

CE00998 Coding and Transformations
Assignment 1: Fourier Series
Please submit by 3.30pm Monday 19th October 2009

1. For each of the functions below find the Fourier series in the form

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^N [a_n \cos(n\pi x) + b_n \sin(n\pi x)]$$

expressing the coefficients as simple as possible.

[12 marks]

Plot each series on the same axes as the function for $N = 2, 5$ and 25 (three separate plots for each series).

[12 marks]

(a) $f(x) = \begin{cases} -1 & -1 < x \leq 0 \\ -1 + 2 \sin(\pi x) & 0 < x \leq 1 \end{cases}$

(b) $f(x) = \begin{cases} x+1 & -1 < x \leq 0 \\ x-1 & 0 < x \leq 1 \end{cases}$

(c) $f(x) = x(x^2 - 1) \quad -1 < x \leq 1$

(d) $f(x) = \begin{cases} -2(x+1) & -1 < x \leq -1/2 \\ -1 & -1/2 < x \leq 0 \\ 1 & 0 < x \leq 1/2 \\ -2(x-1) & 1/2 < x \leq 1 \end{cases}$

2. Now answer these questions based on your observations from (a). They relate to the convergence of the series – that is how rapidly the series approaches the function as N increases. Carefully written answers will receive more marks.

- (a) Compare and contrast the convergence you observe for
- (i) the function(s) with jumps (discontinuities)
 - (ii) the function(s) that are continuous (no jumps) but have corners
 - (iii) the function(s) that have no jumps and no corners.

[9 marks]

- (b) Discuss in more detail the convergence you observe for the function(s) with a jump and the function(s) with a jump and a corner.

[5 marks]

- (c) Investigate any connection between the formula for the series and its convergence.

[12 marks]

Turn over for important notes

Notes

- **Your answers should be submitted in the form of a summary report no longer than four sides of A4 including graphs. All other working used such as hand calculation and MAPLE worksheets must be included as appendices.**
- **The University views plagiarism (academic dishonesty) very seriously. Although we would not wish to discourage you from talking to other students about this assignment it is essential that the final submission is your own work. Ignoring this advice may lead to disciplinary action being taken.**
- **Any student may be required to undertake a viva (oral examination) to confirm their understanding of work submitted for assessment.**