

Coding and Transformation

Assignment 2: Fourier Transforms

1. Experiment with two factors of $f(t)$, a and b :

a. If,

$$f := t \rightarrow \cos(b t) e^{-a t^2}$$

The Fourier transform of this function is :

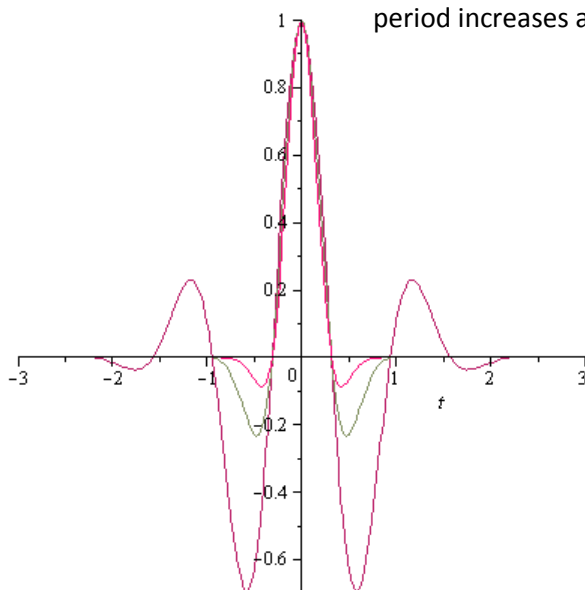
$$\frac{\sqrt{\pi} \cosh\left(\frac{1}{2} \frac{\omega b}{a}\right) e^{-\frac{1}{4} \frac{b^2 + \omega^2}{a}}}{\sqrt{a}}$$

cosh is cosinus hyperbolic, $\cosh(x) = 1/2(\exp(1/2*x) + \exp(-1/2*x))$

b. To determinate influence of a and b on the function, we will firstly study a and secondly b , in order to look at the graph of $f(t)$ and $F(\omega)$.

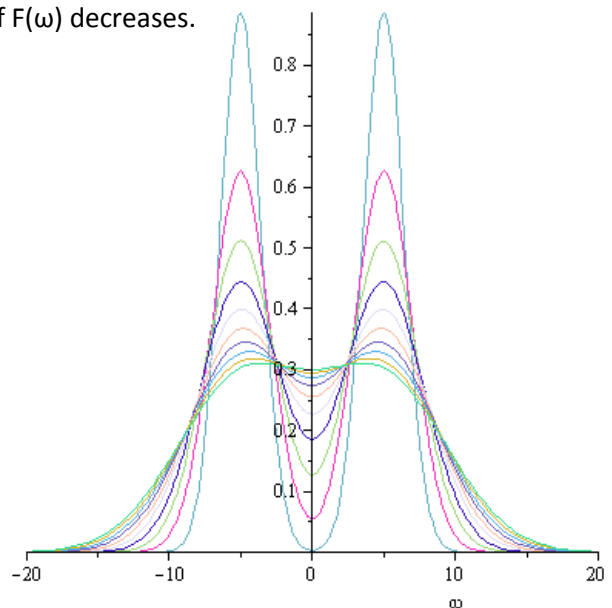
o Influence of a , with $b=5$:

For an easier comprehension we study $f(t)$ for a few value of a : 1,5,10. (For more value refer to the Maple worksheet). For $F(\omega)$ we look the function for : $1 \leq a \leq 10$. We can see that a changes the function shape. With a high value of a , $f(t)$ has less oscillations and their amplitudes are reduced. On $F(\omega)$, we can notice that a influence the amplitude and period. Indeed, if a increases, period increases and amplitude of $F(\omega)$ decreases.



Graph of $f(t)$

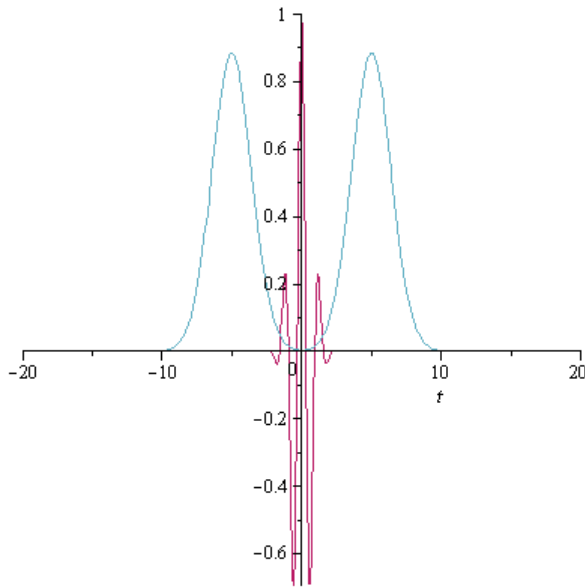
For $a=10$ the function is pink, 5 green, and 1 purple



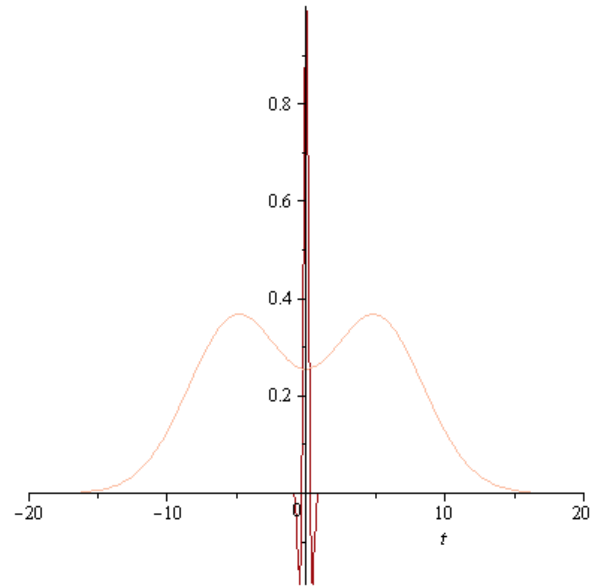
Graph of $F(\omega)$

For $a = 10$, the function is green and sky-blue with $a = 1$

For an easier comprehension, we choose to put on the same axes $f(t)$ and $F(\omega)$ for $\alpha = 1$ and $\alpha = 6$.



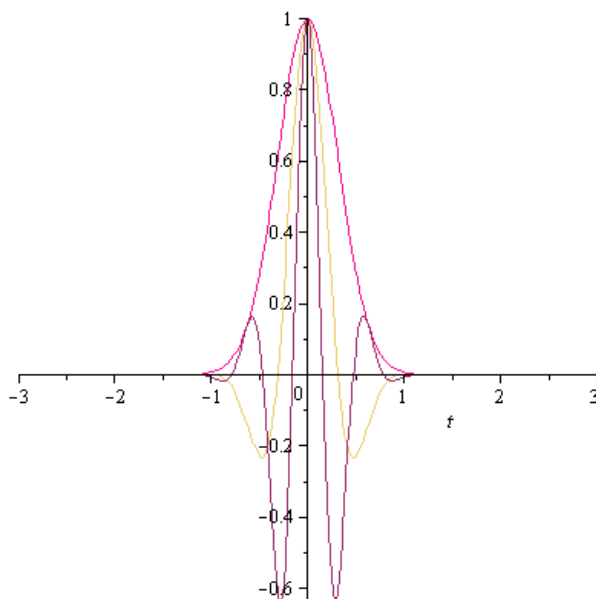
Graphs of $\alpha=1$



Graphs of $\alpha=6$

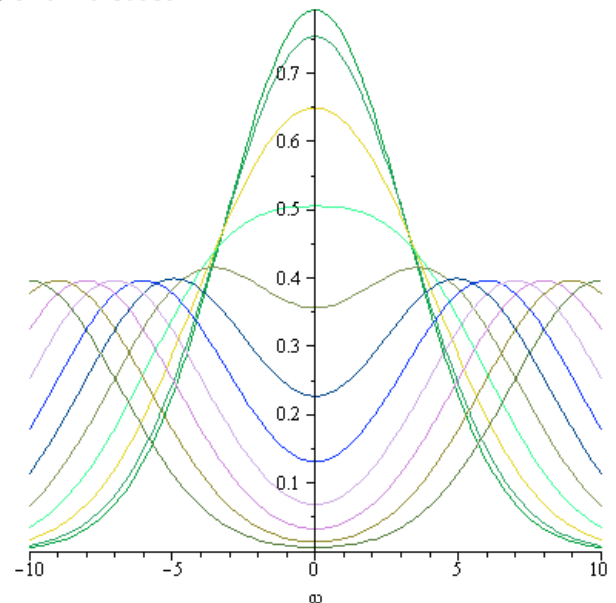
○ Influence of b , with $\alpha=5$:

On $f(t)$ and $F(\omega)$, b has also an influence. When we change the value of b , the frequency of $f(t)$ seems to change. There are more oscillations in a same period, when b increases. As we previously saw, for $f(t)$ we choose just 3 values of b : 0,5 and 10. For $F(\omega)$, we conclude that spicks move away from 0, when the value of b increases.



Graph of $f(t)$

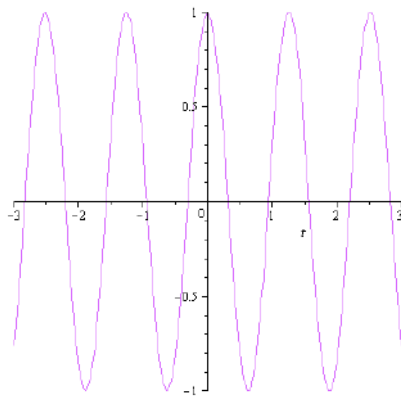
For $b=0$ the function is pink, 5 yellow, and 10 purple



Graph of $F(\omega)$

For $b = 2$, the function is yellow and purple with $b=8$

- Special case, if $a=0$:

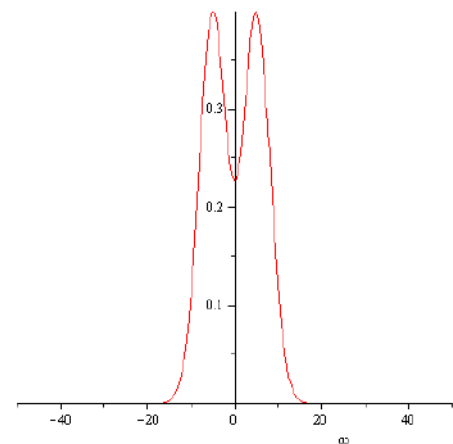
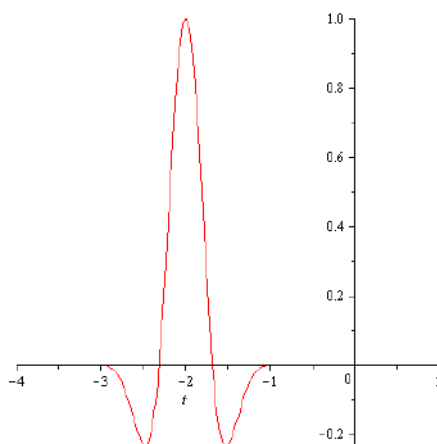


We can see that there is no attenuation for $f(t)$. This function is regular. Whereas $F(\omega)$ can't be plotted because we can't divide by 0!

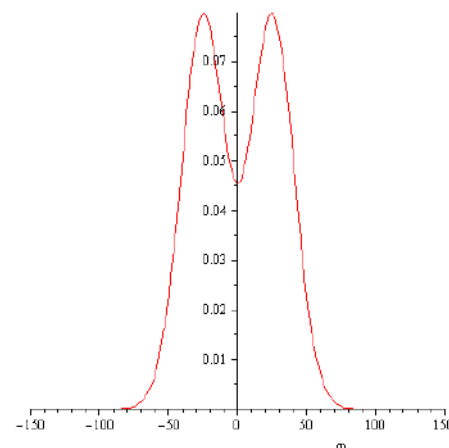
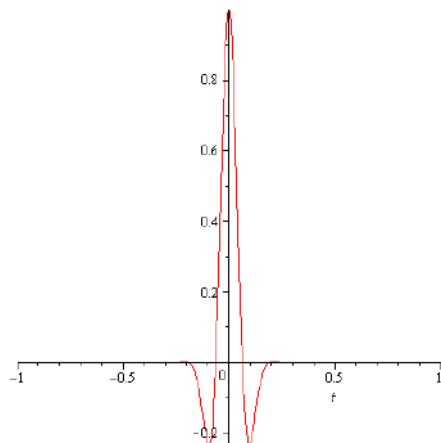
To conclude: We can suppose that $F(\omega)$, is a "specter" of $f(t)$. Its amplitude depends on the intensity of $f(t)$: the value of a . And its spicks depend of the frequencies of $f(t)$: the value of b .

2. Manipulation of Fourier transforms :

- a. Observation of a **delay**, $f(t+2)$: $f(t)$ moves to $t=-2$, $F(\omega)$ doesn't change.

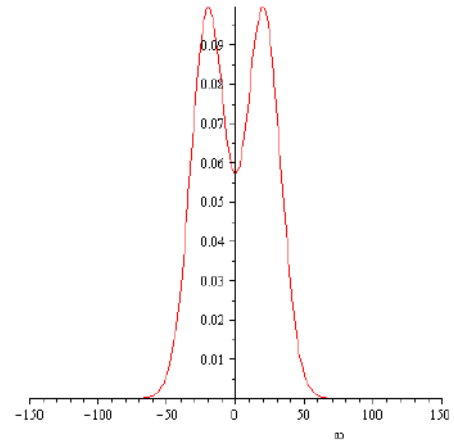
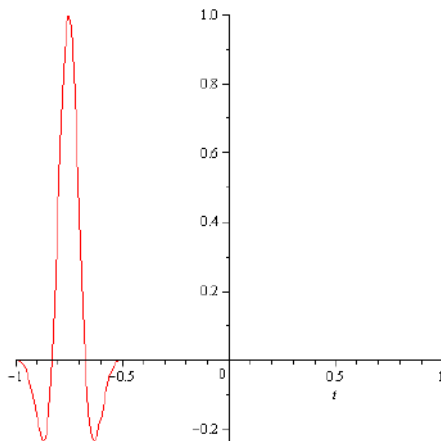


- b. Observation of a **scaling**, $f(5t)$: the period of $f(t)$ is divided by 5. The period of $F(\omega)$ is multiplied by 5 and the amplitude divided by 5.



- c. Observation of an association of **scaling** and **delay** : $f(4t+3)$. We can make an assumption before studying this case thanks to the precedent transformation. So, $f(t)$ will move on the right by $\frac{3}{4}$ and its period will be divided by 4. For $F(\omega)$, its amplitude and period will be divided by 4. Because:

$$f(4t + 3) = f(4(t - (-\frac{3}{4})))$$



Assomptions are verified on the graphs

- d. The latest transformation is a **differentiation**, $\frac{df}{dt}$: We can notice that the period of $f(t)$ is unchanged, but its shape and its amplitude are totally modified. For $F(\omega)$, just the amplitude is changed. The amplitude of $f(t)$ is modified by ± 4.5 and $F(\omega)$ by 2.3.

