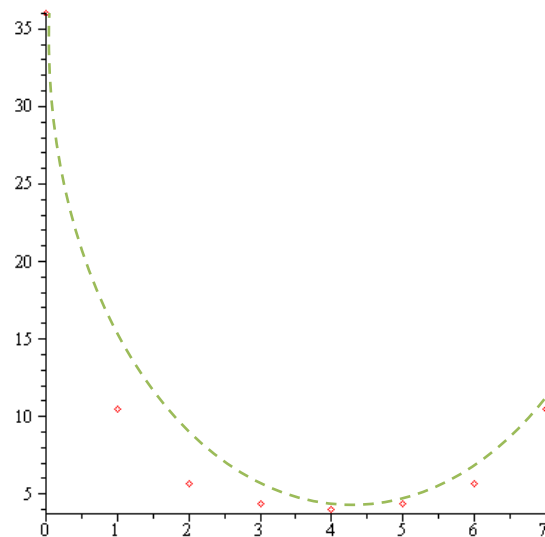


1. Code to find the Fast Fourier Transform with N=8

On the Mapple Sheet "Assigment3_1.ws"

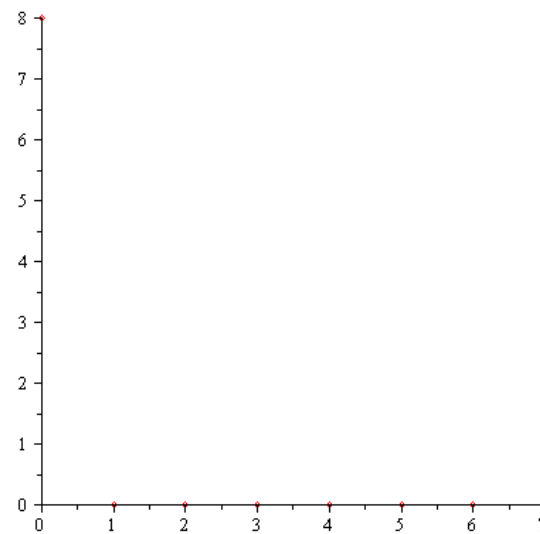
2. Special cases

(a) Case 1 : $f[j] = j + 1$, $j = 0 \dots 7$



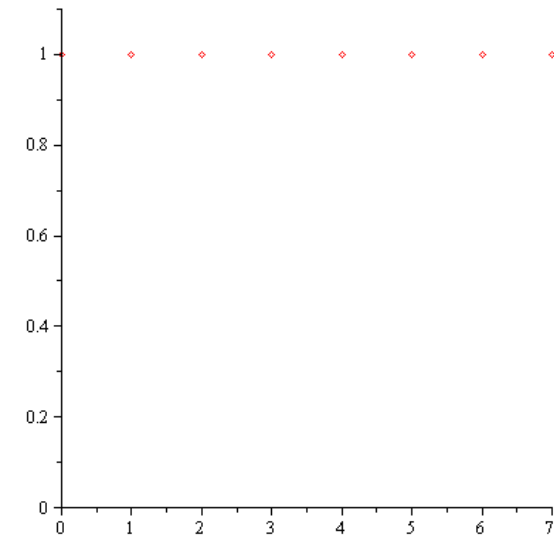
a) This case is a typical Fourier Tranform with several values of $F[j]$. We can see that values are organized in a curve, having the shap of an exponencial function.

(b) Case 2 : $f[j] = 1$, $j = 0 \dots 7$



b) In this case, $j=1$, that's a well known Time Fonction and its Fourier Transform is $2\pi\delta(\omega)$. $\delta(\omega)$ has an infinite value on 0, nought otherwise.

(c) Case 3 : $f[1] = 1$ and $f[j] = 0$, $j \neq 1$



c) In this case, we recognize the function $\partial(t)$. As we can see in the formulae book, the Fourier Transform of this kind of function is always 1. If we look at the graph, we can confirm these results.

3. Algorithms for $N > 3$

Cooley–Tukey FFT algorithm

It re-expresses the DFT of an arbitrary composite size $N = N_1 * N_2$ in terms of smaller DFTs with sizes N_1 and N_2 with the following formula:

$$X_{N_2 k_1 + k_2} = \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} x_{N_1 n_2 + n_1} e^{-\frac{2\pi i}{N_1 N_2} * (N_1 n_2 + n_1) * (N_2 k_1 + k_2)}$$

Prime-factor FFT algorithm :

It re-expresses the DFT of a size $N = N_1 * N_2$ as a two-dimensional $N_1 * N_2$ DFT, but only for the case N_1 and N_2 are prime. By applying the Prime Factor Algorithm recursively, these smaller transforms of size N_1 and N_2 can then be evaluated.

$$X_{N_2 N_2^{-1} k_1 + N_1 N_1^{-1} k_2} = \sum_{n_1=0}^{N_1-1} \left(\sum_{n_2=0}^{N_2-1} x_{N_2 n_1 + N_1 n_2} e^{-\frac{2\pi i}{N_2} * (n_2 k_2)} \right) e^{-\frac{2\pi i}{N_1} n_1 k_1}$$

Bruun's FFT algorithm :

Based on the ordinary Cooley-Tukey FFT algorithm, this algorithm uses a recursive polynomial-factorization approach, proposed for powers of 2. Its operations are involved only with real coefficients. Besides, Bruun's algorithm may be less precise than Cooley-Tukey with a finite numerical precision.

$$\Phi_{N,\alpha}(z) = \begin{cases} z^{2N} - 2 \cos(2\pi\alpha) z^N + 1 & \text{if } 0 < \alpha < 1 \\ z^{2N} - 1 & \text{if } \alpha = 0 \end{cases}$$

With $\varphi_N, \alpha(z)$ are polynomials , N are positive integers, α between $[0,1]$.