

Exercice 1: $\vec{E}(x, t) = 2 \cdot 10^4 \cos\left(\frac{\pi}{3} 10^3 x - \omega t\right) \vec{e}_y$

① La propagation se fait sur la direction (Ox) car :

* $\vec{E}$ dépend de x .

* La phase montre que $\vec{k}$ a une composante : $\begin{pmatrix} k_x \\ 0 \\ 0 \end{pmatrix}$

On a $E(x, t) = E_0 \cdot \cos(k_x x - \omega t) \vec{e}_y$

D'où par identification

$$k_x = \frac{\pi}{3} \cdot 10^3 \rightarrow \vec{k} = \begin{pmatrix} \frac{\pi}{3} \cdot 10^3 \\ 0 \\ 0 \end{pmatrix} \quad \text{d'où } \boxed{k = \frac{\pi}{3} \cdot 10^3 \text{ rad/m}}$$

Calcul de λ :

$$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{k} = \boxed{6 \cdot 10^{-3} \text{ m}}$$

$$\text{Calcul de } T: c = \frac{\lambda}{T} \Rightarrow T = \frac{\lambda}{c} = \frac{6 \cdot 10^{-3}}{3 \cdot 10^8} = \boxed{2 \cdot 10^{-11} \text{ s}}$$

$$T = \frac{1}{f} \Rightarrow \boxed{f = 5 \cdot 10^{10} \text{ Hz}}$$

② On utilise la 4^e pte des ondes planes :

Lo ($\vec{E}$, $\vec{B}$, $\vec{k}$) trièdre direct.



Ondes planes sinusoidales $\Rightarrow E_0 = c_0 \cdot B_0$

$$\Rightarrow B_0 = \frac{E_0}{c_0} = \frac{2 \cdot 10^4}{3 \cdot 10^8} = \boxed{6,7 \cdot 10^{-5} \text{ Tola}}$$

$$\vec{B}(x, t) = \frac{\pi}{3} 10^3 \left(\frac{\pi}{3} 10^3 x - \omega t \right) \vec{e}_z$$

③ $\vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0}$

$$\vec{S} = \frac{1}{\mu_0} \begin{pmatrix} 0 \\ E_y \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ B_z \end{pmatrix} = \begin{pmatrix} E_y B_z \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\mu_0} E_0 B_0 \cos^2\left(\frac{\pi}{3} 10^3 x - \omega t\right) \vec{e}_x$$

$$S_0 = \frac{E_0 \cdot B_0}{\mu_0} = \frac{2 \cdot 10^4 \times 6,7 \cdot 10^{-5}}{4\pi \cdot 10^{-7}} = \boxed{1,06 \cdot 10^6 \text{ W/m}^2}$$

Exercice 2 $\vec{B}(y,t) = 10^{-3} \cos(ky - \omega t) \vec{e}_z$

- ① Propagation sur (Oy) .

k a une composante : $\begin{pmatrix} 0 \\ k_y \\ 0 \end{pmatrix}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0,1 \cdot 10^{-2}} = \boxed{6,28 \cdot 10^3 \text{ rad/m}}$$

$$\omega = kc = 6,28 \cdot 10^3 \times 3 \cdot 10^8 = \boxed{1,88 \cdot 10^{16} \text{ rad/s}}$$

- ② 4^e ptes $(\vec{E}, \vec{B}, \vec{k})$ trièdre direct. $\vec{E} \begin{pmatrix} E_x \\ 0 \\ 0 \end{pmatrix}$

$$E_0 = B_0 c = 10^{-3} \cdot 3 \cdot 10^8 = \boxed{3 \cdot 10^5 \text{ V/m}}$$

③ $\vec{S} = \frac{\vec{E}_m \wedge \vec{B}}{\mu_0}$

$$\vec{S} = \frac{1}{\mu_0} \begin{pmatrix} E_x \\ 0 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ B_z \end{pmatrix} = \frac{1}{\mu_0} \begin{pmatrix} 0 \\ E_x B_z \\ 0 \end{pmatrix}$$

$$S_0 = \frac{E_0 B_0}{\mu_0} = \frac{3 \cdot 10^5 \times 10^{-3}}{4\pi \cdot 10^{-7}} = \boxed{2,38 \times 10^8 \text{ W/m}^2}$$

Exercice 3 : $\vec{E}(x,t) = E_0 \cos(kx - \omega t) \vec{e}_y$

① $\Delta \vec{E} = \begin{pmatrix} \Delta E_x \\ \Delta E_y \\ \Delta E_z \end{pmatrix} = \frac{\partial^2 E_y}{\partial x^2} = \frac{-k^2 E_0 \sin(kx - \omega t)}{\partial x}$



$$= -k^2 E_0 \cos(kx - \omega t)$$

② $\frac{\partial^2 \vec{E}}{\partial t^2} = -\omega^2 E_0 \cos(kx - \omega t)$

③ D'après D'Alembert : $\Delta \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$

Ici : $-k^2 \vec{E}(x,t) + \mu_0 \epsilon_0 \omega^2 \vec{E}(x,t) = 0$

$$\Leftrightarrow \boxed{\omega^2 = \frac{k^2}{\mu_0 \epsilon_0}}$$

Exercice 4 : $\vec{E}(M, t) = E_0 \cos(kx - \omega t) \vec{e}_y$

$$\textcircled{1} \operatorname{div}(\vec{E}) = \vec{\nabla} \cdot \vec{E} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ E_0 \cos(kx - \omega t) \\ 0 \end{pmatrix} = \frac{E_0 \cos(kx - \omega t)}{\partial y} = 0$$

Confirme la 1^{re} eq de Maxwell : $\operatorname{div}(\vec{E}) = 0$

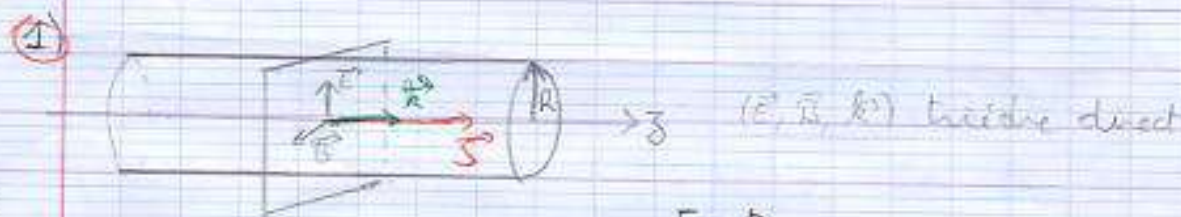
② 4^e ptes d'ondes planes : $(\vec{E}, \vec{B}, \vec{k})$ triade direct
 $\hookrightarrow \vec{B} \begin{pmatrix} 0 \\ 0 \\ B_y \end{pmatrix} = B_0 \cos(kx - \omega t) \vec{e}_y$

$$\textcircled{3} \operatorname{rot} \vec{E} = \vec{\nabla} \wedge \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow i k \wedge \vec{E} = - (-i \omega) \vec{B}$$

$$\Rightarrow k \wedge \vec{E} = \omega \vec{B} \Rightarrow \vec{B} = \frac{k \wedge \vec{E}}{\omega} = \frac{1}{\omega} \begin{pmatrix} k \\ 0 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ E \\ 0 \end{pmatrix} = \frac{k}{\omega} \cos(kx - \omega t) \vec{e}_y$$

Exercice 5 :



$$\textcircled{2} \vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} = \frac{E \cdot B \sin \pi/2}{\mu_0} = \frac{E}{\mu_0} \cdot \frac{E}{c} = \frac{E^2}{\mu_0 c} = \frac{B^2 c}{\mu_0}$$

③ $P_{\text{moy}} = \Phi(S) = \int_S \vec{S} \cdot d\vec{S} = S \cdot \pi \cdot R^2 = \frac{B^2 R^2 c}{\mu_0} \pi = \frac{E^2}{\mu_0 c} \pi R^2$
 $E = E_0 \cos(kx - \omega t)$
 $P_{\text{moy}} = \frac{E_0^2 R^2 \pi}{\mu_0 c} \frac{1}{2} = \frac{(10^3)^2 \times (4 \times 10^{-3})^2 \pi}{4 \pi \cdot 10^{-7} \times 3 \cdot 10^8} \cdot \frac{1}{2} = 1,33 \cdot 10^3 \text{ W} \times \frac{1}{2}$

Exercice 6:

1)



$$2) \vec{E}(z, t) = E_0 \cos(k_x x + k_y y - \omega t) \vec{e}_z = E_0 \cos(k \cos \alpha x + k \sin \alpha y - \omega t) \vec{e}_z$$

$$3) \text{ La 3e eq de Maxwell dit que: } \vec{\nabla} \wedge \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\Leftrightarrow \vec{\nabla} \wedge \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \vec{\nabla} \wedge \vec{E} = - (-i\omega) \vec{B}$$

$$\Rightarrow \vec{\nabla} \wedge \vec{E} = \vec{B} \cdot \omega$$

$$\vec{B} = \frac{1}{\omega} (\vec{\nabla} \wedge \vec{E}) = \frac{1}{\omega} \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ E \end{pmatrix} = \frac{1}{\omega} \begin{pmatrix} \frac{\partial E}{\partial y} \\ - \frac{\partial E}{\partial x} \\ 0 \end{pmatrix}$$

$$\Rightarrow \vec{B} = \frac{1}{\omega} \begin{pmatrix} k \sin \alpha E \\ -k \cos \alpha E \\ 0 \end{pmatrix} \begin{pmatrix} E \sin \alpha \\ -E \cos \alpha \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{k E_0 \sin^2 \alpha \cos \alpha}{\omega} \\ -\frac{k E_0 \sin \alpha \cos^2 \alpha}{\omega} \\ 0 \end{pmatrix}$$

$$B_0 = \frac{E_0}{c_0} = E_0 \times \frac{k}{\omega}$$

Exercice 7:

$$1) k^2 = \frac{\omega^2}{c^2} \left(1 + \frac{i\gamma}{\omega \epsilon_0} \right) = \frac{\omega^2}{c^2} \left(1 + \frac{i}{\omega \epsilon_0} \times \frac{i n_0 e^2}{m \omega} \right)$$

$$k^2 = \frac{\omega^2}{c^2} \left(1 - \frac{n_0 e^2}{m \epsilon_0 \omega^2} \right) = \frac{\omega^2}{c^2} \left(1 - \frac{\omega_p^2}{\omega^2} \right)$$

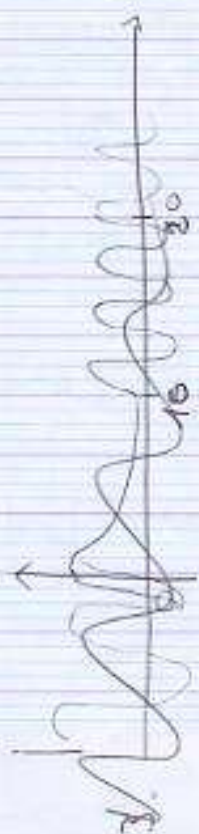
$$2) a) \text{ Pour } k^2 = 0, \text{ on a } \omega = \omega_p \Rightarrow f = f_p = \frac{\omega_p}{2\pi}$$

$$f_c = f_p = \frac{1}{2\pi} \sqrt{\frac{n_0 e^2}{m \epsilon_0}} = \frac{1}{2\pi} 62,8 \sqrt{n_0}$$

$$\Rightarrow f_c = 10^7 \text{ Hz} = \boxed{10 \text{ MHz}}$$

b) Pour $f > f_c$ il y a propagation

Or $f = 5 \text{ MHz} \Rightarrow$ réflexion, car $k^2 < 0$ ni $f < f_c$
 $\hookrightarrow$ atténuation



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Exercice 8

$$a) \vec{E}_1 = \begin{pmatrix} E_{01} \cos(kz - \omega t) \\ E_{01} \sin(kz - \omega t) \end{pmatrix}$$

$$E_y = \tan \alpha \cdot E_x$$

Pas de déphasage entre les phases
Donc polarisation rectiligne.

$$b) \vec{E}_2 = \begin{pmatrix} E_{02} \cos(kz - \omega t) \\ E_{02} \sin(kz - \omega t) \end{pmatrix}$$

Les 2 composantes ont la même amplitude

Mais décalage de $\pm \pi/2$ dû au sin.

Donc polarisation circulaire. $\sin \alpha = \cos(\alpha - \pi/2)$
décalage de $-\pi/2$ donc à droite

$$c) \vec{E}_3 = \begin{pmatrix} E_{03} \cos(kz - \omega t) \\ -E_{03} \sin(kz - \omega t) \end{pmatrix}$$

$$-\sin \alpha = -\cos(\alpha - \pi/2)$$

$$-\cos \alpha = \cos(\alpha + \pi)$$

$$\Rightarrow \begin{pmatrix} E_{03} \cos(kz - \omega t) \\ E_{03} \sin(kz - \omega t + \pi/2) \end{pmatrix} \text{ Donc } \Rightarrow -\sin \alpha = \cos(\alpha + \pi/2)$$

Déphasage de $+\pi/2$ donc à gauche.
Donc polarisation circulaire $\Rightarrow$ gauche.

$$2) \vec{E} = \begin{pmatrix} E_{03} (2 \cos(kz - \omega t)) \\ 0 \end{pmatrix}$$

$$\begin{cases} -\cos \alpha = \cos(\alpha + \pi) \\ -\cos(\alpha - \pi/2) = \cos(\alpha - \pi/2 + \pi) \end{cases}$$

Conclusion ? E_0 colinéaire à $\vec{x}$.

Exercice 9 :

$$① \vec{E} \Rightarrow \begin{cases} E_x = 0 \\ E_y = E_0 \cos(kx - \omega t) \\ E_z = E_0 \cos(kx - \omega t + \pi/3) \end{cases}$$

$$E_y = \tan \alpha \cdot E_z \Rightarrow \frac{E_y}{E_z} = \frac{\sin \pi/3}{\cos \pi/3}$$

$$\hookrightarrow \vec{E}_1 = \begin{pmatrix} 0 \\ E_0 \cos \pi/3 \cos(kx - \omega t) \\ E_0 \sin \pi/3 \end{pmatrix}$$

② $\vec{E} = \begin{cases} E_x = E_{0x} \cos(kx - \omega t + \pi/2) \\ E_y = 0 \\ E_z = E_{0x} \times 3 \cos(kx - \omega t + \pi/2) \end{cases} \Rightarrow \begin{cases} E_{0x} \cos(ky - \omega t) \\ 0 \\ 3E_{0x} \cos(ky - \omega t + \pi/2) \end{cases}$

③ $\vec{E} = \begin{cases} E_x = E_{0x} \cos(kx - \omega t + \pi/4) \\ E_y = 0 \\ E_z = E_{0y} \cos(kx - \omega t) \end{cases} \Rightarrow \begin{cases} 0 \\ E_{0y} \cos(\frac{\pi}{4}x + kx - \omega t) \vec{e}_y \\ 0 \end{cases}$

Exercice 10

1. $E_x = E_{0x} \cos(kz - \omega t)$
 $E_y = E_{0y} \sin(kz - \omega t)$

$$\frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} = \frac{E_{0x}^2 \cos^2(\dots)}{E_{0x}^2} + \frac{E_{0y}^2 \sin^2(\dots)}{E_{0y}^2}$$

$$= \cos^2(\dots) + \sin^2(\dots)$$

$$= 1 \quad \checkmark \quad \text{eq d'une ellipse.}$$

E_{0x} demi-grand axe.

E_{0y} demi-petit axe.

$$\sin(kz - \omega t) = \cos(kz - \omega t - \pi/2).$$

Donc polarisé elliptique droite.

2. $E_x = E_0 \cos(kz - \omega t)$
 $E_y = E_0 \sin(kz - \omega t)$

$$E_x^2 + E_y^2 = E_0^2 \cos^2(kz - \omega t) + E_0^2 \sin^2(kz - \omega t)$$

$$\Rightarrow E_x^2 + E_y^2 = E_0^2 \quad \checkmark \quad \text{eq d'un cercle.}$$

$$E_y = E_0 \sin(kz - \omega t) = E_0 \cos(kz - \omega t - \pi/2) \Rightarrow \text{polarisé circulaire droite}$$

les axes ?

Exercice 11

$$\frac{1}{\omega} \left(\frac{\partial \vec{E}}{\partial t} \right) = \frac{-k E_0}{\omega} \cos(kx - \omega t) \vec{e}_y$$

$$\textcircled{1} \quad \vec{E}(x, t) = E_0 \cos(kx - \omega t) \vec{e}_y$$

$$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = \frac{2\pi \cdot 10^8}{2\pi} = 10^8 \text{ Hz}$$

$$\textcircled{2} \quad \text{rot } \vec{E} = \vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow \frac{\partial}{\partial t} \vec{E} = -(-i\omega) \vec{B} \Rightarrow \vec{B} = \frac{\vec{\nabla} \wedge \vec{E}}{\omega}$$



$$B_0 = \frac{k E_0}{\omega} = \frac{2\pi/\lambda E_0}{\omega} = \frac{E_0}{c} = \frac{10^4}{3 \cdot 10^8} = \boxed{\frac{1}{3} \cdot 10^{-4} \text{ Tesla}}$$

$$\textcircled{3} \quad \vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} = \frac{1}{\mu_0} \begin{pmatrix} +EB \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\mu_0} \begin{pmatrix} 0 \\ 0 \\ E \end{pmatrix} \wedge \begin{pmatrix} 0 \\ B \\ 0 \end{pmatrix}$$

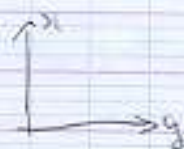
$$S_0 = \frac{E_0 B_0}{\mu_0} = \frac{10^4 \cdot \frac{1}{3} \cdot 10^{-4}}{4\pi \cdot 10^{-7}}$$

$$\Rightarrow \boxed{S_0 = 2,65 \cdot 10^9 \text{ W/m}^2}$$

Exercice 12:

$$\textcircled{1} \quad \vec{E}(z, t) = E_0 \cos(kz - \omega t) \vec{e}_y$$

triebe direct $\Rightarrow \vec{B}_0 = B_0 \cos(kz - \omega t) \vec{e}_x$
(E, B, k)



$$\textcircled{2} \quad \vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} = \frac{1}{\mu_0} \begin{pmatrix} 0 \\ E \\ 0 \end{pmatrix} \wedge \begin{pmatrix} B \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ EB \end{pmatrix} \times \frac{1}{\mu_0}$$

$$\textcircled{3} \quad P_{\text{moy}} = \Phi(\Omega) = \iint_S \vec{S} \cdot d\vec{S} = S \times a^2$$

$$P_{\text{moy}} = \frac{EB}{\mu_0} a^2 = \boxed{\frac{E^2}{\mu_0 c} a^2}$$

Exercise 13

⊙ PPS - $E_0 = 10^3 \text{ V/m}$, ω pulsé.

$$1) E(x, t) = E_0 \cos(kx - \omega t) \vec{e}_y$$

$$2) \vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} = \frac{1}{\mu_0} \begin{pmatrix} E \\ 0 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ -B \end{pmatrix} = \frac{1}{\mu_0} \begin{pmatrix} EB \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{S} = \frac{E_0 B_0 \cos^2(kx - \omega t)}{\mu_0} \vec{e}_x = \frac{E_0 B_0 \cos^2(kx - \omega t)}{\mu_0} \vec{e}_x$$

$$S_0 = \frac{E_0 B_0}{\mu_0} = \frac{E_0^2 \times c}{\mu_0}$$

$$3) P_{\text{moy}} = \langle \Phi(S) \rangle = S 4\pi R^2 \Rightarrow P_{\text{moy}} = \frac{E_0^2 \times c \cos^2}{\mu_0} \times \pi R^2 \times 4$$

$$P_{\text{moy}} = \frac{1}{2} \frac{E_0^2 \times \pi R^2 \times 4}{\mu_0}$$