

Coding and Transformations

Assignment 2: Fourier Transforms

11/8/2010

User name: s004431a

Table of Contents

Conclusion.....	7
Appendices.....	8

Introduction

Function 1

The first function is

Function 2

Function 3

Conclusion

Appendices

Maple worksheet: function 1

```
> restart;
> with(plots):
> with(inttrans):
> alias(u=Heaviside):
> f:=t->2/(4+t^2);
```

$$f:=t\rightarrow\frac{2}{4+t^2}$$

```
> F(omega):=fourier(f(t),t,omega);
```

$$F(\omega):=\pi\left(e^{-2\omega}u(\omega)+e^{2\omega}u(-\omega)\right)$$

```
> alpha:=2:
```

```
> beta:=6:
```

```
> G(omega):=F(omega)*(u(omega+alpha)-u(omega-alpha));
```

$$G(\omega):=\pi\left(e^{-2\omega}u(\omega)+e^{2\omega}u(-\omega)\right)(u(\omega+2)-u(\omega-2))$$

```
> H(omega):=F(omega)*(u(omega-beta)+u(-omega-beta));
```

$$H(\omega):=\pi\left(e^{-2\omega}u(\omega)+e^{2\omega}u(-\omega)\right)(u(\omega-6)+u(-\omega-6))$$

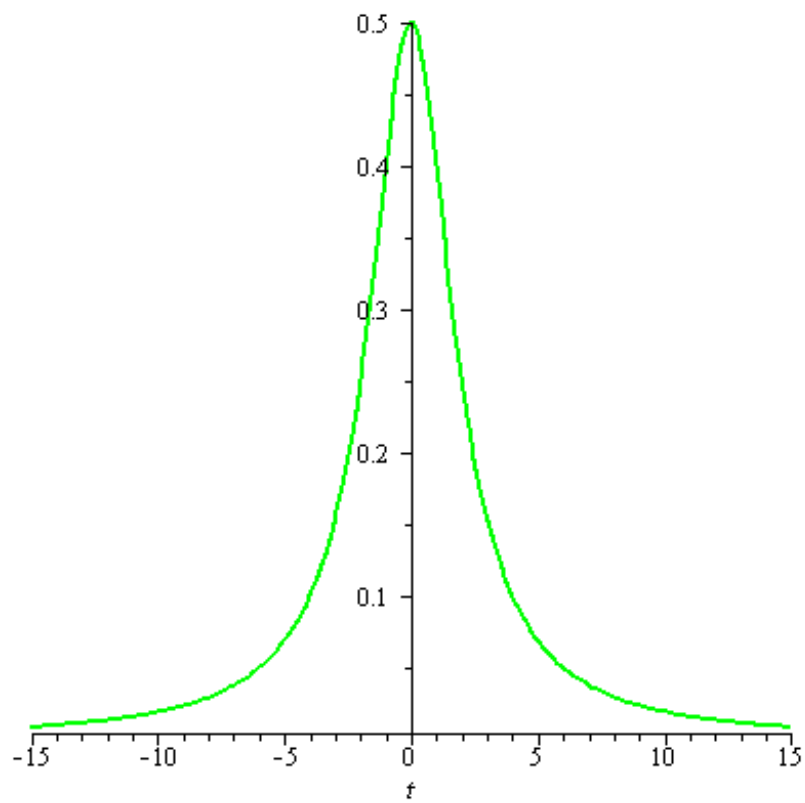
```
> g(t):=(1/2*Pi)*integrate(G(omega)*exp(I*omega*t),omega=-
infinity..infinity);
```

$$g(t):=\frac{1}{2}\pi^2\left(-\frac{1}{1t-2}+\frac{e^{21t-4}}{1t-2}-\frac{e^{-21t-4}}{1t+2}+\frac{1}{1t+2}\right)$$

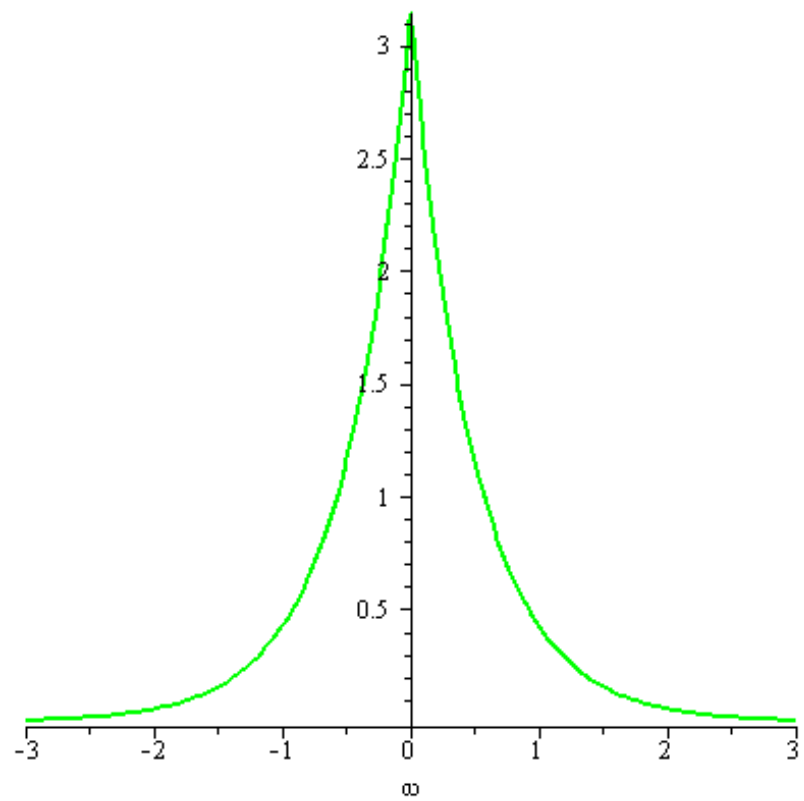
```
> h(t):=(1/2*Pi)*integrate(H(omega)*exp(I*omega*t),omega=-
infinity..infinity);
```

$$h(t):=\frac{1}{2}\pi^2\left(-\frac{e^{61t-12}}{1t-2}+\frac{e^{-61t-12}}{1t+2}\right)$$

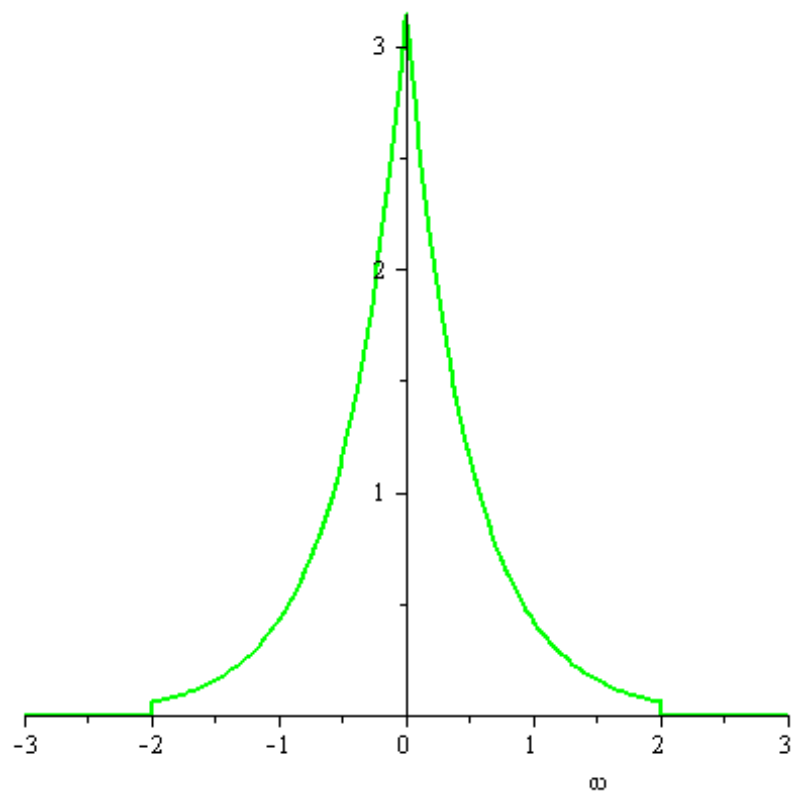
```
> plot(f(t), t=-15..15, colour=green, thickness=2);
```



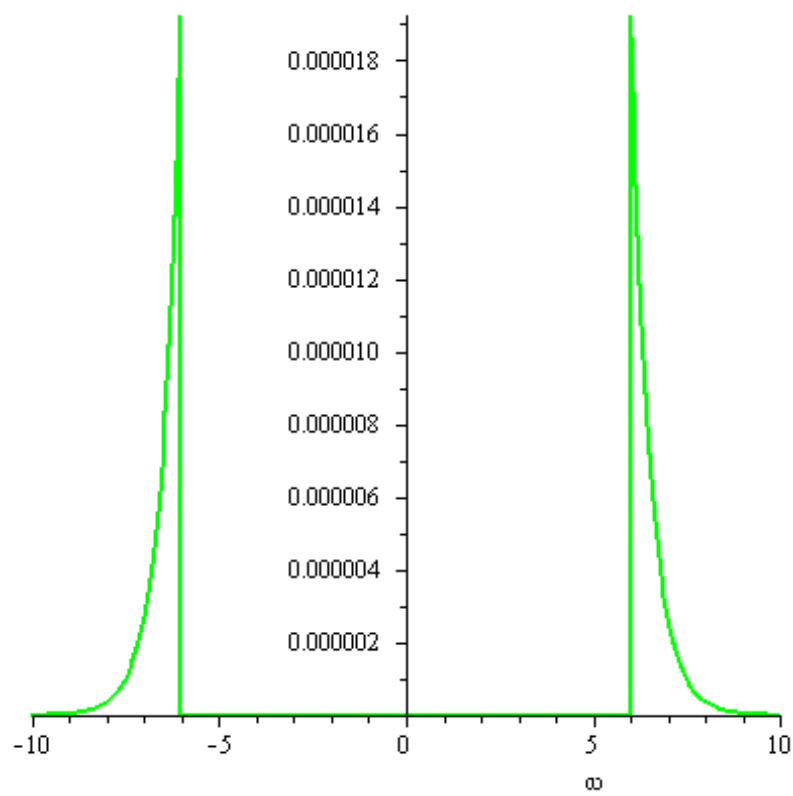
```
> plot(F(omega), omega=-3..3, colour=green, thickness=2);
```



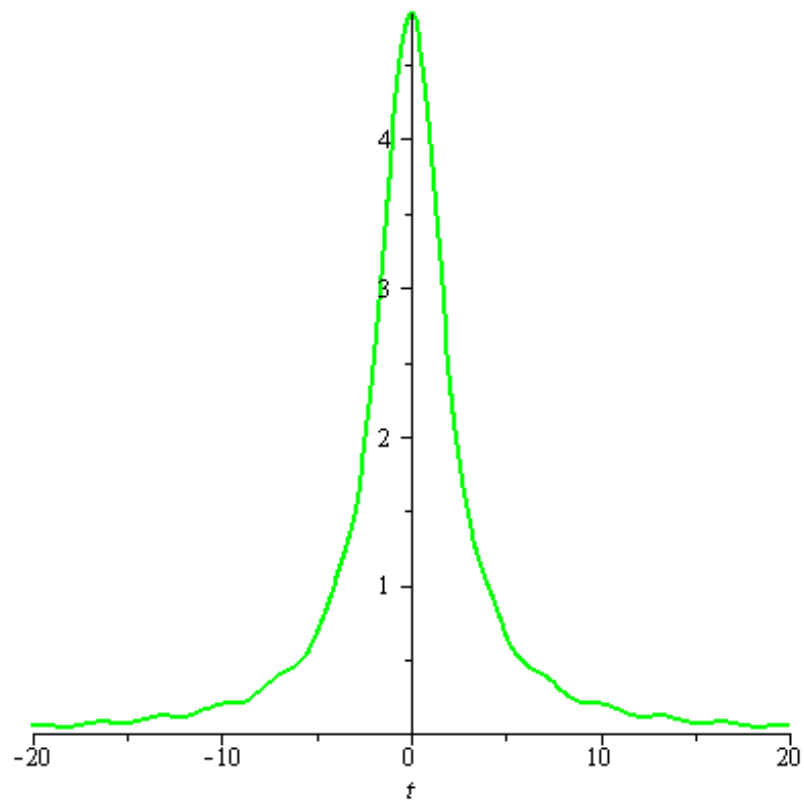
```
> plot(G(omega), omega=-3..3, colour=green, thickness=2);
```



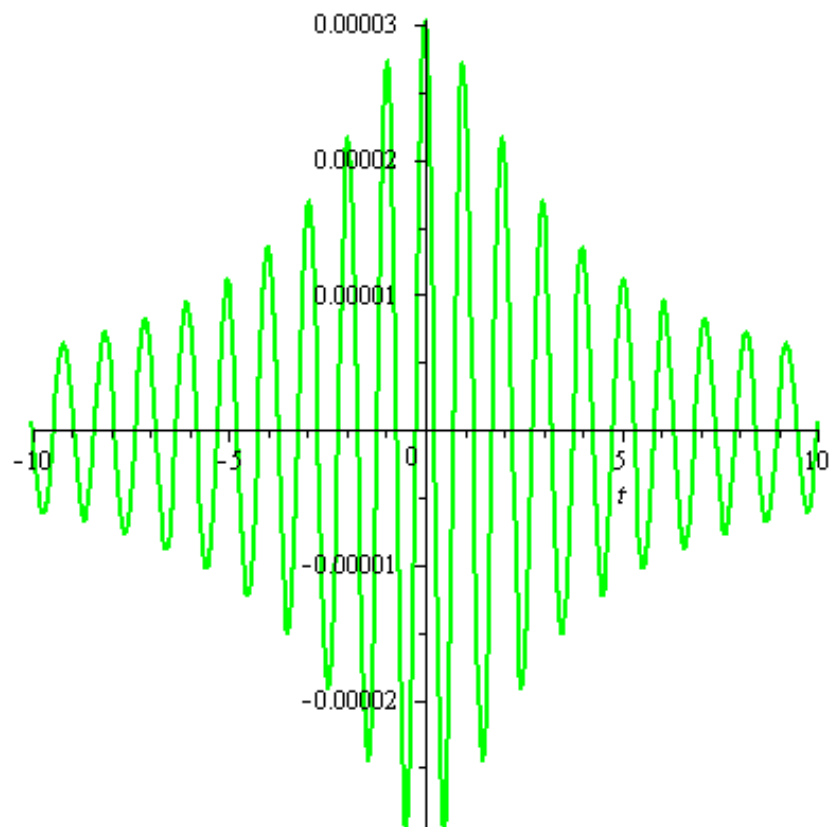
```
> plot(H(omega), omega=-10..10, colour=green, thickness=2);
```



```
> plot(g(t), t=-20..20, colour=green, thickness=2);
```



```
> plot(h(t), t=-10..10, colour=green, thickness=2);
```



Maple worksheet: function 2

```
> restart;
> with(plots):
> with(inttrans):
> alias(u=Heaviside):
> f:=t->(3*(1-abs(t)/2))*(u(t+2)-u(t-2));
```

$$f := t \rightarrow 3 \left(1 - \frac{1}{2} |t| \right) (u(t+2) - u(t-2))$$

```
> F(omega):=integrate(f(t)*exp(-I*omega*t), t=-
infinity..infinity);
```

$$\begin{aligned} F(\omega) := & -\frac{3Ie^{2I\omega}}{\omega} + \frac{3Ie^{-2I\omega}}{\omega} - \frac{3}{2} \frac{(1-2I\omega)e^{2I\omega}}{\omega^2} \\ & - \frac{3}{2} \frac{e^{-2I\omega} + 2Ie^{-2I\omega}\omega - 2}{\omega^2} + \frac{1}{2} \frac{6I\omega - 3(e^{I\omega})^2 + 3}{\omega^2} \\ & + \frac{1}{2} \frac{-6I\omega + 3(e^{I\omega})^2 - 3}{\omega^2} \end{aligned}$$

```
> alpha:=2:
> beta:=6:
> G(omega):=F(omega)*(u(omega+alpha)-u(omega-alpha));
```

$$\begin{aligned} G(\omega) := & \left(-\frac{3Ie^{2I\omega}}{\omega} + \frac{3Ie^{-2I\omega}}{\omega} - \frac{3}{2} \frac{(1-2I\omega)e^{2I\omega}}{\omega^2} \right. \\ & - \frac{3}{2} \frac{e^{-2I\omega} + 2Ie^{-2I\omega}\omega - 2}{\omega^2} + \frac{1}{2} \frac{6I\omega - 3(e^{I\omega})^2 + 3}{\omega^2} \\ & \left. + \frac{1}{2} \frac{-6I\omega + 3(e^{I\omega})^2 - 3}{\omega^2} \right) (u(\omega+2) - u(\omega-2)) \end{aligned}$$

```
> H(omega):=F(omega)*(u(omega-beta)+u(-omega-beta));
```

$$\begin{aligned} H(\omega) := & \left(-\frac{3Ie^{2I\omega}}{\omega} + \frac{3Ie^{-2I\omega}}{\omega} - \frac{3}{2} \frac{(1-2I\omega)e^{2I\omega}}{\omega^2} \right. \\ & - \frac{3}{2} \frac{e^{-2I\omega} + 2Ie^{-2I\omega}\omega - 2}{\omega^2} + \frac{1}{2} \frac{6I\omega - 3(e^{I\omega})^2 + 3}{\omega^2} \\ & \left. + \frac{1}{2} \frac{-6I\omega + 3(e^{I\omega})^2 - 3}{\omega^2} \right) (u(\omega-6) + u(-\omega-6)) \end{aligned}$$

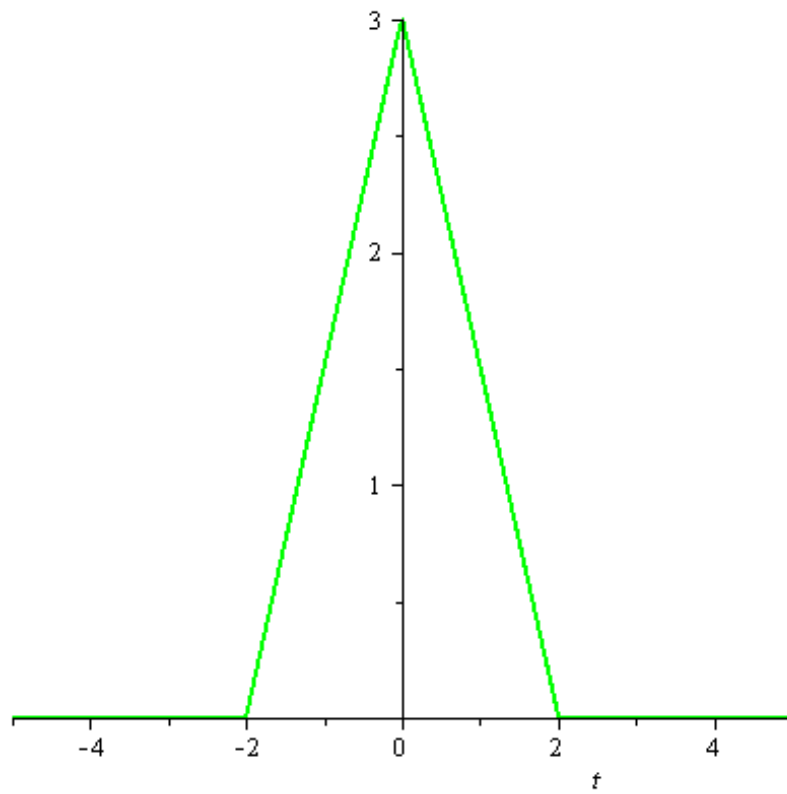
```
> g(t):=(1/2*Pi)*integrate(G(omega)*exp(I*omega*t), omega=-
infinity..infinity);
```

$$\begin{aligned}
g(t) := & \frac{1}{2} \pi \left(\frac{3}{2} (It + 2I) \left(\frac{1}{2} \frac{e^{-2It-4I}}{It + 2I} - \text{Ei}(1, 2It + 4I) \right) \right. \\
& - \frac{3}{2} (It + 2I) \left(-\frac{1}{2} \frac{e^{2It+4I}}{It + 2I} - \text{Ei}(1, -2It - 4I) \right) + \frac{3}{2} (It \\
& - 2I) \left(\frac{1}{2} \frac{e^{-2It+4I}}{It - 2I} - \text{Ei}(1, 2It - 4I) \right) - \frac{3}{2} (It - 2I) \left(\right. \\
& - \frac{1}{2} \frac{e^{2It-4I}}{It - 2I} - \text{Ei}(1, -2It + 4I) \left. \right) - 3It \left(-\frac{\frac{1}{2} I e^{-2It}}{t} \right. \\
& \left. - \text{Ei}(1, 2It) \right) + 3It \left(\frac{\frac{1}{2} I e^{2It}}{t} - \text{Ei}(1, -2It) \right) \\
& + \frac{1}{4} \frac{1}{(e^{It})^2 (e^I)^4} \left(12I \text{Ei}(1, 2It) t (e^{It})^2 (e^I)^4 - 12I \text{Ei}(1, 2It \right. \\
& + 4I) (e^{It})^2 (e^I)^4 + 12I \text{Ei}(1, 2It - 4I) (e^{It})^2 (e^I)^4 - 6I \text{Ei}(1, \\
& 2It - 4I) t (e^{It})^2 (e^I)^4 - 6I \text{Ei}(1, 2It + 4I) t (e^{It})^2 (e^I)^4 \\
& - 6It \ln(It + 2I) (e^{It})^2 (e^I)^4 + 12It \ln(It) (e^{It})^2 (e^I)^4 \\
& - 6It \ln(It - 2I) (e^{It})^2 (e^I)^4 + 12I \ln(It - 2I) (e^{It})^2 (e^I)^4 \\
& - 12I \ln(It + 2I) (e^{It})^2 (e^I)^4 + 3 + 3 (e^I)^8 - 6 (e^I)^4 \left. \right) \\
& + \frac{1}{4} \frac{1}{(e^{It})^2 (e^I)^4} \left(-3 - 3 (e^I)^8 + 6 (e^I)^4 - 12I \text{Ei}(1, \right. \\
& 2It) t (e^{It})^2 (e^I)^4 + 6I \text{Ei}(1, 2It - 4I) t (e^{It})^2 (e^I)^4 + 6I \text{Ei}(1, \\
& 2It + 4I) t (e^{It})^2 (e^I)^4 + 6It \ln(-It + 2I) (e^{It})^2 (e^I)^4 \\
& - 12It \ln(-It) (e^{It})^2 (e^I)^4 + 6It \ln(-It - 2I) (e^{It})^2 (e^I)^4 \\
& + 12I \text{Ei}(1, 2It + 4I) (e^{It})^2 (e^I)^4 - 12I \text{Ei}(1, 2It \\
& - 4I) (e^{It})^2 (e^I)^4 + 12I \ln(-It - 2I) (e^{It})^2 (e^I)^4 - 12I \ln(-It \\
& + 2I) (e^{It})^2 (e^I)^4 \left. \right)
\end{aligned}$$

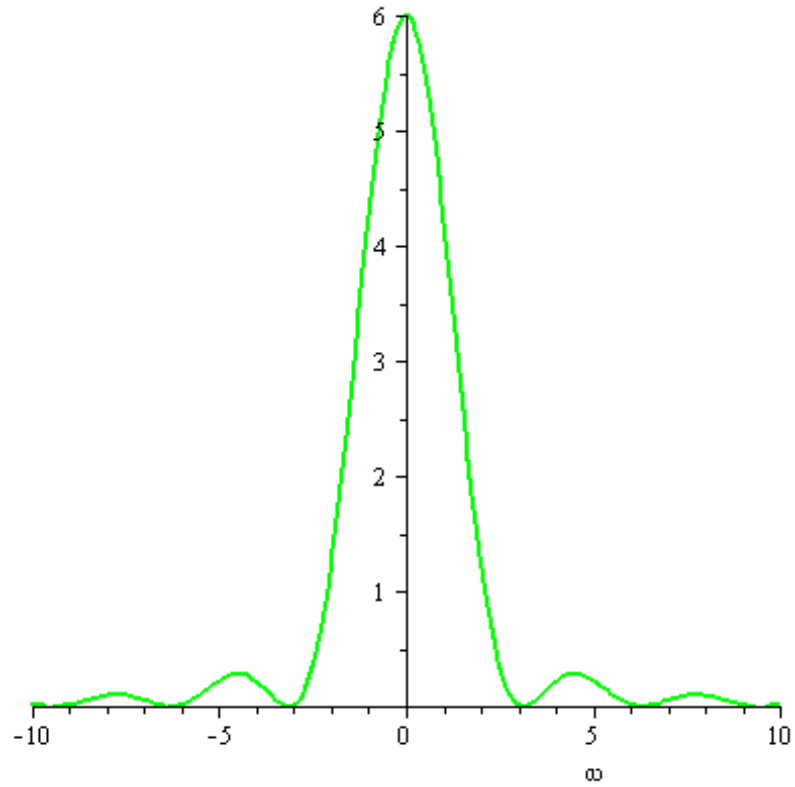
> h(t):=(1/2*Pi)*integrate(H(omega)*exp(I*omega*t),omega=-infinity..infinity);

$$\begin{aligned}
h(t) := & \frac{1}{2} \pi \left(\left(-\frac{3}{2} I t - 3 I \right) \left(\operatorname{Iargument}(-t-2) - I \left(\right. \right. \right. \\
& -\frac{1}{4} (\operatorname{csgn}(I(t+2)) + \operatorname{csgn}(I(2+\overline{t}))) \pi + \frac{1}{2} \pi + \pi \operatorname{signum}(t \\
& + 2) \left. \left. \left. \right) \right) + 3 I t \operatorname{Ei}(1, -6 I t) - 3 I \operatorname{Ei}(1, -6 I t - 12 I) \right. \\
& -\frac{1}{4} e^{6 I(t+2)} + \frac{3}{2} I t \operatorname{Ei}(1, 6 I t - 12 I) - \frac{3}{2} I t \operatorname{Ei}(1, -6 I t \\
& + 12 I) - \frac{1}{4} e^{-6 I(t+2)} + \left(-\frac{3}{2} I t + 3 I \right) \left(\operatorname{Iargument}(-t+2) \right. \\
& - I \left(-\frac{1}{4} (\operatorname{csgn}(I(t-2)) + \operatorname{csgn}(I(-2+\overline{t}))) \pi + \frac{1}{2} \pi \right. \\
& + \pi \operatorname{signum}(t-2) \left. \left. \left. \right) \right) - 3 I t \operatorname{Ei}(1, 6 I t) + 3 I t \left(\operatorname{Iargument}(-t) \right. \\
& + \frac{1}{4} I \pi \operatorname{csgn}(I t) + \frac{1}{4} I \pi \operatorname{csgn}(I \overline{t}) - \frac{1}{2} I \pi - I \pi \operatorname{signum}(t) \left. \right) \\
& -\frac{1}{4} e^{6 I(t-2)} + 3 I \operatorname{Ei}(1, 6 I t + 12 I) + 3 I \operatorname{Ei}(1, -6 I t + 12 I) \\
& -\frac{1}{4} e^{-6 I(t-2)} - 3 I t \left(\operatorname{Iargument}(t) - \frac{1}{4} I \pi \operatorname{csgn}(I t) \right. \\
& -\frac{1}{4} I \pi \operatorname{csgn}(I \overline{t}) - \frac{1}{2} I \pi + I \pi \operatorname{signum}(t) \left. \right) + \frac{1}{2} e^{6 I t} \\
& -\frac{3}{2} I t \operatorname{Ei}(1, -6 I t - 12 I) + \frac{1}{2} e^{-6 I t} + \frac{3}{2} I t \operatorname{Ei}(1, 6 I t + 12 I) \\
& + \left(\frac{3}{2} I t + 3 I \right) \left(\operatorname{Iargument}(t+2) - I \left(\frac{1}{4} (\operatorname{csgn}(I(t+2)) \right. \right. \\
& + \operatorname{csgn}(I(2+\overline{t}))) \pi + \frac{1}{2} \pi - \pi \operatorname{signum}(t+2) \left. \left. \right) \right) + \left(\frac{3}{2} I t \right. \\
& - 3 I \left. \right) \left(\operatorname{Iargument}(t-2) - I \left(\frac{1}{4} (\operatorname{csgn}(I(t-2)) + \operatorname{csgn}(I(-2 \\
& + \overline{t}))) \pi + \frac{1}{2} \pi - \pi \operatorname{signum}(t-2) \right) \right) - 3 I \operatorname{Ei}(1, 6 I t - 12 I) \left. \right)
\end{aligned}$$

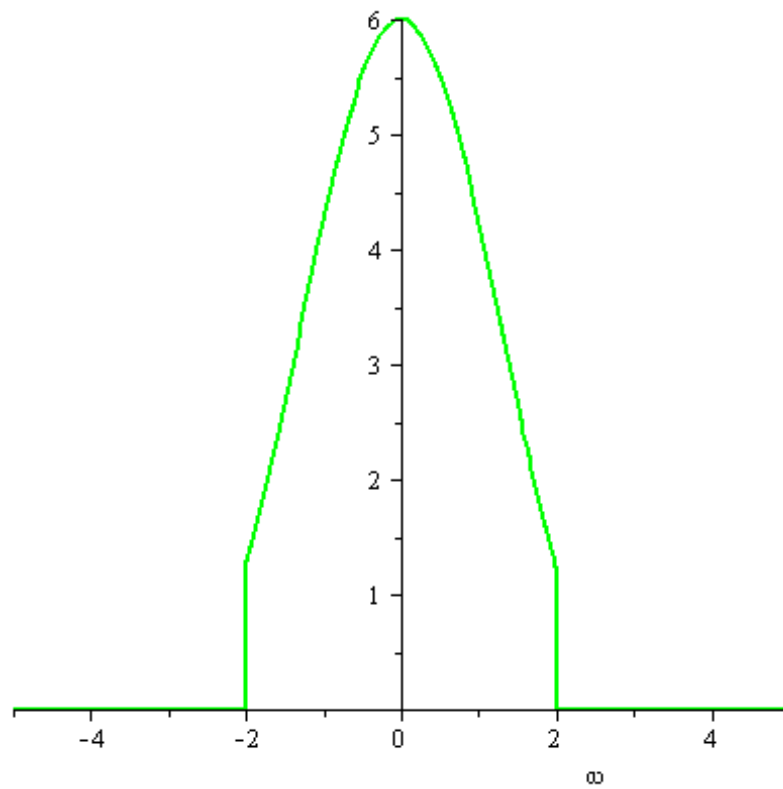
> plot(f(t), t=-5..5, colour=green, thickness=2) ;



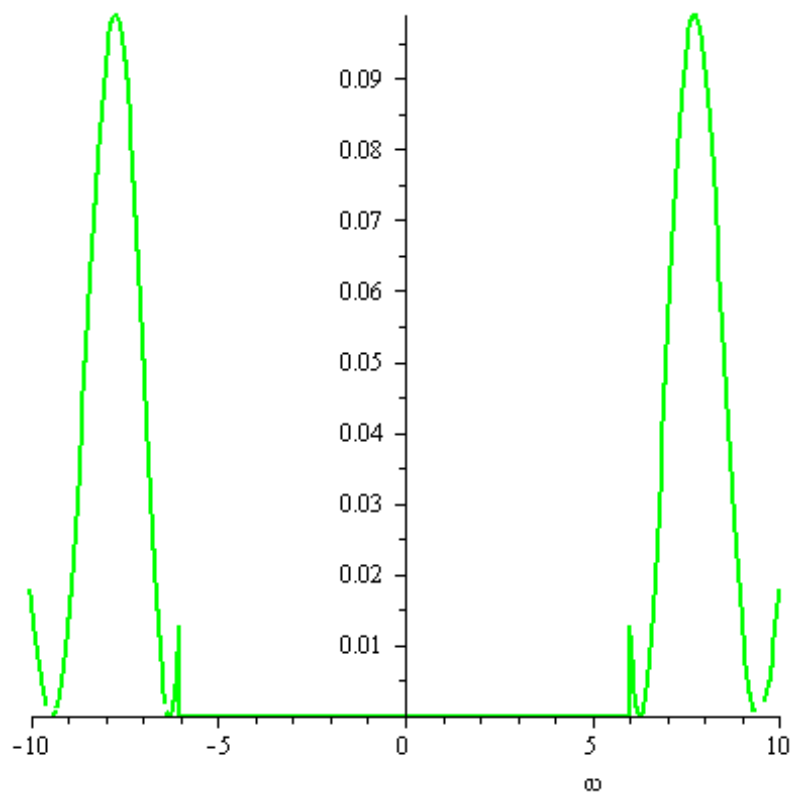
```
> plot(F(omega) , omega=-10..10 , colour=green , thickness=2) ;
```



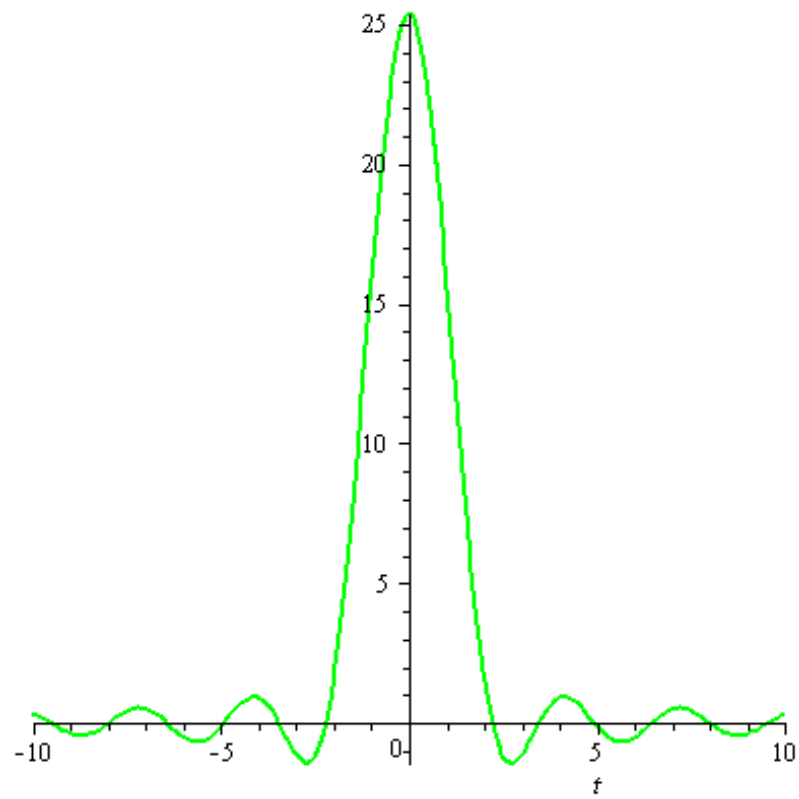
```
> plot(G(omega) , omega=-5..5 , colour=green , thickness=2) ;
```



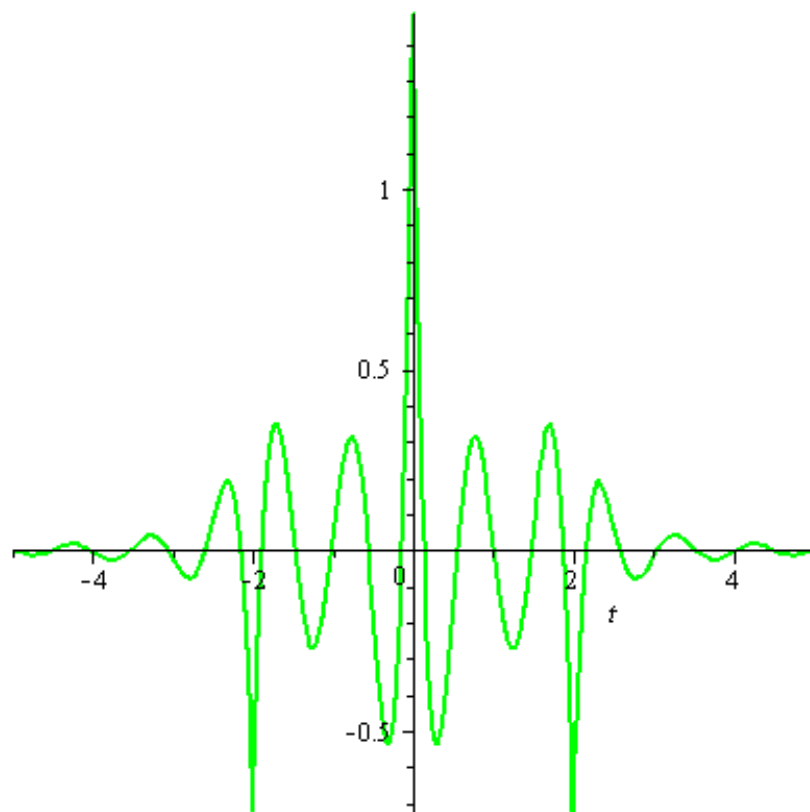
```
> plot(H(omega), omega=-10..10, colour=green, thickness=2);
```



```
> plot(Re(g(t)), t=-10..10, colour=green, thickness=2);
```



```
> plot(Re(h(t)), t=-5..5, colour=green, thickness=2);
```



Maple worksheet: function 3

```
> restart;
> with(plots):
> with(inttrans):
> alias(u=Heaviside):
> f:=t->(1/3)*(u(t+3)-u(t-3));
```

$$f:=t \rightarrow \frac{1}{3} u(t+3) - \frac{1}{3} u(t-3)$$

```
> F(omega):=fourier(f(t),t,omega);
```

$$F(\omega) := \frac{2}{3} \frac{\sin(3\omega)}{\omega}$$

```
> alpha:=2:
```

```
> beta:=6:
```

```
> G(omega):=F(omega)*(u(omega+alpha)-u(omega-alpha));
```

$$G(\omega) := \frac{2}{3} \frac{\sin(3\omega) (u(\omega+2) - u(\omega-2))}{\omega}$$

```
> H(omega):=F(omega)*(u(omega-beta)+u(-omega-beta));
```

$$H(\omega) := \frac{2}{3} \frac{\sin(3\omega) (u(\omega-6) + u(-\omega-6))}{\omega}$$

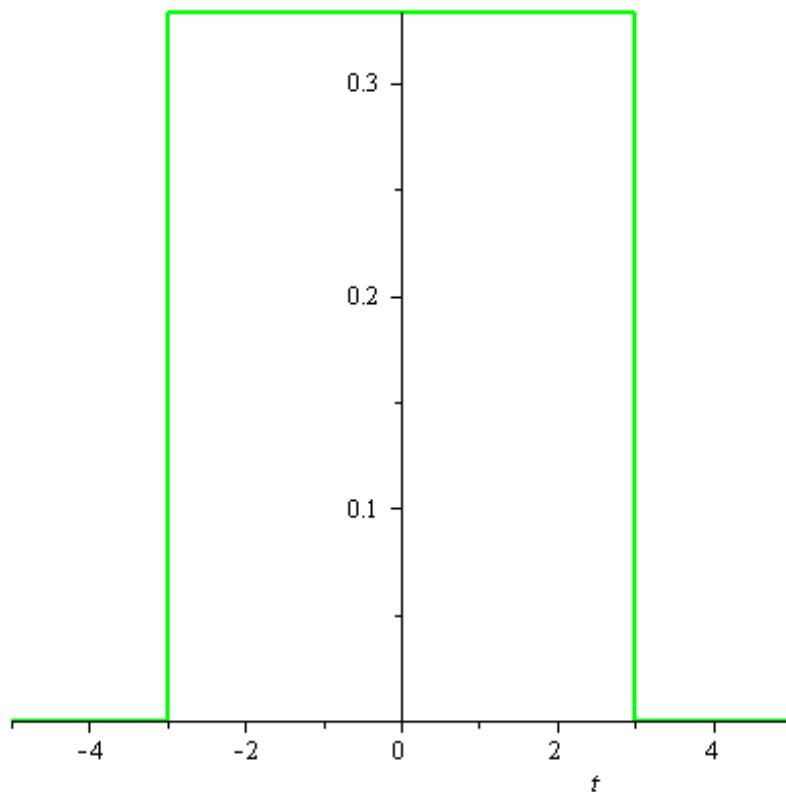
```
> g(t):=(1/2*Pi)*integrate(G(omega)*exp(I*omega*t),omega=-
infinity..infinity);
```

$$\begin{aligned} g(t) := & \frac{1}{2} \pi \left(-\frac{1}{3} \operatorname{Ei}(1, 2It + 6I) + \frac{1}{3} \operatorname{Ei}(1, 2It - 6I) \right. \\ & + \frac{1}{3} \operatorname{Ei}(1, -2It - 6I) - \frac{1}{3} \operatorname{Ei}(1, -2It + 6I) - \frac{1}{3} I \ln(It \\ & + 3I) + \frac{1}{3} I \ln(It - 3I) + \frac{1}{3} I \ln(-It - 3I) - \frac{1}{3} I \ln(-It \\ & \left. + 3I) \right) \end{aligned}$$

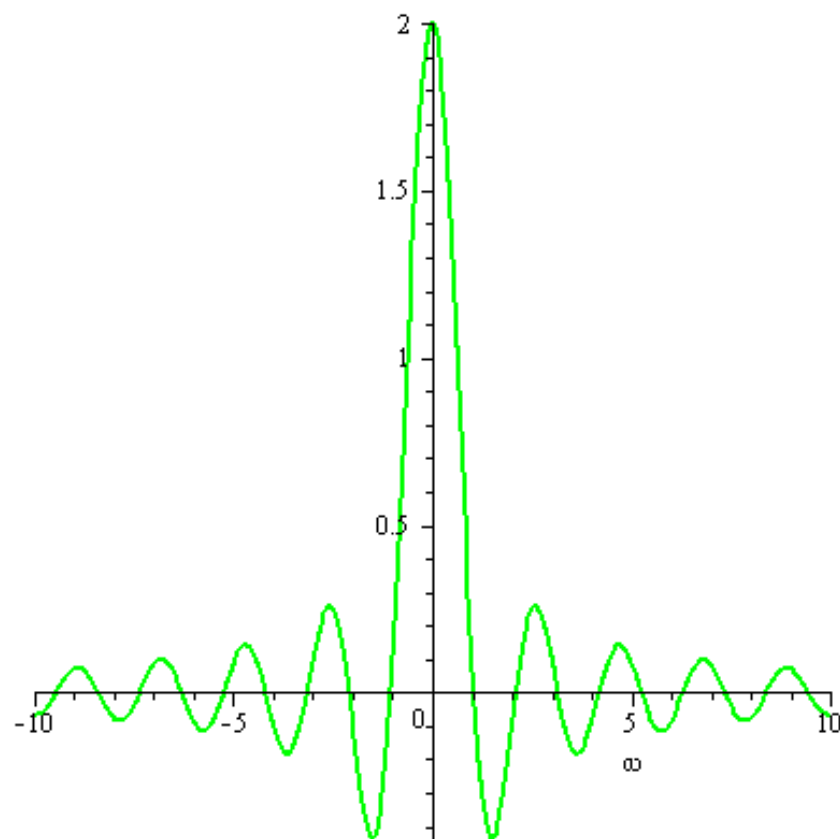
```
> h(t):=(1/2*Pi)*integrate(H(omega)*exp(I*omega*t),omega=-
infinity..infinity);
```

$$\begin{aligned}
 h(t) := & \frac{1}{2} \pi \left(\frac{1}{3} I \left(-I \arg(-t-3) + I \left(-\frac{1}{4} (\operatorname{csgn}(I(t+3)) \right. \right. \right. \\
 & \left. \left. \left. + \operatorname{csgn}(I(3+\overline{t})) \right) \pi + \frac{1}{2} \pi + \pi \operatorname{signum}(t+3) \right) \right) - \frac{1}{3} I \left(\right. \\
 & \left. -I \arg(-t+3) + I \left(-\frac{1}{4} (\operatorname{csgn}(I(t-3)) + \operatorname{csgn}(I(-3+ \right. \right. \\
 & \left. \left. \overline{t})) \right) \pi + \frac{1}{2} \pi + \pi \operatorname{signum}(t-3) \right) \right) - \frac{1}{3} \operatorname{Ei}(1, -6It - 18I) \\
 & + \frac{1}{3} \operatorname{Ei}(1, -6It + 18I) + \frac{1}{3} \operatorname{Ei}(1, 6It + 18I) - \frac{1}{3} \operatorname{Ei}(1, \\
 & 6It - 18I) - \frac{1}{3} I \left(-I \arg(t+3) + I \left(\frac{1}{4} (\operatorname{csgn}(I(t+3)) \right. \right. \\
 & \left. \left. + \operatorname{csgn}(I(3+\overline{t})) \right) \pi + \frac{1}{2} \pi - \pi \operatorname{signum}(t+3) \right) \right) + \frac{1}{3} I \left(\right. \\
 & \left. -I \arg(t-3) + I \left(\frac{1}{4} (\operatorname{csgn}(I(t-3)) + \operatorname{csgn}(I(-3+ \right. \right. \\
 & \left. \left. \overline{t})) \right) \pi + \frac{1}{2} \pi - \pi \operatorname{signum}(t-3) \right) \right) \Big)
 \end{aligned}$$

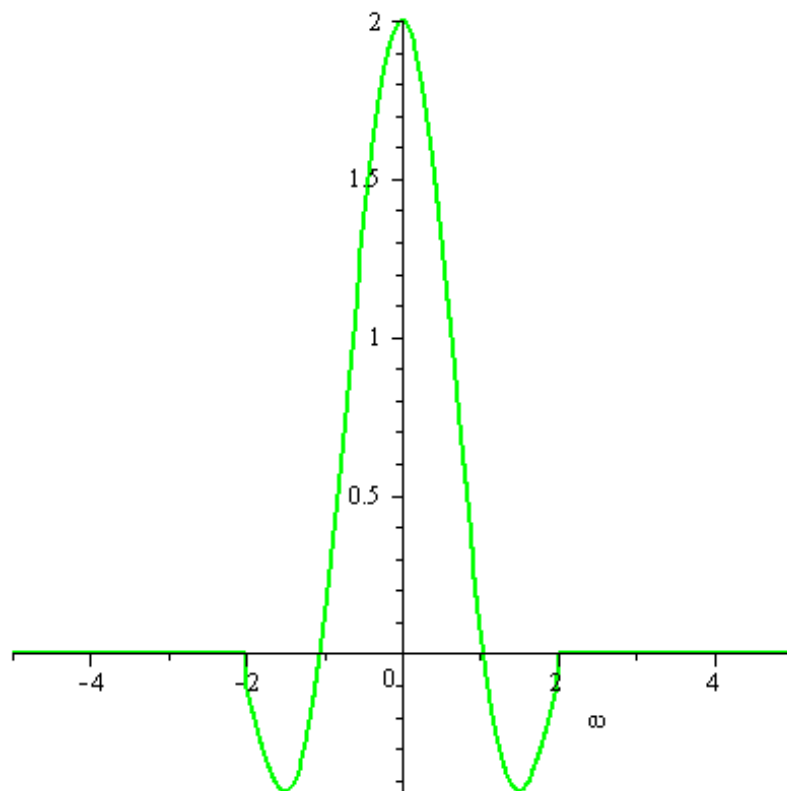
```
> plot(f(t), t=-5..5, colour=green, thickness=2);
```



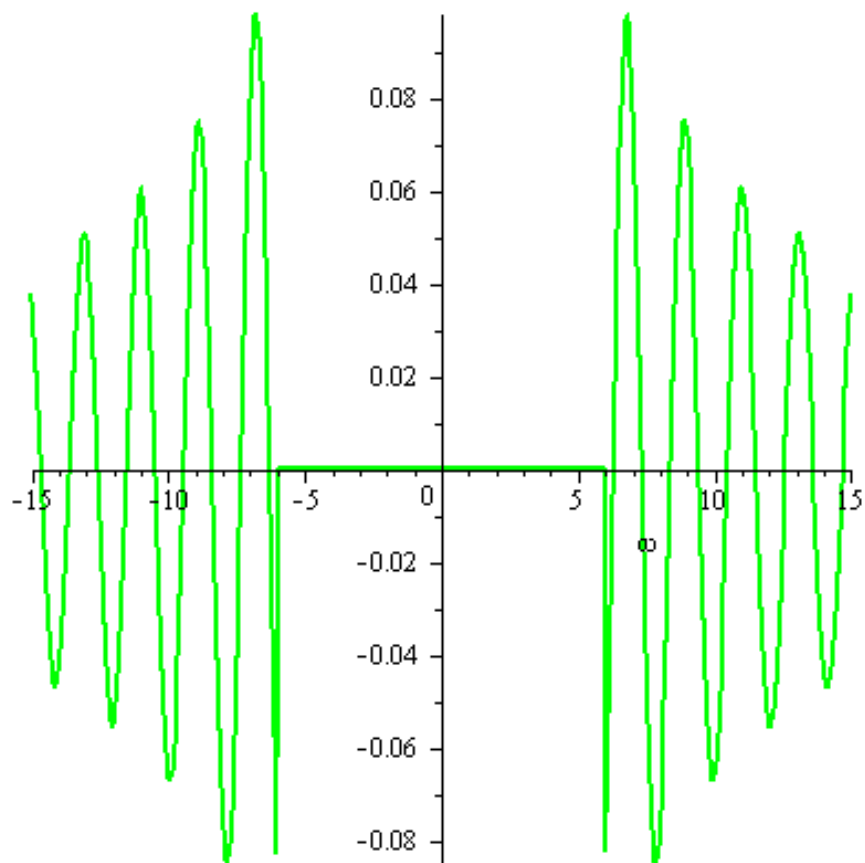
```
> plot(F(omega), omega=-10..10, colour=green, thickness=2);
```



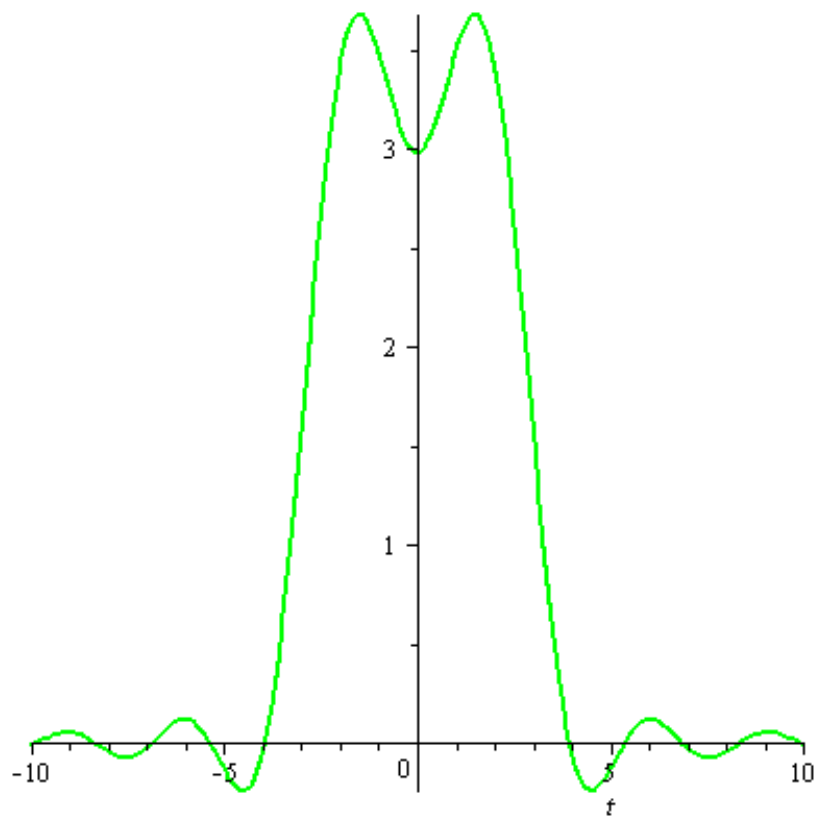
```
> plot(G(omega), omega=-5..5, colour=green, thickness=2);
```



```
> plot(H(omega), omega=-15..15, colour=green, thickness=2);
```



```
> plot(g(t), t=-10..10, colour=green, thickness=2);
```



```
> plot(h(t), t=-5..5, colour=green, thickness=2);
```

