

Exercice 1: $\vec{E}(x, t) = 2 \cdot 10^4 \cos\left(\frac{\pi}{3} \cdot 10^7 x - \omega t\right) \vec{e}_y$

① La propagation se fait sur la direction (Ox) car :

* $\vec{E}$ dépend de x .

* La phase montre que $\vec{k}$ a une composante : $\begin{pmatrix} k_x \\ 0 \\ 0 \end{pmatrix}$

On a $E(x, t) = E_0 \cdot \cos(k_x x - \omega t) \vec{e}_y$

D'où par identification

$$k_x = \frac{\pi}{3} \cdot 10^7 \rightarrow \vec{k} = \begin{pmatrix} \pi/3 \cdot 10^7 \\ 0 \\ 0 \end{pmatrix} \quad \text{d'où} \quad \boxed{k = \frac{\pi}{3} \cdot 10^7 \text{ rad/m}}$$

Calcul de λ :

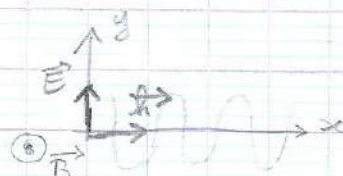
$$k = 2\pi/\lambda \Rightarrow \lambda = 2\pi/k = \boxed{6 \cdot 10^7 \text{ m}}$$

Calcul de T : $c = \frac{\lambda}{T} \Rightarrow T = \frac{\lambda}{c} = \frac{6 \cdot 10^{-7}}{3 \cdot 10^8} = \boxed{2 \cdot 10^{-15} \text{ s}}$

$$T = 1/f \Rightarrow \boxed{f = 5 \cdot 10^{14} \text{ Hz}}$$

② On utilise la 4^e ppte des ondes planes :

$\hookrightarrow (\vec{E}, \vec{B}, \vec{k})$ trièdre direct.



Ondes planes sinusoïdales $\Rightarrow E_0 = c_0 \cdot B_0$

$$\Rightarrow B_0 = \frac{E_0}{c_0} = \frac{2 \cdot 10^4}{3 \cdot 10^8} = \boxed{6,7 \cdot 10^{-5} \text{ Tesla}}$$

③ $\vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0}$

$$\vec{S} = \frac{1}{\mu_0} \begin{pmatrix} 0 \\ E_y \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ B_z \end{pmatrix} = \begin{pmatrix} E_y B_z \\ 0 \\ 0 \end{pmatrix}$$

$$S_0 = \frac{E_0 \cdot B_0}{\mu_0} = \frac{2 \cdot 10^4 \times 6,7 \cdot 10^{-5}}{4\pi \cdot 10^{-7}} = \boxed{1,06 \cdot 10^6 \text{ W/m}^2}$$

Exercice 2 $\vec{B}(y,t) = 10^{-3} \cos(k \cdot y - \omega t) \vec{e}_z$

- ①. Propagation sur (Oy) .
 k a une composante : $\begin{pmatrix} 0 \\ k_y \\ 0 \end{pmatrix}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0,1 \cdot 10^{-6}} = \boxed{6,28 \cdot 10^7 \text{ rad/m}}$$

$$\omega = k \cdot c = 6,28 \cdot 10^7 \times 3 \cdot 10^8 = \boxed{1,88 \cdot 10^{16} \text{ rad/s}}$$

- ②. 4^e ptes $(\vec{E}, \vec{B}, k)$ trièdre direct. $\vec{E} \begin{pmatrix} E_x \\ 0 \\ 0 \end{pmatrix}$

$$E_0 = B_0 \cdot c_0 = 10^{-3} \cdot 3 \cdot 10^8 = \boxed{3 \cdot 10^5 \text{ V/m}}$$

③ $\vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0}$

$$\vec{S} = \frac{1}{\mu_0} \begin{pmatrix} E_x \\ 0 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ -B_z \end{pmatrix} = \frac{1}{\mu_0} \begin{pmatrix} 0 \\ +E_x \cdot B_z \\ 0 \end{pmatrix}$$

$$S_0 = \frac{E_0 B_0}{\mu_0} = \frac{3 \cdot 10^5 \times 10^{-3}}{4\pi \cdot 10^{-7}} = \boxed{2,38 \times 10^8 \text{ W/m}^2}$$

Exercice 3 : $\vec{E}(x,t) = E_0 \cos(kx - \omega t) \vec{e}_y$

① $\vec{\Delta E} = \begin{pmatrix} \Delta E_x \\ \Delta E_y \\ \Delta E_z \end{pmatrix} = \frac{\partial^2 E_y}{\partial x^2} = \frac{-k \cdot E_0 \sin(kx - \omega t)}{\partial x} = -k^2 E_0 \cos(kx - \omega t)$

② $\frac{\partial^2 \vec{E}}{\partial t^2} = -\omega^2 E_0 \cos(kx - \omega t)$

③ D'après D'Alembert : $\Delta \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$

Ici : $-k^2 \vec{E}(x,t) + \mu_0 \epsilon_0 \omega^2 \vec{E}(x,t) = 0$

$$\Leftrightarrow \boxed{\omega^2 = \frac{k^2}{\mu_0 \epsilon_0}}$$

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Exercice 4 : $\vec{E}(M, t) = E_0 \cos(kx - \omega t) \vec{e}_y$

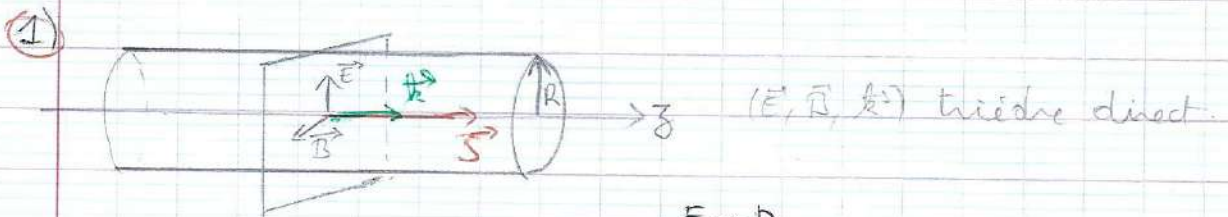
$$① \operatorname{div}(\vec{E}) = \vec{\nabla} \cdot \vec{E} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ E_0 \cos(kx - \omega t) \\ 0 \end{pmatrix} = \frac{E_0 \cos(kx - \omega t)}{\delta y} = 0$$

Confirmer la 1^{re} eq de Maxwell : $\operatorname{div}(\vec{E}) = 0$.

② 4^e ptes d'ondes planes : $(\vec{E}, \vec{B}, \vec{k})$ trièdre direct
 $\hookrightarrow \vec{B} \begin{pmatrix} 0 \\ 0 \\ B_z \end{pmatrix}$

③ $\operatorname{rot} \vec{E} = \vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
 $\Rightarrow i k \wedge \vec{E} = -(-i\omega) \vec{B}$
 $\Rightarrow k \wedge \vec{E} = \omega \vec{B} \Rightarrow \vec{B} = \frac{k \wedge \vec{E}}{\omega}$

Exercice 5 :



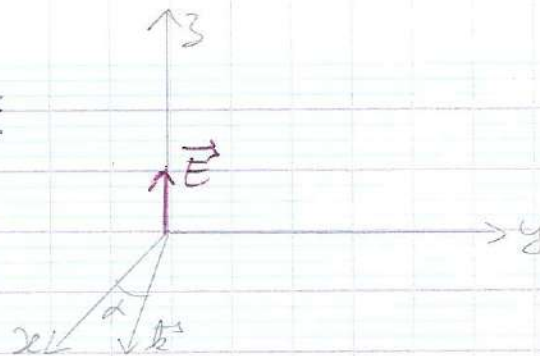
② $\vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} = \frac{E \cdot B \cdot \sin \pi/2}{\mu_0} \xrightarrow{E=cB} \frac{E}{\mu_0} \cdot \frac{E}{c} = \frac{E^2}{\mu_0 c} = \frac{B^2 c}{\mu_0}$

③ $P_{\text{min}} = \Phi(S) = \iint_S \vec{S} \cdot d\vec{S} = S \cdot \pi \cdot R^2 = \frac{B^2 R^2 c}{\mu_0} \pi = \frac{E^2}{\mu_0 c} \pi R^2$

$P_{\text{moy}} = \frac{E_0^2}{\mu_0 c} R^2 \pi = \frac{(10^3)^2 \times (4 \times 10^{-3})^2 \pi}{4 \pi \cdot 10^{-7} \times 3 \cdot 10^8} = 1,33 \cdot 10^2 \text{ W}$

Exercice 6:

1)



$$2) \vec{E}(x, y, t) = E_0 \cos(k_x x + k_y y - \omega t) \vec{e}_z = E_0 \cos(k \cos \alpha x + k \sin \alpha y - \omega t) \vec{e}_z$$

3) La 3^e eq de Maxwell dit que: $\vec{\nabla} \wedge \vec{E} = - \frac{\partial \vec{B}}{\partial t}$

$$\Rightarrow \vec{\nabla} \wedge \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow i k \wedge \vec{E} = - (-i \omega) \vec{B}$$

$$\Rightarrow k \wedge \vec{E} = \vec{B} \cdot \omega$$

$$\vec{B} = \frac{1}{\omega} (k \wedge \vec{E}) = \frac{1}{\omega} \begin{pmatrix} k_x \\ k_y \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ E \end{pmatrix} = \frac{1}{\omega} \begin{pmatrix} k \sin \alpha E - 0 \\ 0 - k \cos \alpha E \\ 0 \end{pmatrix}$$

$$\Rightarrow \vec{B} = \frac{1}{\omega} \begin{pmatrix} k \cos \alpha E \\ -k \sin \alpha E \\ 0 \end{pmatrix} \begin{pmatrix} E \sin \alpha \\ -E \cos \alpha \\ 0 \end{pmatrix}$$

$$B_0 = \frac{E_0}{c_0} = E_0 \times \frac{k}{\omega}$$

Exercice 7:

$$1) k^2 = \frac{\omega^2}{c^2} \left(1 + \frac{i \gamma}{\omega \epsilon_0} \right) = \frac{\omega^2}{c^2} \left(1 + \frac{i}{\omega \epsilon_0} \times \frac{(n_0 e^2)}{m \omega} \right)$$

$$k^2 = \frac{\omega^2}{c^2} \left(1 - \frac{n_0 e^2}{m \epsilon_0 \omega^2} \right) = \frac{\omega^2}{c^2} \left(1 - \frac{\omega_p^2}{\omega^2} \right)$$

$$2) a/ \text{ Pour } k^2 = 0, \text{ on a } \omega = \omega_p \Rightarrow f = f_p = \frac{\omega_p}{2\pi}$$

$$f_c = f_p = \frac{1}{2\pi} \sqrt{\frac{n_0 e^2}{m \epsilon_0}} = \frac{1}{2\pi} 62,8 \sqrt{n_e}$$

$$\Rightarrow f_c = 10^7 \text{ Hz} = \boxed{10 \text{ MHz}}$$

b/

2

Exercice 8

$$a) \vec{E}_1 = \begin{pmatrix} E_{01} \cos(kz - \omega t) \\ E_{01} \sin(kz - \omega t) \end{pmatrix}$$

Pas de déphasage entre les phases
Donc polarisation rectiligne.

$$b) \vec{E}_2 = \begin{pmatrix} E_{02} \cos(kz - \omega t) \\ E_{02} \sin(kz - \omega t) \end{pmatrix}$$

Les 2 composantes ont la même amplitude
Mais décalage de $\pm \pi/2$ dû au sin.
Donc polarisation circulaire.

$$c) \vec{E}_3 = \begin{pmatrix} E_{03} \cos(kz - \omega t) \\ -E_{03} \sin(kz - \omega t) \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} E_{03} \cos(kz - \omega t) \\ E_{03} \sin(kz - \omega t + \pi) \end{pmatrix}$$

Déphasage de $\pm \pi$
Donc polarisation circulaire.

$$d) \vec{E} = \begin{pmatrix} E_{03} (2 \cos(kz - \omega t)) \\ 0 \end{pmatrix}$$

Conclusion ?

Exercice 9 :

$$\textcircled{1} \vec{E} \Rightarrow \begin{cases} E_x = 0 \\ E_y = E_{0y} \cos(kx - \omega t) \\ E_z = E_{0z} \cos(kx - \omega t + \pi/3) \end{cases}$$

2

$$\vec{E} = \begin{cases} E_x = E_{0x} \cos(kx - \omega t + \pi/2) \\ E_y = 0 \\ E_z = E_{0x} \times 3 \cos(kx - \omega t + \pi/2) \end{cases}$$

3

$$\vec{E} = \begin{cases} E_x = E_{0x} \cos(kx - \omega t + \pi/4) \\ E_y = 0 \\ E_z = E_{0y} \cos(kx - \omega t) \end{cases}$$

Exercice 10

1.
$$\begin{aligned} E_x &= E_{0x} \cos(kz - \omega t) \\ E_y &= E_{0y} \sin(kz - \omega t) \end{aligned}$$

$$\begin{aligned} \frac{E_x^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} &= \frac{E_{0x}^2 \cos^2(\text{---})}{E_{0x}^2} + \frac{E_{0y}^2 \sin^2(\text{---})}{E_{0y}^2} \\ &= \cos^2(\text{---}) + \sin^2(\text{---}) \\ &= 1 \quad \checkmark \quad \text{eq d'une ellipse.} \end{aligned}$$

E_{0x} demi-grand axe.

E_{0y} demi-petit axe.

$$\sin(kz - \omega t) = \cos(kz - \omega t - \pi/2).$$

Donc polarisé elliptique droite.

2.
$$\begin{aligned} E_x &= E_0 \cos(kz - \omega t) \\ E_y &= E_0 \sin(kz - \omega t) \end{aligned}$$

$$\begin{aligned} E_x^2 + E_y^2 &= E_0^2 \cos^2(kz - \omega t) + E_0^2 \sin^2(kz - \omega t) \\ \Rightarrow E_x^2 + E_y^2 &= E_0^2 \quad \checkmark \quad \text{eq d'un cercle.} \end{aligned}$$

$$E_y = E_0 \sin(kz - \omega t) = E_0 \cos(kz - \omega t - \pi/2) \Rightarrow \text{polarisé circulaire droite.}$$

les axes ?

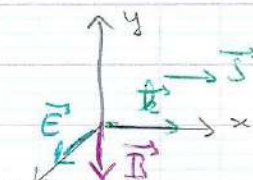
Exercice 11

$$\textcircled{1} \quad \vec{E}(x,t) = E_0 \cos(kx - \omega t) \vec{e}_z$$

$$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = \frac{2\pi \cdot 10^{14}}{2\pi} = 10^{14} \text{ Hz}$$

$$\textcircled{2} \quad \text{rot } \vec{E} = \vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow i k \wedge \vec{E} = -(-i\omega) \vec{B} \Rightarrow \vec{B} = \frac{k \wedge \vec{E}}{\omega}$$



$$B_0 = \frac{k E_0}{\omega} = \frac{\omega/c E_0}{\omega} = \frac{E_0}{c} = \frac{10^6}{3 \cdot 10^8} = \boxed{\frac{1}{3} 10^{-2} \text{ Tesla}}$$

$$\textcircled{3} \quad \vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} = \frac{1}{\mu_0} \begin{pmatrix} +EB \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\mu_0} \begin{pmatrix} 0 \\ 0 \\ E \end{pmatrix} \wedge \begin{pmatrix} 0 \\ B \\ 0 \end{pmatrix}$$

$$S_0 = \frac{E_0 B_0}{\mu_0} = \frac{10^6 \cdot \frac{1}{3} \cdot 10^{-2}}{4\pi \cdot 10^{-7}}$$

$$\Rightarrow \boxed{S_0 = 2,65 \cdot 10^9 \text{ W/m}^2}$$

Exercice 12:

$$\textcircled{1} \quad \vec{E}(x,t) = E_0 \cos(kz - \omega t) \vec{e}_y$$

triedre direct $\Rightarrow \vec{B} = B_0 \cos(kz - \omega t) \vec{e}_x$
 $(\vec{E}, \vec{B}, \vec{k})$

$$\textcircled{2} \quad \vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} = \frac{1}{\mu_0} \begin{pmatrix} 0 \\ E \\ 0 \end{pmatrix} \wedge \begin{pmatrix} B \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ EB \end{pmatrix} \times \frac{1}{\mu_0}$$

$$\textcircled{3} \quad P_{\text{moy}} = \Phi(\Omega) = \iint_S \vec{S} \cdot d\vec{S} = S \times a$$

$$P_{\text{moy}} = \frac{EB}{\mu_0} a = \boxed{\frac{E^2}{\mu_0 c} a}$$

exercice 1

(1)

on a dans le vide

$$\vec{E}(x,t) = E_0 \cdot 10^4 \cos\left(\frac{\pi}{3} \cdot 10^7 \cdot x - \omega t\right) \vec{e}_y$$

a) - la propagation se fait selon l'axe x

car - $\vec{E}$ dépend de x

- la phase $\cos(\text{phase})$ montre que k a 1 composante : $\vec{k} \begin{pmatrix} k_x \\ 0 \\ 0 \end{pmatrix}$

$$E(x,t) = E_0 \cos(k_x x - \omega t) \vec{e}_y$$

donc, par identification : $E_0 = 8 \cdot 10^4$
 $k_x = \frac{\pi}{3} \cdot 10^7$

on module : $k = \left(\frac{\pi}{3} \cdot 10^7\right)^2 = \frac{\pi}{3} \cdot 10^7 \text{ rad/m}$

$$\boxed{k = \frac{2\pi}{\lambda}} \Rightarrow \lambda = \frac{2\pi}{k} = \lambda = \frac{2\pi}{\frac{\pi}{3} \cdot 10^7} = \frac{6\pi}{10^7} = 6 \cdot 10^{-7} \text{ m}$$

$$\boxed{c = \frac{\lambda}{T}} \Leftrightarrow T = \frac{\lambda}{c}$$

$$\boxed{\frac{f}{T} = \frac{1}{T}} = \frac{1}{4 \cdot 10^{-16} \text{ s}} = \frac{6 \cdot 10^7}{3 \cdot 10^8} = 2 \cdot 10^{-10} \text{ s}$$

1) d'après la 3^e propriété d'ondes planes. (1.2)

$$E = c B. \Rightarrow B = \frac{E}{c}$$

$$\Rightarrow B_0 = \frac{E_0}{c} \quad (\text{onde électromagnétique})$$

$$B_0 = \frac{2 \cdot 10^4}{3 \cdot 10^8} = \frac{2}{3} \cdot 10^{-4} \cdot \text{Tesla}$$

$$3) \quad \vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} \Rightarrow S_0 = \frac{E_0 B_0}{\mu_0} = \frac{\frac{2}{3} \cdot 10^{-4} \times 2 \cdot 10^4}{4\pi \cdot 10^{-7}}$$

$$= \frac{1}{6\pi} \times 10^7 \quad \text{W/m}^2$$

exercice 22-1

$$B(y, t) = 10^{-3} \cos(k \cdot y - \omega t) \vec{e}_z$$

1) la propagation se fait selon l'axe y .

car : - $\vec{E}$ dépend de y

- la phase cos(phase) montre que k a

1 composante $\vec{k} \begin{pmatrix} 0 \\ k_y \\ 0 \end{pmatrix}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0,1 \cdot 10^{-3}} = 0,2\pi \cdot 10^{10}$$

$$\omega = 2\pi f$$

$$f = \frac{1}{T}$$

$$c = \frac{\lambda}{T}$$

$$\omega T = \frac{\lambda}{c}$$

$$\Rightarrow \omega = \frac{2\pi c}{\lambda} = \frac{2\pi \cdot 3 \cdot 10^8}{0,1 \cdot 10^{-3}}$$

$$\omega = 6 \cdot 10^{18}$$

2)

$$\boxed{B = \frac{E_0}{c}} \Rightarrow E_0 = B_0 c = 10^{-3} \cdot 3 \cdot 10^8 = \underline{\underline{3 \cdot 10^5}}$$

$$3) \quad \vec{S} = \frac{\vec{B} \wedge \vec{E}}{\mu_0} \Rightarrow S = \frac{B_0 \cdot E_0}{\mu_0} = \frac{3 \cdot 10^5 \times 10^{-3}}{4\pi \cdot 10^{-7}}$$

$$\boxed{S = \frac{3}{4\pi} 10^8}$$

exercice 3

(3-1)

$$\vec{E}(x,t) = E_0 \cos(ky - \omega t) \vec{e}_z$$

$$\begin{aligned} 1) \Delta \vec{E} &= 0 + 0 - E_0 \omega^2 \sin(ky - \omega t) \vec{e}_z \\ &= -\omega^2 \vec{E}(x,t). \end{aligned}$$

$$\vec{E}(x,t) = 0 + 0 + -k^2 \vec{E}$$

$$= -k^2 \vec{E} = \Delta \vec{E} \Rightarrow \frac{\Delta \vec{E}}{-k^2} = \vec{E}$$

$$2) \frac{\partial^2 \vec{E}}{\partial t^2} = -\omega^2 \vec{E}$$

$$\begin{aligned} 3) \vec{E} &= \frac{1}{\omega^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{\Delta \vec{E}}{k^2} \\ \Delta \vec{E} &= -\frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \end{aligned}$$

$$3) \quad \vec{E} = -\frac{1}{\omega^2} \frac{d^2 \vec{E}}{dt^2}$$

$$\frac{d\vec{E}}{-k^2} = \vec{E}$$

$$\Rightarrow \frac{d\vec{E}}{-k^2} = -\frac{1}{\omega^2} \frac{d^2 \vec{E}}{dt^2}$$

$$\Rightarrow -\frac{k^2}{\omega^2} \frac{d^2 \vec{E}}{dt^2} + d\vec{E} = 0$$

$$\frac{1}{c^2}$$

$$\Rightarrow \frac{k^2}{\omega^2} = \frac{1}{c^2}$$

$$c = \frac{\omega}{k} \Rightarrow \underline{\underline{\omega = kc}}$$

exercice 4

(h-1)

$$\vec{E}(\vec{r}, t) = E_0 \cos(kx - \omega t) \cdot \vec{e}_y$$

1. div ($\vec{E}$).

$$\text{div}(\vec{E}) = -E_0 k \sin(kx - \omega t) \vec{e}_y$$

$k \cdot E = 0$ car ils sont perpendiculaires.

$$\Rightarrow \text{div}(\vec{E}) = 0$$

2. ~~$\vec{E} = c \vec{B}$~~ $\|\vec{E}_0\| = c \|\vec{B}\|$

can vector
transverse
with
 $\frac{\omega}{k}$

$$\vec{B}(\vec{r}, t) = B_0 \cos(kx - \omega t) \vec{e}_z$$

$$= \frac{E_0}{c} \cos(kx - \omega t) \vec{e}_z \Rightarrow B_0 = \frac{E_0}{c}$$

3.

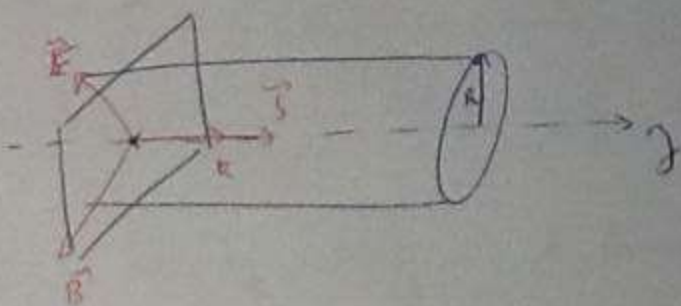
exercice 5

(5-1)

$$E_0 = 10^7 \text{ V/m}$$

faisceau: rayon R .
axe Oy .

1. OPEB $\rightarrow \vec{E}$ et $\vec{S}$ sont colinéaires à l'axe de propagation
 $\vec{B}$ et $\vec{S}$ perpendiculaires à l'axe
en ave ?



$$E_0 = c B_0 \rightarrow B_0 = \frac{E_0}{c} = \frac{10^7}{3 \cdot 10^8} = \frac{1}{3} \times 10^{-1}$$

$$\vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} = \frac{E_0 B_0}{\mu_0} = \frac{10^7 \times \frac{1}{3} \times 10^{-1}}{4\pi \cdot 10^{-7}} = \frac{1}{12} \times 10^7 \text{ Tesla}$$

3. puissance moyenne de rayonnement

(5.2)

par def:

$$P_{\text{moy}} = \oint (\vec{S}) = \iint_S \vec{S} \cdot d\vec{\Sigma} = \iint_S S d\Sigma = S \iint d\Sigma = S \Sigma = S \pi R^2$$

$$\boxed{P_{\text{moy}} = S \pi R^2}$$

Or, $\vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0}$ et $\vec{E} \perp \vec{B}$ car onde plane.

$$\Rightarrow \|\vec{S}\| = \frac{E B \sin(\frac{\pi}{2})}{\mu_0} = \frac{E \cdot E}{\mu_0 c} \quad \text{car } E = cB.$$

Or, $E = E_0 \cos(ky - \omega t)$ car le faisceau se propage selon $\vec{Oy}$

$$P = \frac{1}{\mu_0 c} E^2 = \frac{E_0^2}{\mu_0 c} \cos^2(ky - \omega t) \pi R^2.$$

$$P_{\text{moy}} = \frac{\pi R^2 E_0^2}{\mu_0 c} \underbrace{\langle \cos^2(ky - \omega t) \rangle}_{{1/2}} = \frac{1}{2} \frac{E_0^2 \pi R^2}{\mu_0 c}$$

$$= \frac{1}{2} \frac{(10^3)^2 \pi (10^{-3})^2}{4\pi \cdot 10^{-7} \cdot 3 \cdot 10^8}$$

exercice 6

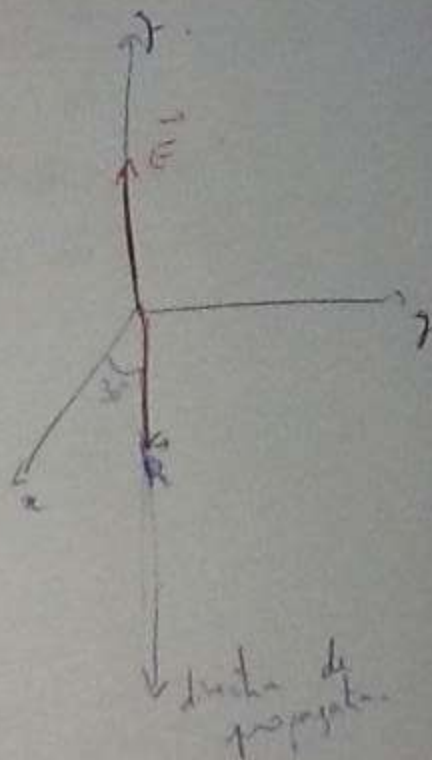
$$c = 3 \cdot 10^8 \text{ m/s}$$

plan (xoy)

$\alpha = 30^\circ \Rightarrow$ angle entre $\vec{E}$ et Ox .

$\vec{E}$ with Oz ($z > 0$)

1.



$$2. \vec{E}(x, y, t) = E_0 \cos\left(\left(\frac{\sqrt{2}}{2}x + \frac{1}{2}y\right) - \omega t\right) \vec{e}_z$$

exercice 10

$$1) \quad E(z, t) = E_0 \cos(kz - \omega t) \vec{e}_z$$

$$\omega = 2\pi f \Rightarrow f = \frac{\omega}{2\pi} = 10^{14} \text{ Hz}$$

$$2) \quad \frac{\partial \vec{E}}{\partial t} = - \frac{\partial \vec{B}}{\partial t}$$

$$\|\vec{E}\| = c \|\vec{B}\|$$

$$E_0 = c B_0$$

$$B_0 = \frac{E_0}{c}$$

$$= \frac{10^6}{3 \cdot 10^8}$$

$$B_0 = \frac{1}{3} \cdot 10^{-2}$$

$$\vec{E}(r, t) = \vec{E}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$\vec{B}(r, t) = \vec{B}_0 e^{i(\vec{k} \cdot \vec{r} - \omega t)}$$

$$3) \quad \vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} \quad \text{car pour une onde}$$

$$\vec{E} \wedge \vec{B} = c \times B_0 \times \vec{u}$$

$\Rightarrow$ direction de propagation $= \vec{x}$

$$S = \frac{E_0 \times B_0}{\mu_0} = \frac{1}{3} \times \frac{10^{-2} \times 10^{16}}{4\pi \cdot 10^{-7}}$$

exercice 4

$$1. E(z, t) = E_0 \cos(kz - \omega t) \vec{e}_y$$

$$\|\vec{E}\| = c \|\vec{B}\|$$

$$E_0 = c B_0 \Rightarrow B_0 = \frac{E_0}{c}$$

$$B(z, t) = \frac{E_0}{c} \cos(kz - \omega t) \vec{e}_x$$

$$2. \vec{S} = \frac{\vec{B} \wedge \vec{E}}{\mu_0}$$

$$3. P_{\text{moy}} = \varphi(\vec{S}) = \iint_S \vec{S} \cdot d\vec{\Sigma} = \iint_S S d\Sigma$$
$$= S \iint_S d\Sigma$$

$$\text{Or } \vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} \quad \vec{E} \perp \vec{B}$$

$$= S \Sigma$$
$$= S a^2$$

$$\|\vec{S}\| = \frac{E \cdot B}{\mu_0} \Rightarrow S = \frac{E \cdot B}{\mu_0}$$

$$\text{or } E_0 = c B_0 \Rightarrow B_0 = \frac{E_0}{c}$$

$$S = \frac{E^2}{\mu_0 c} \quad \text{avec } E = E_0 \cos(kz - \omega t)$$

$$P = \frac{1}{\mu_0 c} E_0^2 \cos^2(kz - \omega t) a^2$$
$$P_{\text{moy}} = \frac{1}{2} \frac{E_0^2}{\mu_0 c} a^2$$

exercice 12

$$1) \quad E(y, t) = E_0 \cos(ky - \omega t) \vec{e}_x$$

$$2) \quad \vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu_0} \quad \|\vec{S}\| = \frac{E \wedge B}{\mu_0} \vec{e}_z$$

$$S = \frac{E \wedge B}{\mu_0} \quad \text{avec} \quad E = cB$$

$$B = \frac{E}{c}$$

$$S = \frac{E^2}{\mu_0 c}$$

$$= \frac{E_0^2}{\mu_0 c} \cos^2(ky - \omega t)$$

$$3) \quad P = \iint_S \vec{S} \cdot d\vec{\Sigma}$$

$$= S \int d\Sigma \quad \rightarrow \quad P = \frac{E_0^2 \pi R^2}{\mu_0 c} \cos^2(ky - \omega t)$$

$$= S \Sigma$$

$$= S \pi R^2$$

$$P_{\text{moy}} = \frac{E_0^2 \pi R^2}{\mu_0 c} \langle \cos^2(ky - \omega t) \rangle$$

$$= \frac{1}{2} \frac{E_0^2 \pi R^2}{\mu_0 c}$$

Ex 8:

1)
 $E(x,t) = E_0 \cos(ky - \omega t)$

$\vec{e}_x$

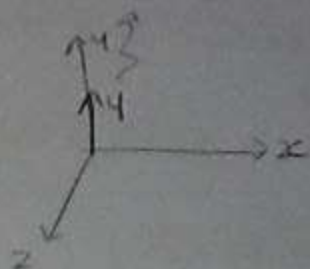


$\rightarrow E_0, k, \omega, \vec{e}_x$

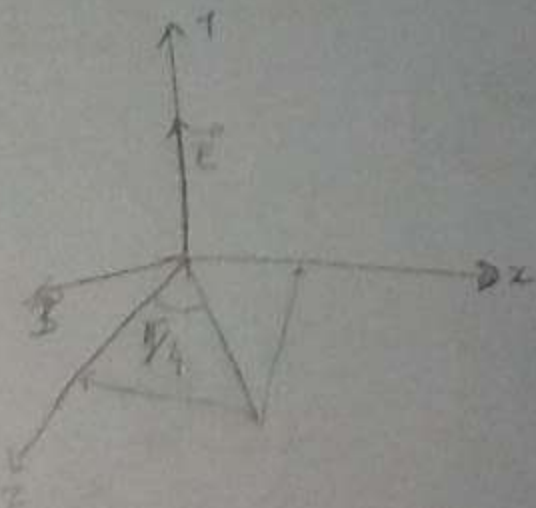
$E_0 \cos\left(\frac{\sqrt{3}}{2}y + \frac{1}{2}z - \omega t\right) \vec{e}_x$

$\Phi(t, r) = E_0 \cos(\alpha x + \beta y + \gamma z - \omega t)$

2



3



$\vec{E}(y,t) = E_0 \cos\left(\frac{\sqrt{3}}{2}y - \omega t\right) \vec{e}_x + E_0 \frac{\sqrt{3}}{2} \cos(ky - \omega t) \vec{e}_y$

Ex 7

a. $\vec{E}_1 \begin{pmatrix} E_0 \cos(\alpha t) \cos(k_2 z - \omega t) \\ E_0 \sin(\alpha t) \cos(k_2 z - \omega t) \end{pmatrix}$

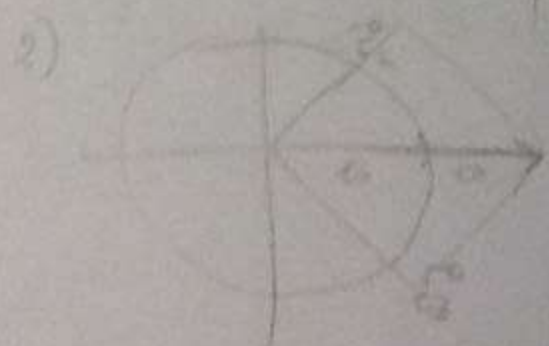
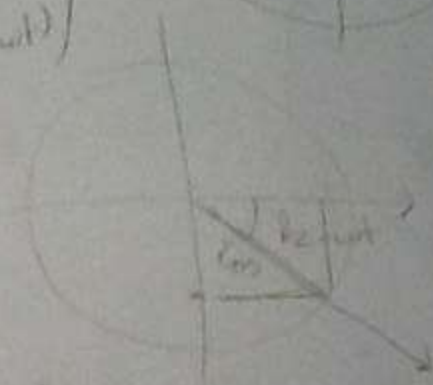
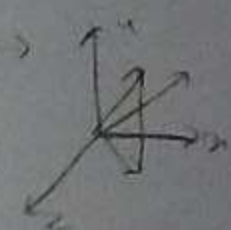
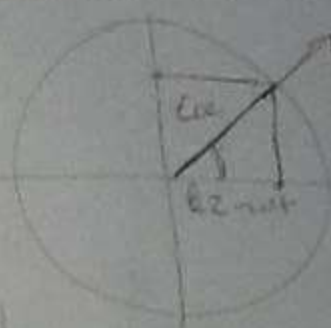
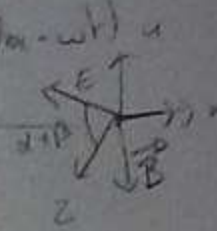
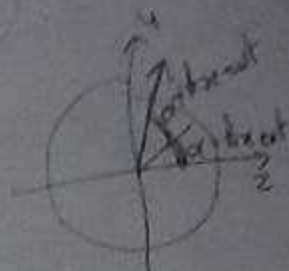
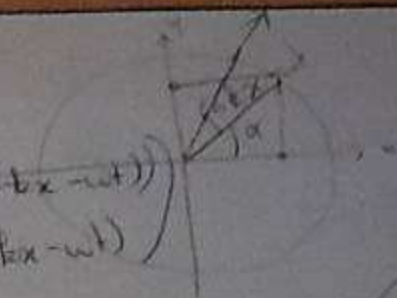
$\vec{E}_1 \begin{pmatrix} E_0 \frac{1}{2} (\cos(\alpha t - k_2 z - \omega t) + \cos(\alpha t + k_2 z - \omega t)) \\ E_0 \frac{1}{2} (\sin(\alpha t - k_2 z - \omega t) + \sin(\alpha t + k_2 z - \omega t)) \end{pmatrix}$

Ar $\vec{E}_1 \begin{pmatrix} E_0 \frac{1}{2} \cos(\alpha t - k_2 z - \omega t) + E_0 \frac{1}{2} \cos(\alpha t + k_2 z - \omega t) \\ E_0 \frac{1}{2} \sin(\alpha t - k_2 z - \omega t) + E_0 \frac{1}{2} \sin(\alpha t + k_2 z - \omega t) \end{pmatrix}$

1/2 est distribué par 2 + addition de $(\alpha t - k_2 z - \omega t) + (\alpha t + k_2 z - \omega t)$

b. $\vec{E}_2 \begin{pmatrix} E_0 \cos(k_2 z - \omega t) \\ E_0 \sin(k_2 z - \omega t) \end{pmatrix}$

c. $\vec{E}_3 \begin{pmatrix} E_0 \cos(k_2 z - \omega t) \\ -E_0 \sin(k_2 z - \omega t) \end{pmatrix}$



$\vec{E}_1 + \vec{E}_2 = \vec{E}_3$

FICHE PROPAGATION ELECTROMAGNETIQUE

OUTILS MATHÉMATIQUES

$\vec{\text{grad}}(f) = \begin{pmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{pmatrix}$
 $\vec{\text{div}}(\vec{P}) = \vec{\nabla} \cdot \vec{P} = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}$

↳ vecteur

↳ scalaire

$\vec{\text{rot}}(\vec{P}) = \vec{\nabla} \wedge \vec{P} = \begin{pmatrix} \frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z} \\ \frac{\partial v_1}{\partial z} - \frac{\partial v_3}{\partial x} \\ \frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y} \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \wedge \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

↳ vecteur

$\Delta \text{ Laplacien } f(x, y, z) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

ONDES ELECTROMAGNETIQUES:

• selon (Oz)

$$\vec{E} = \frac{\partial^2 \vec{E}}{\partial x^2} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \vec{0} \Rightarrow \Delta \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\vec{B} = \frac{\partial^2 \vec{B}}{\partial x^2} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = \vec{0} \Rightarrow \Delta \vec{B} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\lambda = \frac{1}{k} = \frac{2\pi}{k} = \frac{v}{f} = \frac{c}{f}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$T = \frac{c}{v}$$

EQUATIONS DE MAXWELL:

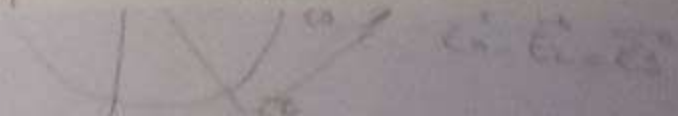
• $\text{div } \vec{E} = \frac{\rho}{\epsilon_0}$

• $\text{div } \vec{B} = 0$ → les lignes de $\vec{B}$ ne ont pas borne

• $\text{rot } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

• $\text{rot } \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$

→ dérivée des courants



PROPAGATION DANS LE VIDE

$$\begin{aligned} \cdot \operatorname{div} \vec{E} &= 0 & \cdot \operatorname{rot} \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \cdot \operatorname{div} \vec{B} &= 0 & \cdot \operatorname{rot} \vec{B} &= \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \end{aligned}$$

EQUATIONS ET SOLUTIONS de d'ALEMBERT

$$\begin{aligned} \Delta \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} &= \vec{0} \\ \Delta \vec{B} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} &= \vec{0} \end{aligned}$$

$$\frac{\partial^2 p}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0 \quad \Rightarrow \quad p(x, t) = F(x - ct) + G(x + ct)$$

le signal se propage vers $x > 0$ le signal se propage vers $x < 0$
le onde progressive le onde régressive

→ Onde PROGRESSIVE $\Rightarrow E(x - ct), B(x - ct)$ (sur (Ox))

→ Onde RÉGRESSIVE $\Rightarrow E(x + ct), B(x + ct)$ (sur (Ox))

ONDES PLANES

$\vec{E}(x, t) = \vec{E}(x - ct)$ → Onde progressive

- $\vec{E}$ et $\vec{B}$ perpendiculaires à l'axe de propagation
- $\|\vec{E}\| = c \|\vec{B}\|$
- $\vec{E}$ perpendiculaire à $\vec{B}$
- $(\vec{E}, \vec{B}, \vec{a})$: triade directe

ONDES PLANES PROGRESSIVES SINUSOIDALES

$$\begin{aligned} \vec{E}(x - ct) &= \vec{E}_0 \cos(kx - \omega t) \\ &= \vec{E}_0 \cos(kx - \omega t) \quad \omega = kc \end{aligned}$$

$$\begin{aligned} f &= \frac{1}{T} & \omega &= \frac{2\pi}{T} = kc & c &= \frac{\lambda}{T} & \lambda &= cT = \frac{c}{f} \\ & & k &= \frac{2\pi}{\lambda} \end{aligned}$$

Vecteurs de Poynting

$$\vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu}$$

Pour une OPM $\vec{E} \wedge \vec{B} = EB \vec{u} \Rightarrow \vec{S} = \frac{EB}{\mu} \vec{u}$
avec $\vec{u}$ direction de propagation

$$S_0 = \frac{E_0^2}{\mu_0 c} = \epsilon_0 c \cdot E_0^2 = \frac{E_0 B_0}{\mu_0}$$

$$I = \frac{S_0}{2}$$

PROPAGATION DANS UN MILIEU MATÉRIEL

$$k^2 = \frac{\omega^2}{c^2} \left(1 + \frac{j}{\omega \tau} \right)$$

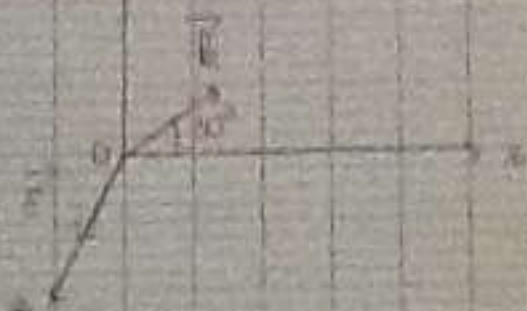
• $k^2 > 0 \Rightarrow k$ réel : onde progressive
 $E(x, t) = E_0 \cos(kx - \omega t)$

• $k^2 < 0 \Rightarrow k$ imaginaire : onde atténuée
 $E(x, t) = E_0 e^{-kx} \cos(\omega t) \quad k = -ik'$

• k^2 complexe $\Rightarrow k$ complexe : onde amortie
 $E(x, t) = E_0 e^{-kx} \cos(kx - \omega t) \quad k = k' + jk''$

Exercice n° 6.

1)



$$2) E(x, y, t) = E_0 \cos\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}y - \omega t\right) \vec{e}_z$$

$$3) \text{rot } \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{car } \vec{k}_n \vec{E} = j \omega \vec{B}$$

$$\vec{B} = \frac{1}{\omega} \vec{k}_n \vec{E} = \frac{1}{\omega} \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ E_0 \cos\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}y - \omega t\right) \end{pmatrix}$$

$$\vec{B} = \frac{1}{\omega} E_0 \cos\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}y - \omega t\right) \vec{e}_x - \frac{\sqrt{3}}{\omega} E_0 \cos\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}y - \omega t\right) \vec{e}_y$$

$$\vec{B} = \frac{1}{\omega} E_0 \cos\left(\frac{\sqrt{3}}{2}x + \frac{1}{2}y - \omega t\right) [\vec{e}_x - \sqrt{3} \vec{e}_y]$$

Exercice n° 7:

$$1. a. \vec{E}_1 = \begin{pmatrix} E_{01} \cos(\alpha) \cos(k_1 z - \omega t) \\ E_{01} \sin(\alpha) \cos(k_1 z - \omega t) \end{pmatrix}$$

E_{01}

Il n'y a pas de déphasage $\Rightarrow$ Polarisation linéaire

$$b. \vec{E}_2 = \begin{pmatrix} E_{02} \cos(k_2 z - \omega t) \\ E_{02} \sin(k_2 z - \omega t) \end{pmatrix}$$

déphasage de $-\pi/2 \Rightarrow$ Polarisation circulaire à droite.

$$c. \vec{E}_3 = \begin{pmatrix} E_{03} \cos(k_3 z - \omega t) \\ -E_{03} \sin(k_3 z - \omega t) \end{pmatrix}$$

déphasage de $\pi/2 \Rightarrow$ Polarisation circulaire à gauche.

$$2. \vec{E} = \vec{E}_1 + \vec{E}_2 \quad \text{avec } E_{01} = E_{02} = E_0$$

$$\vec{E} = \begin{pmatrix} 2E_0 \cos(ky - \omega t) \\ 0 \\ 0 \end{pmatrix}$$

$\Rightarrow$ Polarisation linéaire (vecteur à une composante)

Ex 8:

$$1. \vec{E}_1 = \begin{pmatrix} E_{01} \frac{\sqrt{2}}{2} \cos(kx - \omega t) \\ 0 \\ E_{01} \frac{\sqrt{2}}{2} \cos(kx - \omega t) \end{pmatrix}$$

$$2. \vec{E}_2 = \begin{pmatrix} E_{02} \cos(ky - \omega t) \\ 0 \\ E_{02}' \cos(ky - \omega t + \varphi) \end{pmatrix} \quad E_{02} = 1 \quad E_{02}' = 3$$

$$\varphi = \frac{\pi}{2}$$

$$\vec{E}_2 = \begin{pmatrix} \cos(ky - \omega t) \\ 0 \\ 3 \cos(ky - \omega t + \pi/2) \end{pmatrix} = \begin{pmatrix} \cos(ky - \omega t) \\ 0 \\ -3 \sin(ky - \omega t) \end{pmatrix}$$

$$3. \vec{E} = \begin{pmatrix} 0 \\ E_0 \cos(\frac{\sqrt{2}}{2}x + \frac{\sqrt{2}}{2}z - \omega t) \\ 0 \end{pmatrix}$$



Ex 9:

$$\vec{E} = \begin{pmatrix} E_{0x} \cos(kz - \omega t) \\ E_{0y} \sin(kz - \omega t) \\ 0 \end{pmatrix} \quad E_{0x} > E_{0y}$$

Il s'agit d'une vibration elliptique car il y a un déphasage de $-\pi/2$ et $E_{0x} \neq E_{0y}$

Il s'agit d'une polarisation elliptique à droite car $-\frac{\pi}{2} = \frac{3\pi}{2}$, $\pi < \frac{3\pi}{2} < 2\pi$

d'axe x porte le grand axe de l'ellipse.

$$\frac{E_z^2}{E_{0x}^2} + \frac{E_y^2}{E_{0y}^2} = 1$$

$$\frac{E_0 x^2 \cos^2(kz - \omega t)}{E_0 x^2} + \frac{E_0 y^2 \sin^2(kz - \omega t)}{E_0 y^2} = \cos^2(kz - \omega t) + \sin^2(kz - \omega t)$$

$$= 1 \quad \rightarrow \text{vérifié!}$$

2. Il s'agit d'une vibration circulaire car il y a un déphasage de $-\pi/2$ et que $E_x = E_y = E_0$.

Exercice n° 10

1) $\vec{E}(x, t) = E_0 \cos(kx - \omega t) \vec{e}_z$

$$\omega = 2\pi f \quad \Leftrightarrow f = \frac{\omega}{2\pi} = 10^{14} \text{ s}^{-1} \text{ ou Hz}$$

2) $\vec{B} = \frac{1}{\omega} \vec{k} \wedge \vec{E} = \frac{1}{\omega} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ E_0 \cos(kx - \omega t) \end{pmatrix}$

$$\vec{B} = \begin{pmatrix} \frac{\omega}{4\pi c} E_0 \cos(kx - \omega t) \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{B} = \frac{1}{\frac{2\pi \cdot 10^{14}}{3 \cdot 10^8}} \cdot 10^6 \cos\left(\frac{2\pi \cdot 10^{14}}{3 \cdot 10^8} x - 2\pi \cdot 10^{14} t\right) \vec{e}_y$$

$$\vec{B} = \frac{1}{2\pi} \cdot 10^{-8} \cos\left(\frac{2\pi}{3} x - 2\pi \cdot 10^{14} t\right) \vec{e}_y$$

$$E_0 = c B_0 \quad \Leftrightarrow B_0 = \frac{E_0}{c} = \frac{10^6}{3 \cdot 10^8} = \frac{1}{3} \times 10^{-2}$$

3) $\vec{S} = \frac{\vec{E} \wedge \vec{B}}{\mu} = \frac{E B \times \vec{k}}{\mu} = \frac{E_0 \cos(kx - \omega t) \cdot B_0 \cos(kx - \omega t) \cdot \vec{k}}{\mu}$

$$\vec{S} = \frac{E_0 B_0 \cos^2(kx - \omega t)}{\mu} \vec{k} \quad \text{car } \vec{k} = \vec{e}_x$$

$$S_0 = \frac{E_0 \cdot B_0}{\mu_0} = \frac{E_0^2}{\mu_0 c} = \frac{10^{12}}{4\pi \cdot 10^{-7} \cdot 3 \cdot 10^8} = \frac{1}{12\pi} \times 10^{11}$$

Exercice 11.

$$1) \vec{E} = E_0 \cos(kz - \omega t) \vec{e}_y$$

$$\vec{B} = B_0 \cos(kz - \omega t) \vec{e}_x$$

$$2) \vec{S} = \frac{\vec{E} \times \vec{B}}{\mu} = \frac{E B \vec{e}_z}{\mu} = \frac{E_0 B_0 \cos^2(kz - \omega t) \vec{e}_z}{\mu}$$

$$S_0 = \frac{E_0 B_0}{\mu_0}$$

$$3) P_{\text{moy}} = \iint_S \vec{S} \cdot d\vec{\Sigma} = \frac{E_0^2 \times \frac{1}{2} a^2}{\mu c}$$

$$P_{\text{moy}} = \frac{(E_0 a)^2}{2\mu c}$$

Exercice 12.

$$1) \vec{E}(x, t) = E_0 \cos(kx - \omega t) \vec{e}_y$$

$$B(x, t) = B_0 \cos(kx - \omega t) \vec{e}_z$$

$$2) \vec{S} = \frac{E B \vec{e}_x}{\mu} = \frac{E_0 B_0 \cos^2(kx - \omega t) \vec{e}_x}{\mu}$$

$$S_0 = \frac{E_0 B_0}{\mu_0} = \frac{E_0^2}{\mu_0 c}$$

$$3) P_{\text{moy}} = \iint_S \vec{S} \cdot d\vec{\Sigma} = \frac{1}{2} \frac{E_0 B_0}{\mu_0} \times \Sigma$$

$$P_{\text{moy}} = \frac{E_0 B_0}{2\mu_0} \times 4\pi R^2 = \boxed{\frac{E_0^2 4\pi R^2}{2\mu_0 c}}$$

