

Coding and Transformations

Assignment 3: Fast Fourier Transforms

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Question 1

We have to calculate X_0 , X_1 and X_2 , then we can find the Fast Fourier Transforms in the case of $N = 16$.

To find X_0 :

$$X_0(a_2 a_1 a_0 b_0) = \sum_{a_3=0}^1 f(a_3 a_2 a_1 a_0) W^{8a_3 b_0}$$

With Maple:

```
> for a2 from 0 to 1 do
> for a1 from 0 to 1 do
> for a0 from 0 to 1 do
> for b0 from 0 to 1 do
> add(f[8*a3+4*a2+2*a1+a0]*W^(8*a3*b0), a3=0..1);
> X0[8*a2+4*a1+2*a0+b0]:=%;
> od: od: od: od:
```

To find X_1 :

$$X_1(a_1 a_0 b_0 b_1) = \sum_{a_2=0}^1 X_0(a_2 a_1 a_0 b_0) W^{4a_2(2b_1+b_0)}$$

With Maple:

```
> for a1 from 0 to 1 do
> for a0 from 0 to 1 do
> for b0 from 0 to 1 do
> for b1 from 0 to 1 do
> add(X0[8*a2+4*a1+2*a0+b0]*W^(4*a2*(2*b1+b0)), a2=0..1);
> X1[8*a1+4*a0+2*b0+b1]:=%;
> od: od: od: od:
```

To find X_2 :

$$X_2(a_0 b_0 b_1 b_2) = \sum_{a_1=0}^1 X_1(a_1 a_0 b_0 b_1) W^{2a_1(4b_2+2b_1+b_0)}$$

With Maple:

```
> for a0 from 0 to 1 do
> for b0 from 0 to 1 do
> for b1 from 0 to 1 do
> for b2 from 0 to 1 do
> add(X1[8*a1+4*a0+2*b0+b1]*W^(2*a1*(4*b2+2*b1+b0)), a1=0..1);
> X2[8*a0+4*b0+2*b1+b2]:=%;
> od: od: od: od:
```

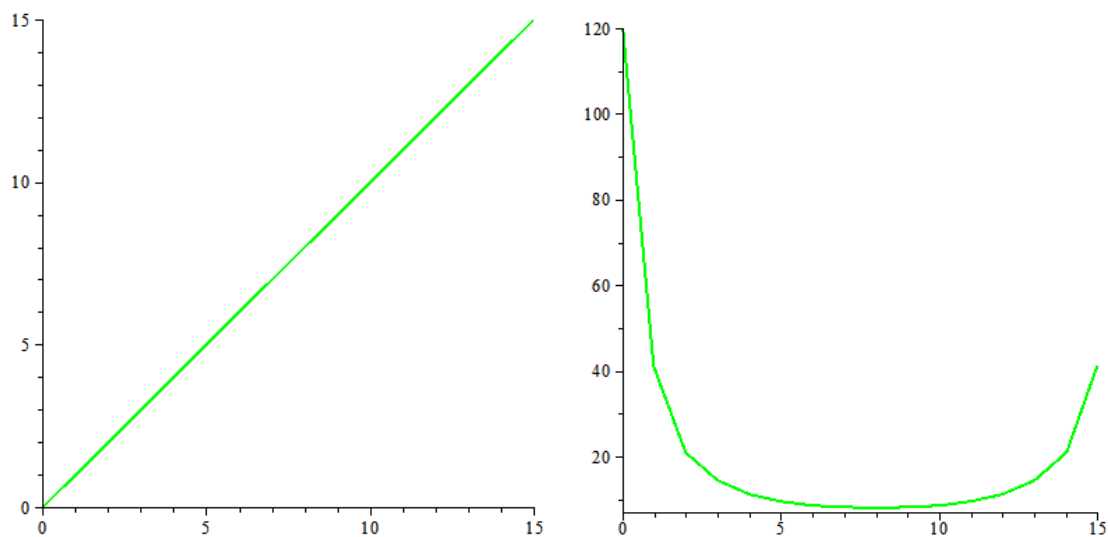
To find the Fast Fourier Transforms:

$$F(b_3 b_2 b_1 b_0) = \sum_{a_0=0}^1 X_2(a_0 b_0 b_1 b_2) W^{a_0(8b_3+4b_2+2b_1+b_0)}$$

With Maple:

```
> for b3 from 0 to 1 do
> for b2 from 0 to 1 do
> for b1 from 0 to 1 do
> for b0 from 0 to 1 do
>
add(X2[8*a0+4*b0+2*b1+b2]*W^(a0*(8*b3+4*b2+2*b1+b0)),a0=0..1);
> F[8*b3+4*b2+2*b1+b0]:=%;
> od: od: od: od:
```

These are the graphs of the function $f(x) = x$ and it Fast Fourier Transforms:



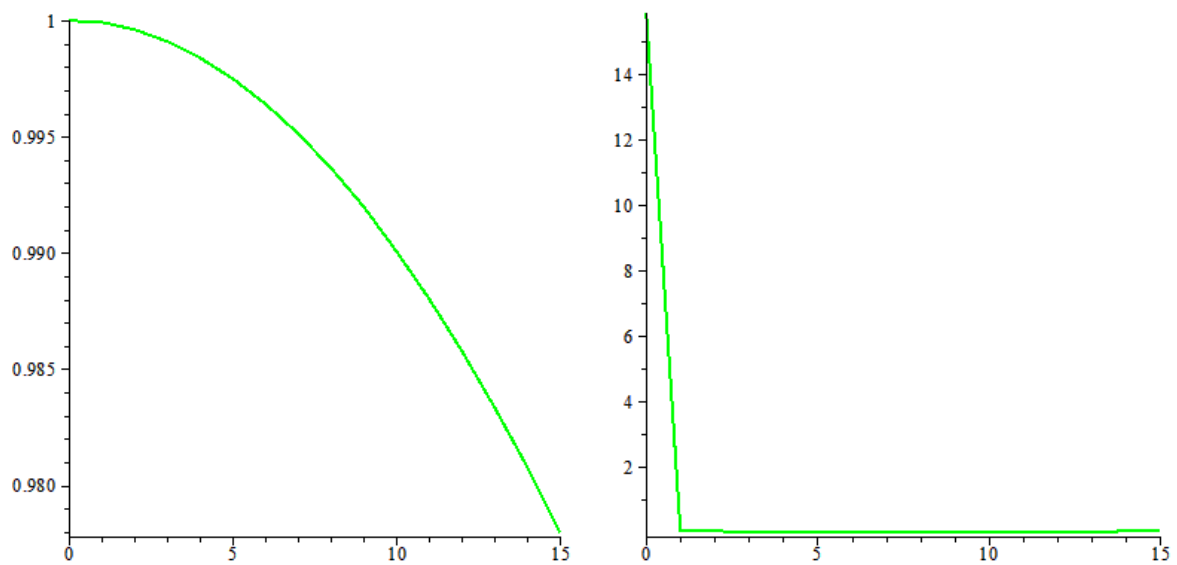
Question 2

To use the function $y(n) = \frac{1+a^2}{1+an^2}$, we just have to define $y(n)$ instead of $f(x) = x$ at the beginning of the program:

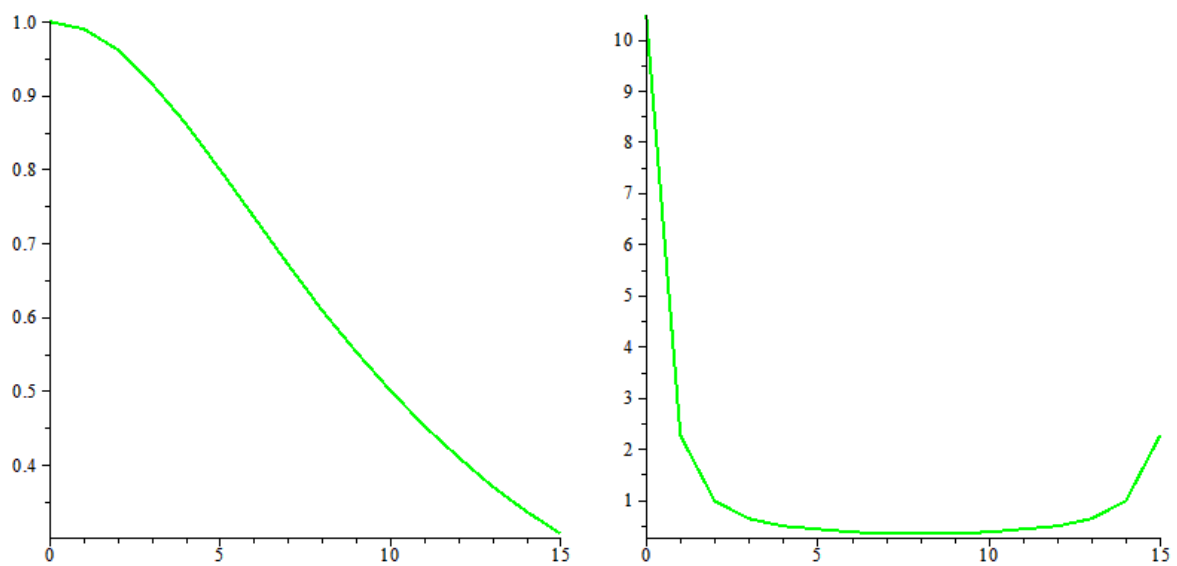
```
> for i from 0 to N-1 do Y[i]:=(1+a^2)/(1+a*i^2);    > od:
```

These are the graphs of $y(n)$ and it Fast Fourier Transforms with different values of a :

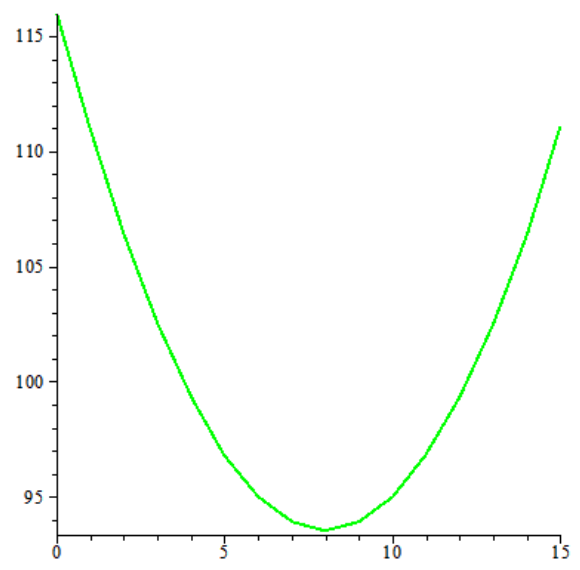
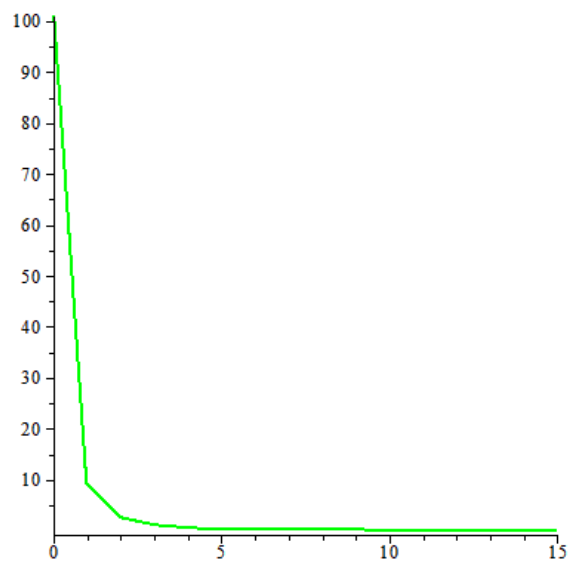
$a = 0.0001$



$a = 0.01$



$$a = 10$$



Question 3

To find the inverse Fast Fourier Transforms in the case $N = 8$, we have to find the Fast Fourier Transforms in the case $N = 8$ with:

$$X_0(a_1 a_0 b_0) = \sum_{a_2=0}^1 f(a_2 a_1 a_0) W^{4a_2 b_0}$$

$$X_1(a_0 b_0 b_1) = \sum_{a_1=0}^1 X_0(a_1 a_0 b_0) W^{2a_1(2b_1 + b_0)}$$

$$F(b_2 b_1 b_0) = \sum_{a_0=0}^1 X_1(a_0 b_0 b_1) W^{a_0(4b_2 + 2b_1 + b_0)}$$

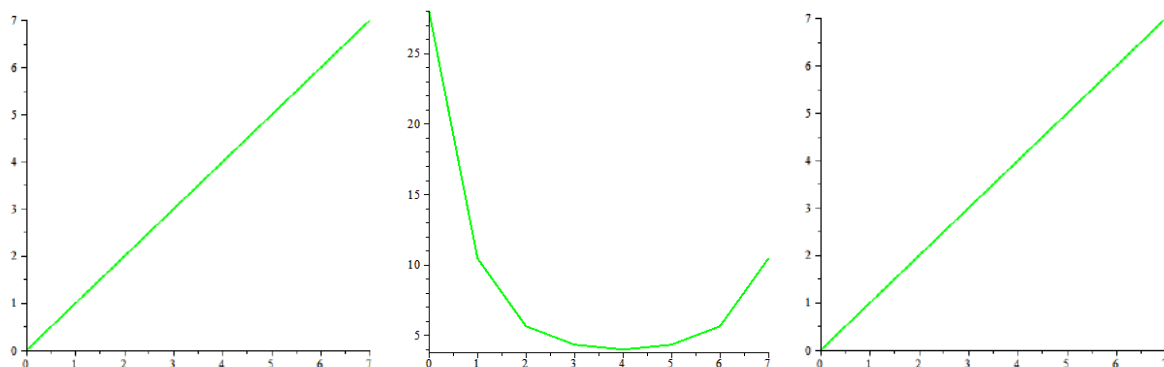
Then we just have to apply the formula:

$$f_j = \frac{1}{N} \sum_{k=0}^{N-1} F_k W_N^{-jk}$$

With Maple:

```
> for i from 0 to N-1 do
> invFFT[i] := (1/N) * sum(F[k] * W^(-i*k), k=0..N-1) :
> od:
```

These are the graphs of $y(n)$, it Fast Fourier Transforms and it inverse:

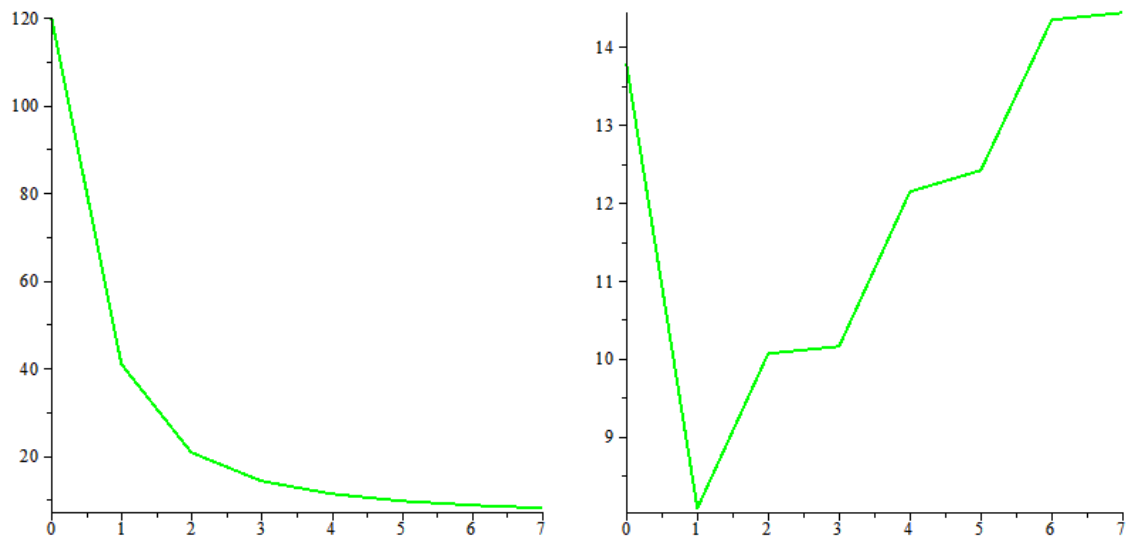


The inverse Fourier Transforms looks like the initial function.

Question 4

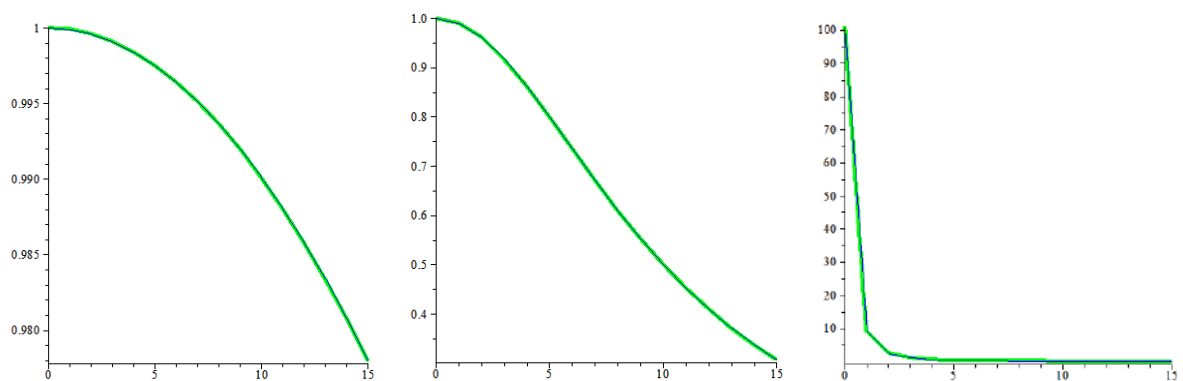
We have to find the inverse Fourier Transform of $G(q, r, s) = F(0, q, r, s)$. Function G is the same as F on the interval $[0; 7]$.

These are the graphs of G and it inverse:



Question 5 and 6

These are the graphs of $y(n)$ and it inverse for different values of a:



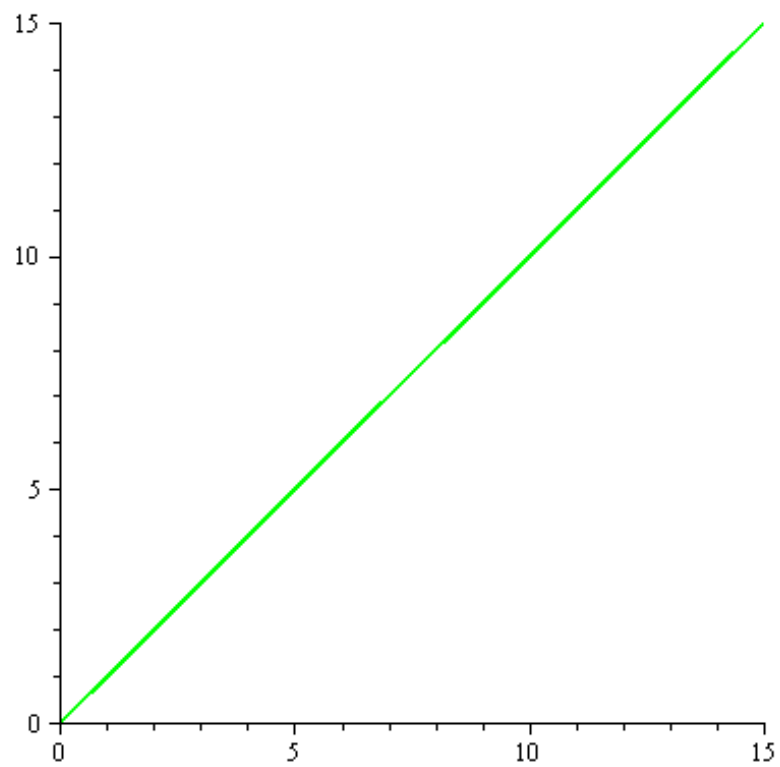
We can see that both $y(n)$ and it inverse are similar.

Appendices

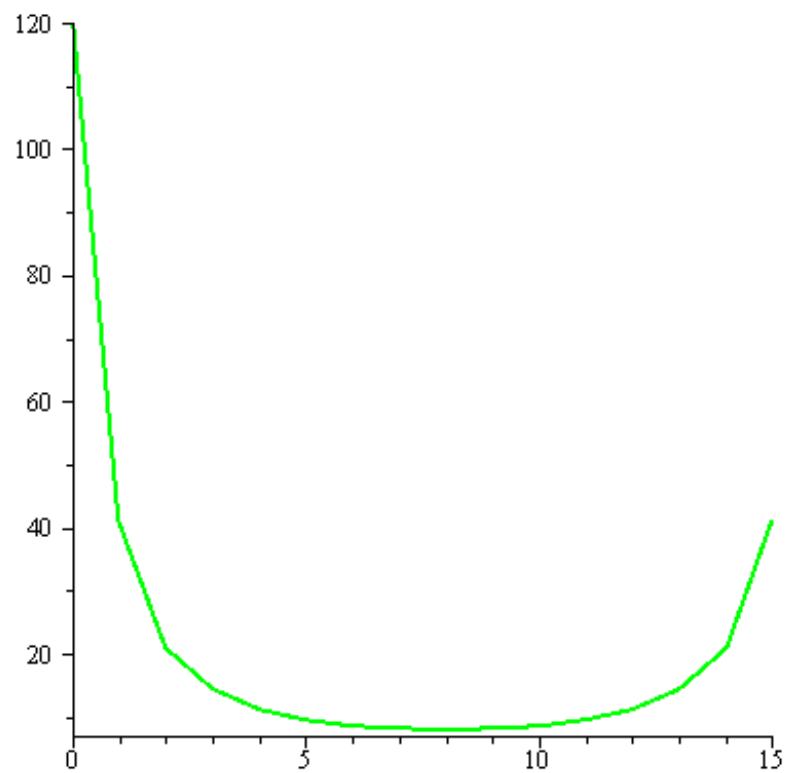
Maple worksheet: question 1

```
> restart:
> N:=16:
> f:=array(0..N-1):
> for i from 0 to N-1 do f[i]:=i;
> od:
> W:=evalf(exp(-2*Pi*I/N)):
> for a2 from 0 to 1 do
> for a1 from 0 to 1 do
> for a0 from 0 to 1 do
> for b0 from 0 to 1 do
> add(f[8*a3+4*a2+2*a1+a0]*W^(8*a3*b0),a3=0..1);
> X0[8*a2+4*a1+2*a0+b0]:=%;
> od: od: od: od:
> for a1 from 0 to 1 do
> for a0 from 0 to 1 do
> for b0 from 0 to 1 do
> for b1 from 0 to 1 do
> add(X0[8*a2+4*a1+2*a0+b0]*W^(4*a2*(2*b1+b0)),a2=0..1);
> X1[8*a1+4*a0+2*b0+b1]:=%;
> od: od: od: od:
> for a0 from 0 to 1 do
> for b0 from 0 to 1 do
> for b1 from 0 to 1 do
> for b2 from 0 to 1 do
> add(X1[8*a1+4*a0+2*b0+b1]*W^(2*a1*(4*b2+2*b1+b0)),a1=0..1);
> X2[8*a0+4*b0+2*b1+b2]:=%;
> od: od: od: od:
> for b3 from 0 to 1 do
> for b2 from 0 to 1 do
> for b1 from 0 to 1 do
> for b0 from 0 to 1 do
>
add(X2[8*a0+4*b0+2*b1+b2]*W^(a0*(8*b3+4*b2+2*b1+b0)),a0=0..1;
> F[8*b3+4*b2+2*b1+b0]:=%;
> od: od: od: od:
```

```
> u:= [seq([i,abs(f[i])],i=0..N-1)]:  
> plot(u,colour=green,thickness=2);
```



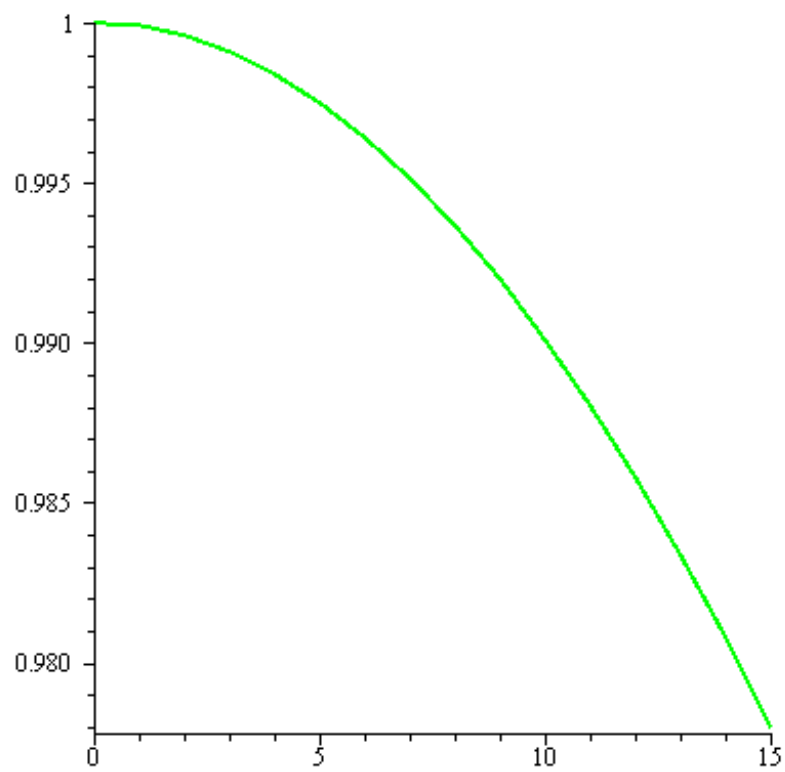
```
> u:= [seq([i,abs(F[i])],i=0..N-1)]:  
> plot(u,colour=green,thickness=2);
```



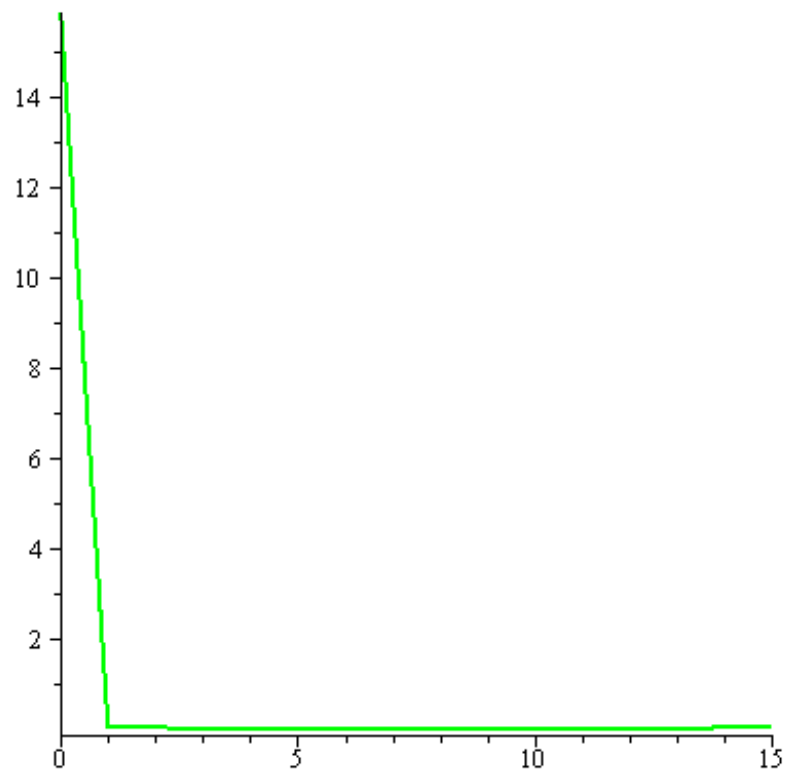
Maple worksheet: question 2

```
> restart:
> N:=16:
> a:=0.0001:
> Y:=array(0..N-1):
> for i from 0 to N-1 do Y[i]:=(1+a^2)/(1+a*i^2);
> od:
> W:=evalf(exp(-2*Pi*I/N)):
> for a2 from 0 to 1 do
> for a1 from 0 to 1 do
> for a0 from 0 to 1 do
> for b0 from 0 to 1 do
> add(Y[8*a3+4*a2+2*a1+a0]*W^(8*a3*b0),a3=0..1);
> X0[8*a2+4*a1+2*a0+b0]:=%;
> od: od: od: od:
> for a1 from 0 to 1 do
> for a0 from 0 to 1 do
> for b0 from 0 to 1 do
> for b1 from 0 to 1 do
> add(X0[8*a2+4*a1+2*a0+b0]*W^(4*a2*(2*b1+b0)),a2=0..1);
> X1[8*a1+4*a0+2*b0+b1]:=%;
> od: od: od: od:
> for a0 from 0 to 1 do
> for b0 from 0 to 1 do
> for b1 from 0 to 1 do
> for b2 from 0 to 1 do
> add(X1[8*a1+4*a0+2*b0+b1]*W^(2*a1*(4*b2+2*b1+b0)),a1=0..1);
> X2[8*a0+4*b0+2*b1+b2]:=%;
> od: od: od: od:
> for b3 from 0 to 1 do
> for b2 from 0 to 1 do
> for b1 from 0 to 1 do
> for b0 from 0 to 1 do
>
> add(X2[8*a0+4*b0+2*b1+b2]*W^(a0*(8*b3+4*b2+2*b1+b0)),a0=0..1);
> F[8*b3+4*b2+2*b1+b0]:=%;
> od: od: od: od:
```

```
> u:= [seq([i,abs(Y[i])],i=0..N-1)]:  
> plot(u,colour=green,thickness=2);
```



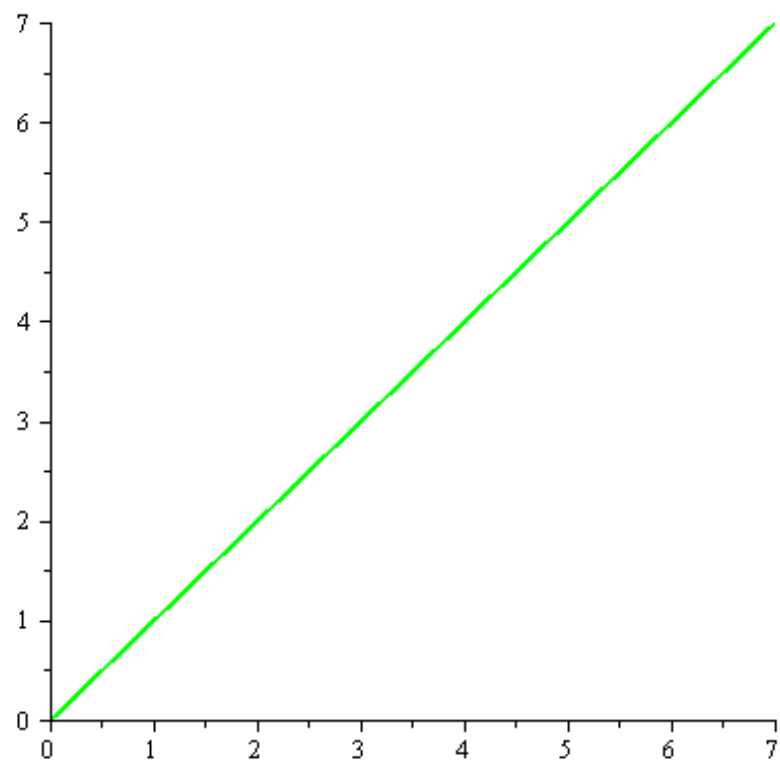
```
> u:= [seq([i,abs(F[i])],i=0..N-1)]:  
> plot(u,colour=green,thickness=2);
```



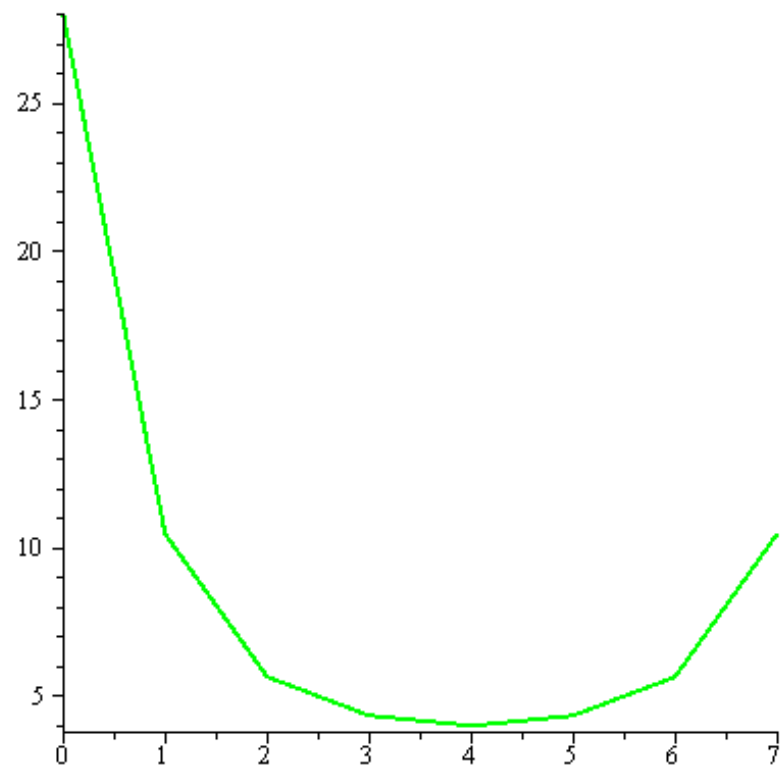
Maple worksheet: question 3

```
> restart:
> N:=8:
> f:=array(0..N-1):
> for i from 0 to N-1 do f[i]:=i;
> od:
> W:=evalf(exp(-2*Pi*I/N)):
> for a1 from 0 to 1 do
> for a0 from 0 to 1 do
> for b0 from 0 to 1 do
> add(f[4*a2+2*a1+a0]*W^(4*a2*b0),a2=0..1);
> X0[4*a1+2*a0+b0]:=%;
> od: od: od:
> for a0 from 0 to 1 do
> for b0 from 0 to 1 do
> for b1 from 0 to 1 do
> add(X0[4*a1+2*a0+b0]*W^(2*a1*(2*b1+b0)),a1=0..1);
> X1[4*a0+2*b0+b1]:=%;
> od: od: od:
> for b2 from 0 to 1 do
> for b1 from 0 to 1 do
> for b0 from 0 to 1 do
> add(X1[4*a0+2*b0+b1]*W^(a0*(4*b2+2*b1+b0)),a0=0..1);
> F[4*b2+2*b1+b0]:=%;
> od: od: od:
> for i from 0 to N-1 do
> invFFT[i]:=(1/N)*sum(F[k]*W^(-i*k),k=0..N-1):
> od:
```

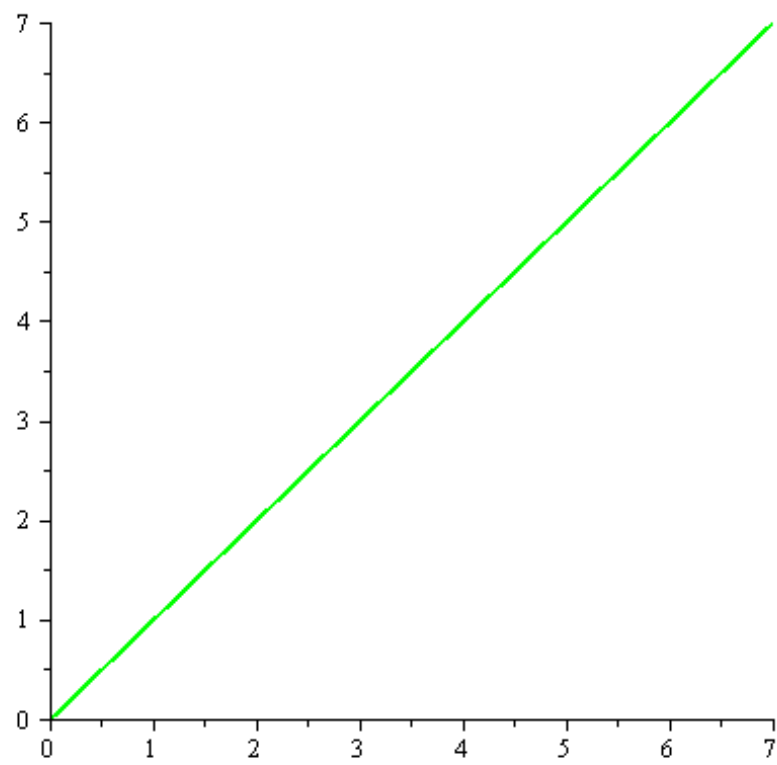
```
> u:= [seq([i,abs(f[i])],i=0..N-1)]:  
> plot(u,colour=green,thickness=2);
```



```
> u:= [seq([i,abs(F[i])],i=0..N-1)]:  
> plot(u,colour=green,thickness=2);
```



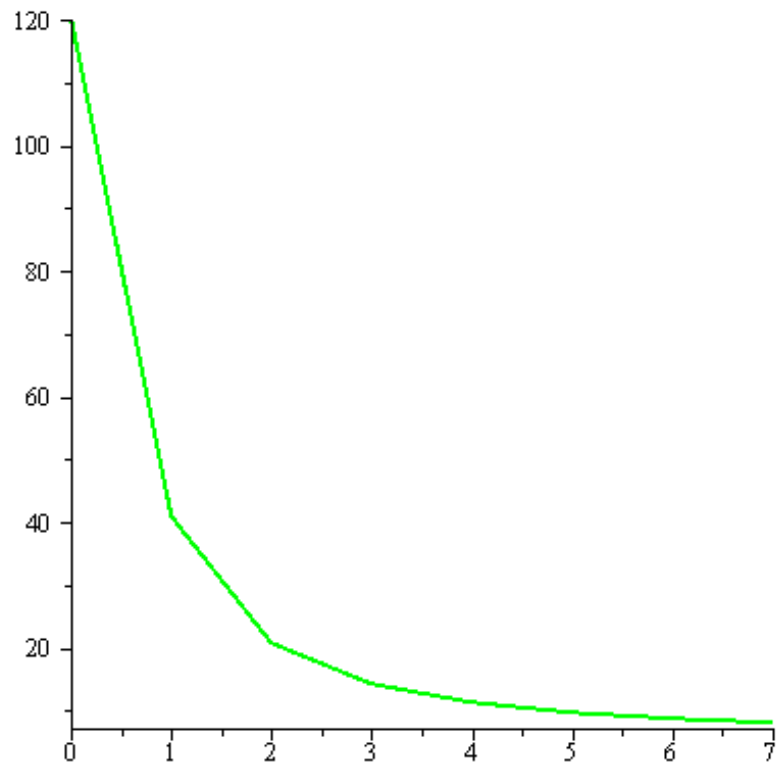
```
> u:= [seq([i,abs(invFFT[i])],i=0..N-1)]:  
> plot(u,colour=green,thickness=2);
```



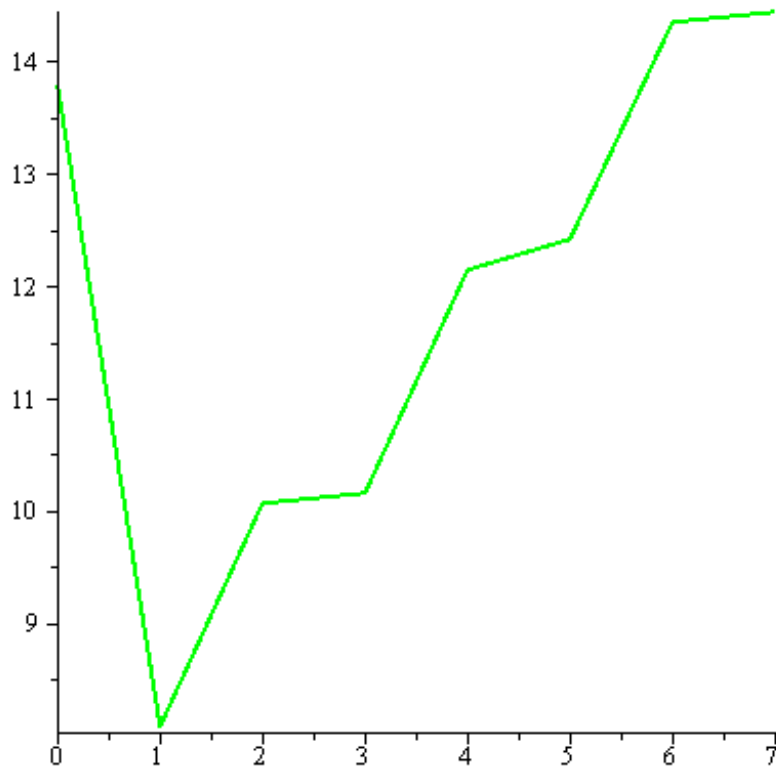
Maple worksheet: question 4

```
> restart:
> N:=16:
> f:=array(0..N-1):
> for i from 0 to N-1 do f[i]:=i;
> od:
> W:=evalf(exp(-2*Pi*I/N)):
> for a2 from 0 to 1 do
> for a1 from 0 to 1 do
> for a0 from 0 to 1 do
> for b0 from 0 to 1 do
> add(f[8*a3+4*a2+2*a1+a0]*W^(8*a3*b0),a3=0..1);
> X0[8*a2+4*a1+2*a0+b0]:=%;
> od: od: od: od:
> for a1 from 0 to 1 do
> for a0 from 0 to 1 do
> for b0 from 0 to 1 do
> for b1 from 0 to 1 do
> add(X0[8*a2+4*a1+2*a0+b0]*W^(4*a2*(2*b1+b0)),a2=0..1);
> X1[8*a1+4*a0+2*b0+b1]:=%;
> od: od: od: od:
> for a0 from 0 to 1 do
> for b0 from 0 to 1 do
> for b1 from 0 to 1 do
> for b2 from 0 to 1 do
> add(X1[8*a1+4*a0+2*b0+b1]*W^(2*a1*(4*b2+2*b1+b0)),a1=0..1);
> X2[8*a0+4*b0+2*b1+b2]:=%;
> od: od: od: od:
> for b3 from 0 to 1 do
> for b2 from 0 to 1 do
> for b1 from 0 to 1 do
> for b0 from 0 to 1 do
>
> add(X2[8*a0+4*b0+2*b1+b2]*W^(a0*(8*b3+4*b2+2*b1+b0)),a0=0..1);
> F[8*b3+4*b2+2*b1+b0]:=%;
> od: od: od: od:
> N:=8:
> for i from 0 to N-1 do
> G[i]:=F[i];
> od:
> for i from 0 to N-1 do
> invFFT[i]:=(1/N)*sum(G[k]*W^(-i*k),k=0..N-1):
> od:
```

```
> u:= [seq([i,abs(G[i])],i=0..N-1)]:  
> plot(u,colour=green,thickness=2);
```



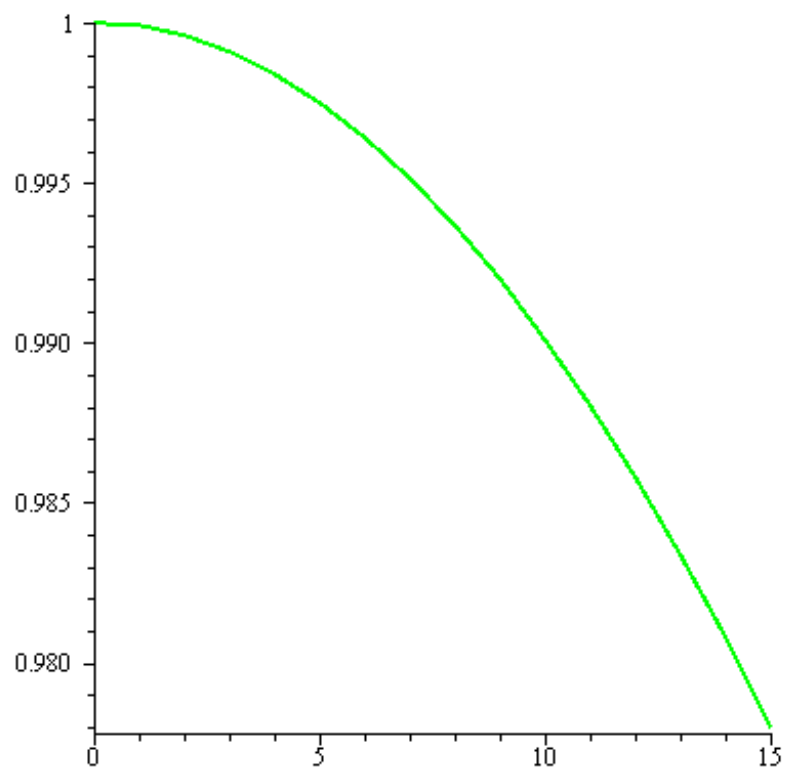
```
> u:= [seq([i,abs(invFFT[i])],i=0..N-1)]:  
> plot(u,colour=green,thickness=2);
```



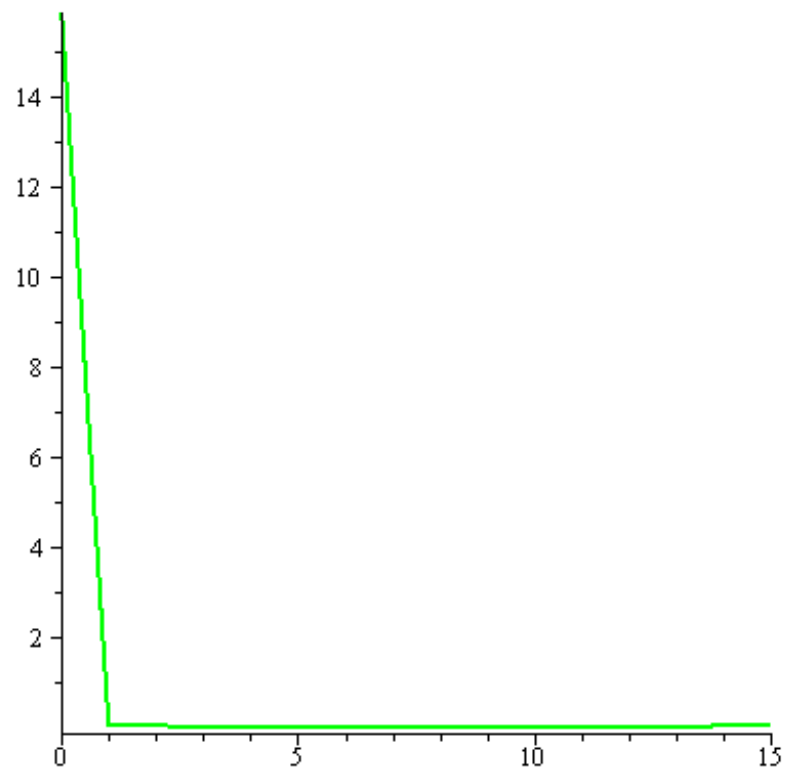
Maple worksheet: question 5

```
> restart:
> N:=16:
> a:=0.0001:
> Y:=array(0..N-1):
> for i from 0 to N-1 do Y[i]:=(1+a^2)/(1+a*i^2);
> od:
> W:=evalf(exp(-2*Pi*I/N)):
> for a2 from 0 to 1 do
> for a1 from 0 to 1 do
> for a0 from 0 to 1 do
> for b0 from 0 to 1 do
> add(Y[8*a3+4*a2+2*a1+a0]*W^(8*a3*b0),a3=0..1);
> X0[8*a2+4*a1+2*a0+b0]:=%;
> od: od: od: od:
> for a1 from 0 to 1 do
> for a0 from 0 to 1 do
> for b0 from 0 to 1 do
> for b1 from 0 to 1 do
> add(X0[8*a2+4*a1+2*a0+b0]*W^(4*a2*(2*b1+b0)),a2=0..1);
> X1[8*a1+4*a0+2*b0+b1]:=%;
> od: od: od: od:
> for a0 from 0 to 1 do
> for b0 from 0 to 1 do
> for b1 from 0 to 1 do
> for b2 from 0 to 1 do
> add(X1[8*a1+4*a0+2*b0+b1]*W^(2*a1*(4*b2+2*b1+b0)),a1=0..1);
> X2[8*a0+4*b0+2*b1+b2]:=%;
> od: od: od: od:
> for b3 from 0 to 1 do
> for b2 from 0 to 1 do
> for b1 from 0 to 1 do
> for b0 from 0 to 1 do
>
> add(X2[8*a0+4*b0+2*b1+b2]*W^(a0*(8*b3+4*b2+2*b1+b0)),a0=0..1);
> F[8*b3+4*b2+2*b1+b0]:=%;
> od: od: od: od:
> for i from 0 to N-1 do
> invFFT[i]:=(1/N)*sum(F[k]*W^(-i*k),k=0..N-1):
> od:
```

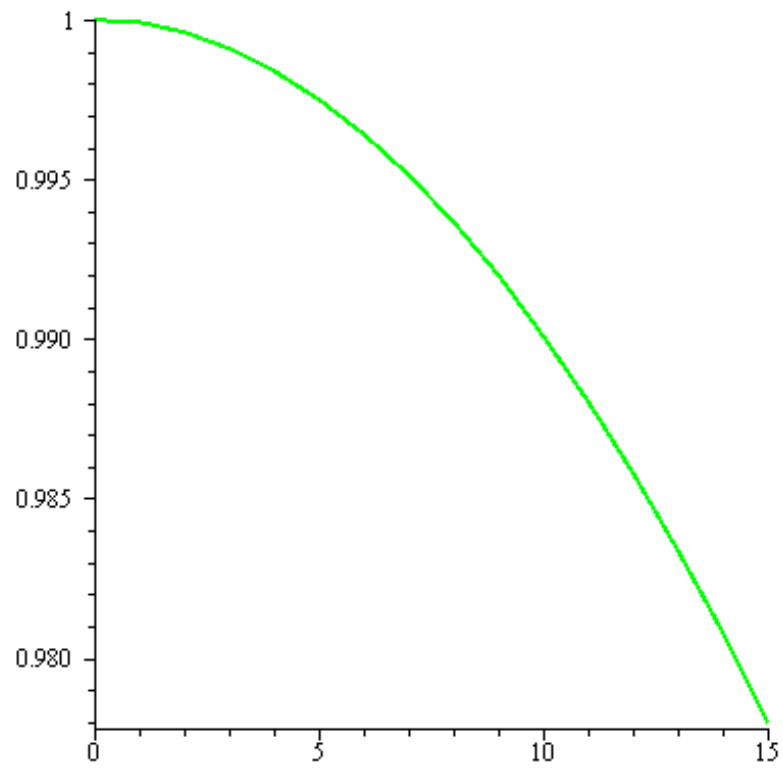
```
> u:= [seq([i,abs(Y[i])],i=0..N-1)]:  
> plot(u,colour=green,thickness=2);
```



```
> fu:= [seq([i,abs(F[i])],i=0..N-1)]:  
> plot(fu,colour=green,thickness=2);
```



```
> iu:=seq([i,abs(invFFT[i])],i=0..N-1):  
> plot(iu,colour=green,thickness=2);
```



```
> plot([u,iu],colour=[green,blue],thickness=[4,1]);
```

