

1. Considering the original function

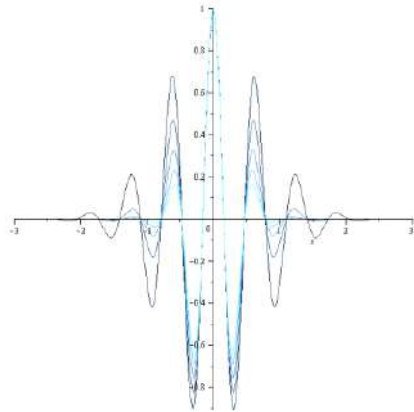
(a) Fourier series :

With the function $f := t \rightarrow \cos(b t) e^{-a t^2}$ $a > 0$ and $b \in \mathbb{R}$; $F(\omega) = \frac{\sqrt{\pi} \cosh\left(\frac{1}{2} \frac{\omega b}{a}\right) e^{-\frac{1}{4} \frac{b^2 + \omega^2}{a}}}{\sqrt{a}}$

(b) Study of the behaviour of $f(t)$ and $F(\omega)$ by changing a and b coefficients (see graphs on the Maple Worksheet) :

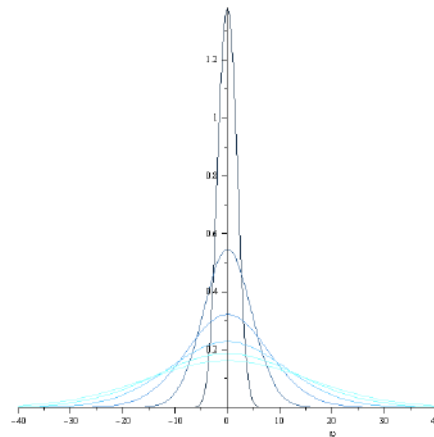
$f(t)$ with $a=1, 2, 3, 4, 5$ and $b=10$:
The **darker** curve is $a=1$, the **lighter** curve is $a=5$.

We can clearly see that if a is big, the amplitude of $f(t)$ gets lower and vice versa. The period always stays unchanged.



$F(\omega)$ with $a=1, 10, 30, 60, 90, 120$ and $b=1$:

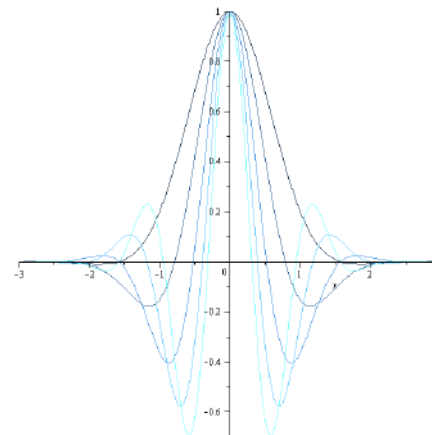
The **darker** curve is $a=1$, the **lighter** curve is $a=120$. We can clearly see that if b is big, the period of $F(\omega)$ gets higher but the amplitude is lower; and vice versa.



$f(t)$ with $b=1, 2, 3, 4, 5$ and $a=1$:

The **darker** curve is $b=1$, the **lighter** curve is $b=5$.

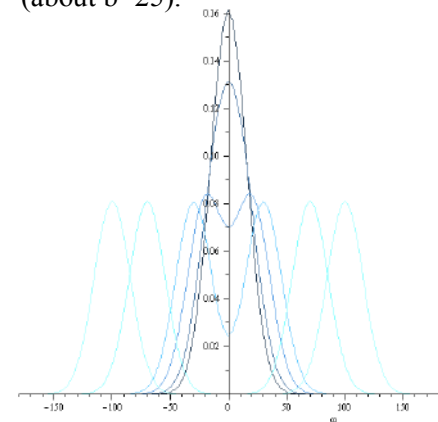
We can clearly see that if b is big, the period of $f(t)$ gets lower but the amplitude is higher; and vice versa.



$F(\omega)$ with $b=1, 10, 20, 30, 70, 100$ and $a=1$:

The **darker** curve is $b=1$, the **lighter** curve is $b=120$. We can clearly see that if b is big, the offset of $F(\omega)$ from zero gets higher ; and vice versa.

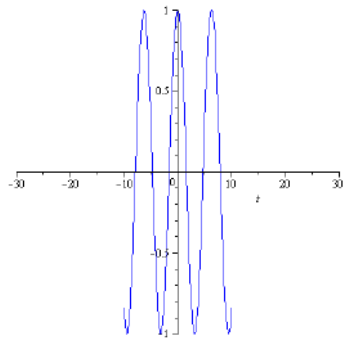
At a certain stage, we observe that the amplitude stays the same (about $b=25$).



Conclusion:

Let f be a function (or signal) periodic with a period T . Joseph Fourier, French mathematician, claimed in a memorandum dated of 1807, that it was possible under certain conditions, to decompose a periodic function f as a infinite sum of sinusoidal signals. These infinite numbers of sinusoidal terms are called harmonics. The curve with equation $y = |F(f(t))|$ is called the **spectrum** of f .

Special case: $a=0$: We can clearly see that when **a is naught**, the $F(\omega)$ curve isn't workable since we have $\sqrt{a}$ on the denominator, $\sqrt{0} = 0$!



2. Changing shape of the original fonction

All functions are studied with $a=1$ and $b=1$

- **$f(t+2)$: Delay** of ($a=-2$), the formulae says that $FT\{f(t-a)\} = e^{-i\omega a} F(\omega)$: shift in t

So the curve moves on the time axe from $t=0$ to $t=-2$

- **$f(5t)$: Scaling** of ($a=5$), the formulae says that $FT\{f(at)\} = \frac{1}{|a|} F(\frac{\omega}{a})$

If $|a|$ is large, then $f(ax)$ is around 0

and $\frac{1}{|a|} |F(\omega)|$ spreads out and flattens.

So the curve shrinks by a factor of $1/5$ of the time axe.

N.b : The case $a = -1$ leads to the time-reversal property

- **$f(4t+3)$: Scaling and Delay** : with a delay of 3, and a scaling of 4.

So the curve shrinks by a factor of $1/4$ and moves to $t=-3$ on the time axe.

- **df/dt : Differentiation** : $FT\left\{\frac{d^n f(t)}{dt^n}\right\} = (i\omega)^n F(\omega)$ so the function's period is unchanged

but the amplitude is changed by the derivative!

