^{-\frac{\alpha x}{2}} \right) \right] \right] dx \\
&= \int_0^{\infty} \left[\psi^* \left(-\hbar^2 \right) N \left[-\alpha e^{-\frac{\alpha x}{2}} + \frac{\alpha^2}{4} x e^{-\frac{\alpha x}{2}} \right] \right] dx
\end{aligned}$$

Exercice n°6 (suite)

TA1 : Mécanique Quantique.

$$\begin{aligned}
\langle p^2 \rangle &= \int_0^{\infty} [\Psi^* (-\hbar^2) N \left[e^{-\frac{\alpha x}{2}} \left(-\alpha + \frac{\alpha^2}{4} x \right) \right]] dx \\
&= (-\hbar^2)^2 N^2 \int_0^{\infty} (-\alpha) e^{-\frac{\alpha x}{2}} + \frac{\alpha^2}{4} x e^{-\frac{\alpha x}{2}} dx \\
&= (-\hbar^2)^2 N^2 \left(-\alpha \int_0^{\infty} e^{-\frac{\alpha x}{2}} dx + \frac{\alpha^2}{4} \int_0^{\infty} x e^{-\frac{\alpha x}{2}} dx \right) \\
&= -\hbar^2 N^2 \left(-\frac{\alpha}{\alpha^2} + \frac{\alpha^2}{4} \left(\frac{2!}{\alpha^3} \right) \right) = -\hbar^2 N^2 \left(-\frac{1}{\alpha} + \frac{1}{2\alpha} \right) \\
&= -\hbar^2 N^2 \left(-\frac{1}{2\alpha} \right) \\
&= -\hbar^2 \frac{\alpha^3}{2} \left(-\frac{1}{2\alpha} \right)
\end{aligned}$$

$$\boxed{\langle p^2 \rangle = \hbar^2 \frac{\alpha^2}{4}}$$

$$3. \Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{12}{\alpha^2} - \frac{9}{\alpha^2}} = \sqrt{\frac{3}{\alpha^2}} = \frac{\sqrt{3}}{\alpha}$$

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \sqrt{\hbar^2 \frac{\alpha^2}{4}} = \hbar \frac{\alpha}{2}$$

$$\Delta x \Delta p = \frac{\hbar \sqrt{3}}{2} \geq \frac{1}{2} \hbar \Rightarrow \boxed{\text{principe d'Heisenberg respecté}}$$

TA1 : Mécanique Quantique

Exercice n°8:

$$E = -\frac{\hbar^2 \alpha^2}{2m}$$

$$\phi(x) = \begin{cases} N x e^{-\alpha x} & \text{si } 0 \leq x < \infty \\ 0 & \text{sinon} \end{cases}$$

N et α des réels > 0

1. Equation de Schrödinger:

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x,t) + V(x,t) \psi(x,t) = i\hbar \frac{\partial}{\partial t} \psi(x,t)$$

$$\Rightarrow E \phi(x) = -\frac{\hbar^2 \alpha^2}{2m} N x e^{-\alpha x} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} N x e^{-\alpha x} + V(x) N x e^{-\alpha x}$$

$$\Rightarrow V(x) = \frac{-\frac{\hbar^2}{2m} N x e^{-\alpha x} + \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} N x e^{-\alpha x}}{N x e^{-\alpha x}}$$

$$\frac{\partial}{\partial x} N x e^{-\alpha x} = N \frac{\partial}{\partial x} x e^{-\alpha x}$$

$$\begin{aligned} u(x) &= x & u'(x) &= 1 \\ v(x) &= e^{-\alpha x} & v'(x) &= -\alpha e^{-\alpha x} \end{aligned}$$

$$= N (e^{-\alpha x} + x(-\alpha) e^{-\alpha x})$$

$$= N e^{-\alpha x} - \alpha x N e^{-\alpha x} = N e^{-\alpha x} (1 - \alpha x)$$

$$\frac{\partial}{\partial x} (N e^{-\alpha x} (1 - \alpha x)) = N \frac{\partial}{\partial x} e^{-\alpha x} - \alpha N \frac{\partial}{\partial x} x e^{-\alpha x}$$

$$= -N \alpha e^{-\alpha x} - \alpha N e^{-\alpha x} (1 - \alpha x)$$

$$= -\alpha N e^{-\alpha x} (-1 + \alpha x - 1)$$

$$= \alpha N e^{-\alpha x} (\alpha x - 2)$$

$$\Rightarrow V(x) = \frac{-\frac{\hbar^2 \alpha^2}{2m} N x e^{-\alpha x} + \frac{\hbar^2}{2m} \alpha N e^{-\alpha x} (\alpha x - 2)}{N x e^{-\alpha x}}$$

$$V(x) = \frac{N e^{-\alpha x} \left(-\frac{\hbar^2 \alpha^2}{2m} x + \frac{\hbar^2}{2m} \alpha (\alpha x - 2) \right)}{N x e^{-\alpha x}}$$

$$V(x) = \frac{-\hbar^2 \alpha^2 x + \hbar^2 \alpha (\alpha x - 2)}{2m x}$$

$$V(x) = \frac{-\hbar^2 \alpha^2 x + \hbar^2 \alpha^2 x - 2\hbar^2 \alpha}{2m x}$$

$$\boxed{V(x) = -\frac{\hbar^2 \alpha}{m x}}$$

$$2. dP(x) = |\psi(x)|^2 dx$$

$$= |N x e^{-\alpha x}|^2 dx$$

$$= N^2 x^2 e^{-2\alpha x} dx$$

$$U(x) = x^2$$

$$U'(x) = 2x$$

$$V(x) = e^{-2\alpha x}$$

$$V'(x) = -2\alpha e^{-2\alpha x}$$

$$= N^2 (2x e^{-2\alpha x} - 2\alpha x^2 e^{-2\alpha x})$$

$$= N^2 2 e^{-2\alpha x} (x - \alpha x^2)$$

$$\Rightarrow dP(x) = 0 \Rightarrow \begin{cases} 2N^2 e^{-2\alpha x} = 0 \\ x - \alpha x^2 = 0 \end{cases} \Rightarrow \begin{cases} x=0 \\ 1 - \alpha x = 0 \end{cases} \Rightarrow \begin{cases} x=0 \\ x = \frac{1}{\alpha} \end{cases}$$

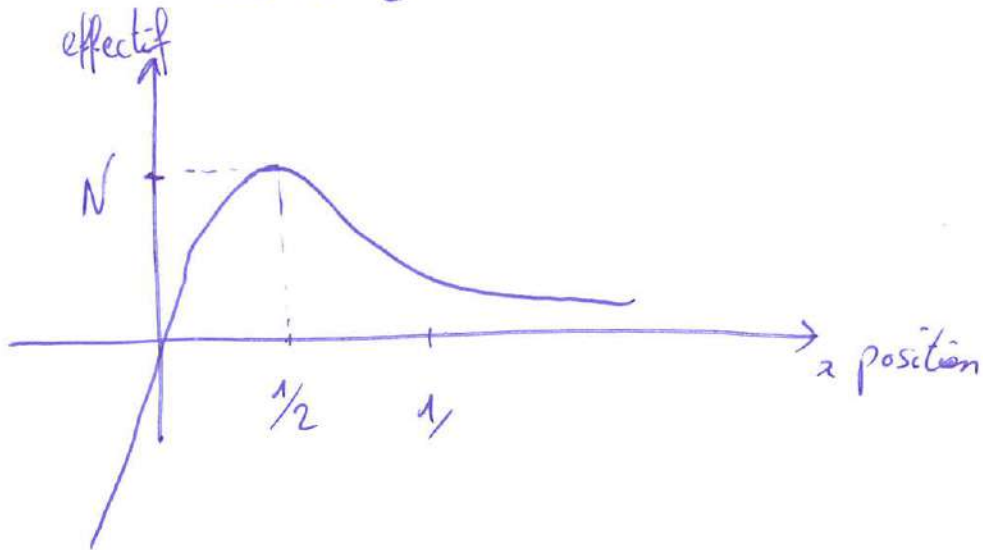
Exercice n°8 (suite)

TAI : Mécanique Quantique

$$\Rightarrow x_{\max} = \frac{1}{\alpha}$$

3. Pour $\alpha = 1$: $x_{\max} = \frac{1}{1} = 1$

Pour $\alpha = 2$: $x_{\max} = \frac{1}{2}$



TA1 : Mécanique Quantique

Exercice n°10

$$V(x) = \frac{1}{2} m \omega^2 x^2 \quad \psi(x, t) = \frac{1}{\sqrt{2}} \left[\phi_0(x) e^{-\frac{iE_0 t}{\hbar}} + \phi_1(x) e^{-\frac{iE_1 t}{\hbar}} \right]$$

$$E_n = \left(n + \frac{1}{2}\right) \hbar \omega$$

1. $E_0 = \frac{1}{2} \hbar \omega$; $E_1 = \frac{3}{2} \hbar \omega \Rightarrow$

$$\boxed{\psi(x, t) = \frac{1}{\sqrt{2}} \left[\phi_0(x) e^{-\frac{i\omega t}{2}} + \phi_1(x) e^{-\frac{3i\omega t}{2}} \right]}$$

$$\psi(x, t) \psi^*(x, t) = \frac{1}{2} \left[\phi_0(x) e^{-\frac{i\omega t}{2}} + \phi_1(x) e^{-\frac{3i\omega t}{2}} \right] \left[\phi_0^*(x) e^{\frac{i\omega t}{2}} + \phi_1^*(x) e^{\frac{3i\omega t}{2}} \right]$$

Or ϕ_0 et ϕ_1 sont réelles $\Rightarrow$

$$= \frac{1}{2} \left[\phi_0(x) e^{-\frac{i\omega t}{2}} + \phi_1(x) e^{-\frac{3i\omega t}{2}} \right] \left[\phi_0(x) e^{\frac{i\omega t}{2}} + \phi_1(x) e^{\frac{3i\omega t}{2}} \right]$$

$$= \frac{1}{2} \left[\phi_0^2(x) + \phi_0(x) \phi_1(x) e^{\frac{2i\omega t}{2}} + \phi_1(x) \phi_0(x) e^{-\frac{2i\omega t}{2}} + \phi_1^2(x) \right]$$

$$= \frac{1}{2} \left[\phi_0^2(x) + \phi_0(x) \phi_1(x) e^{i\omega t} + \phi_1(x) \phi_0(x) e^{-i\omega t} + \phi_1^2(x) \right]$$

$$= \frac{1}{2} \left[\phi_0^2(x) + \phi_1^2(x) + 2\phi_0(x) \phi_1(x) (e^{i\omega t} + e^{-i\omega t}) \right]$$

$$\boxed{\psi(x, t) \psi^*(x, t) = \frac{1}{2} \left[\phi_0^2(x) + \phi_1^2(x) + 2\phi_0(x) \phi_1(x) \cos(\omega t) \right]}$$

Pour montrer que cette fonction est normée, intégrons $\Psi(x,t) \Psi^*(x,t)$

$$\begin{aligned} \int_{-\infty}^{\infty} \Psi(x,t) \Psi^*(x,t) dx &= \int_{-\infty}^{\infty} \frac{1}{2} \left[\phi_0^2(x) + \phi_1^2(x) + 2\phi_0(x)\phi_1(x)\cos(\omega t) \right] dx \\ &= \frac{1}{2} \left(\underbrace{\int_{-\infty}^{\infty} \phi_0^2(x) dx}_{=1} + \underbrace{\int_{\mathbb{R}} \phi_1^2(x) dx}_{=1} + 2\cos(\omega t) \underbrace{\int_{\mathbb{R}} \phi_0(x)\phi_1(x) dx}_{=0 \text{ car } \phi_0 \perp \phi_1} \right) \\ &= \frac{1}{2} \times 2 = 1 \end{aligned}$$

$$\Rightarrow \boxed{\Psi(x,t) \text{ est normée}}$$

$$d. \langle x \rangle = \int_{\mathbb{R}} x \Psi^*(x,t) \Psi(x,t) dx$$

$$= \frac{1}{2} \left[\underbrace{\int_{\mathbb{R}} x \phi_0^2(x) dx}_{=0} + \underbrace{\int_{\mathbb{R}} x \phi_1^2(x) dx}_{=0} + 2\cos(\omega t) \int_{\mathbb{R}} x \phi_0(x)\phi_1(x) dx \right]$$

$$= \cos(\omega t) \underbrace{\int_{\mathbb{R}} x \phi_0(x)\phi_1(x) dx}_{=1}$$

$$\boxed{\langle x \rangle = A \cos(\omega t)}$$

finir la
p3