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Groupe B. DE de Probabilités

Exercice de cours :

a) Pour une loi uniforme sur un intervalle $[a; b]$:

$$E(X) = \frac{a+b}{2}$$

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$$E(X^2) = \int_a^b x^2 f(x) dx = \left[\frac{x^3}{3} \right]_a^b = \frac{b^3 - a^3}{3}$$

$f(x) = ?$

$$V(X) = E(X^2) - (E(X))^2$$

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$$= \frac{b^3 - a^3}{3} - \left(\frac{a+b}{2} \right)^2$$

$$= \frac{b^3 - 3b^2 - a^3 - 3a^2}{6}$$

$$b) \text{Var}(aX+bY) = E((aX+bY)^2) - [E(aX+bY)]^2$$

$$= E(a^2 X^2 + 2ab XY + b^2 Y^2) - [a E(X) + b E(Y)]^2$$

$$= a^2 E(X^2) + 2ab E(XY) + b^2 E(Y^2) - a^2 E(X)^2 - 2ab E(X)E(Y) - b^2 E(Y)^2$$

$$= a^2 V(X) + b^2 V(Y) + 2ab \text{Cov}(X, Y)$$

étant donné que $V(X) = E(X^2) - (E(X))^2$ et que $\text{Cov}(X, Y) = E(XY) - E(X)E(Y)$

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• Exercise 2

$$f(x) = \frac{c}{x^2+1}$$

$$a) \int_{-\infty}^{+\infty} f(x) dx = 1.$$

$$\int_{-\infty}^{+\infty} \frac{c}{x^2+1} dx = 1.$$

$$c \int_{-\infty}^{+\infty} \frac{1}{x^2+1} dx = 1.$$

$$c \lim_{M \rightarrow \infty} \left[\arctan x \right]_{-M}^M = 1.$$

$$c \left[+\frac{\pi}{2} - \left(-\frac{\pi}{2}\right) \right] = 1.$$

$$c \pi = 1.$$

$$c = \frac{1}{\pi}$$

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$$F(x) = \int_{-\infty}^x f(x) dx.$$

$$= \frac{1}{\pi} \int_{-\infty}^x \frac{1}{x^2+1} dx = \frac{1}{\pi} \lim_{M \rightarrow \infty} \left[\arctan x \right]_{-M}^x$$

$$= \frac{1}{\pi} \arctan x + \frac{1}{2}$$

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b) calcul de $P(X > 2)$ et $P(-3 \leq X \leq 4)$.

$$P(X > 2) = 1 - P(X < 2) = \frac{1}{2} - \frac{1}{\pi} \arctan 2$$

$$-3 \leq X \leq 4 = X > -3 \text{ et } X < 4.$$

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$$P(-3 \leq X \leq 4) = P(X < -3 \wedge X < 4)$$

$$= P(X < -3) + P(X < 4)$$

$$= \frac{1}{\pi} (\arctan 3 + \arctan 4)$$

$$= \frac{1}{\pi} (\arctan 3 + \arctan 4)$$

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$$P(-3 \leq X \leq 4) = P(X < 4) - P(-3 < X)$$

$$= \frac{1}{\pi} (\arctan 4 - \arctan 3)$$

c) $P\left(\frac{1}{3} \leq X^2 \leq 1\right)$?

$$\frac{1}{3} \leq X^2 \leq 1 \Leftrightarrow \begin{cases} \frac{\sqrt{3}}{3} < X < 1 \\ -1 < X < -\frac{\sqrt{3}}{3} \end{cases}$$

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$$* \frac{\sqrt{3}}{3} < X < 1.$$

$$P\left(\frac{\sqrt{3}}{3} < X < 1\right) = -P\left(X > \frac{\sqrt{3}}{3}\right) + P(X < 1) = -\frac{1}{\pi} \arctan\left(\frac{\sqrt{3}}{3}\right) + \frac{1}{\pi} \arctan(-1)$$

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$$= -\frac{\pi}{6} \times \frac{1}{\pi} = -\frac{1}{6}$$

$$* -1 \leq X \leq -\frac{\sqrt{3}}{3}$$

$$P\left(-1 \leq X \leq -\frac{\sqrt{3}}{3}\right) = -P(X > -1) + P\left(X < -\frac{\sqrt{3}}{3}\right) = \dots$$

Exercice 3.

On est en présence de variable de Bernoulli, on va utiliser la loi Binomiale.

$$\begin{cases} X_i = 1 & \text{si defectueuse} \\ X_i = 0 & \text{sinon} \end{cases}$$

~~X~~

$$P(X=x) = \binom{N}{x} p^x (1-p)^{N-x}$$

p étant la probabilité de succès
ici, une vss normale.

$$P(X=1) = \binom{N}{1} p^1 (1-p)^{N-1} = Np (1-p)^{N-1}$$

~~$$= \frac{N!}{(N-1)!} p (1-p)^{N-1} = Np (1-p)^{N-1}$$~~

Attention!

Vous confondez
 n et N

~~$$E(X=1) = \sum_{x=0}^N x P(X=x)$$~~

~~$$= 1 \times P(X=1)$$~~

~~$$= 1 \times \frac{N!}{(N-1)!} p (1-p)^{N-1} = \frac{N(Np)(1-p)^{N-1}}{(1-p)^N}$$~~

~~$$V(X) = E(X^2) - (E(X))^2$$~~

~~$$E(X) = Np = T$$~~

~~$$V(X) = Np(1-p) = T(1-p)$$~~