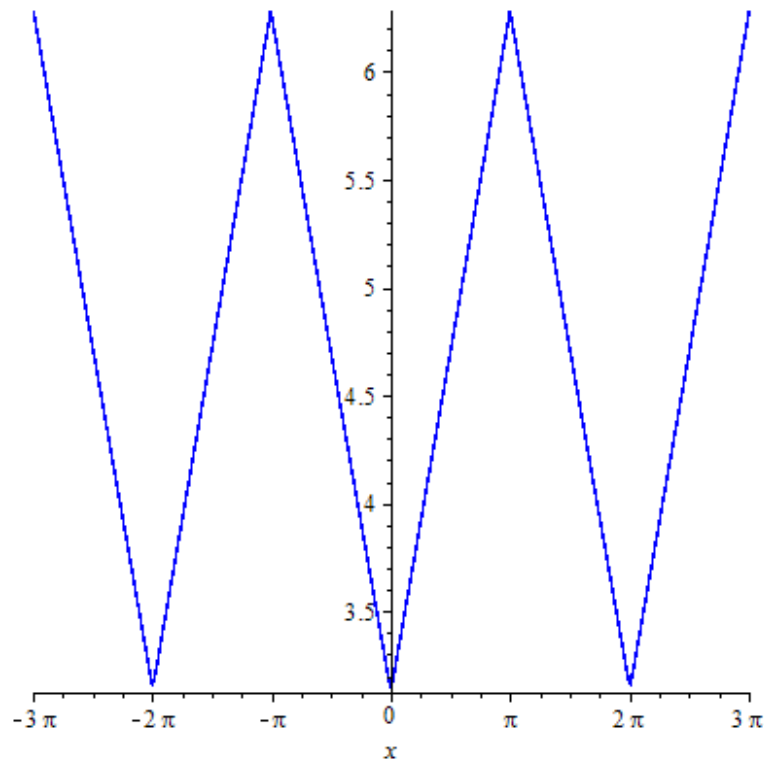


Fourier Series Assignment

Question 1

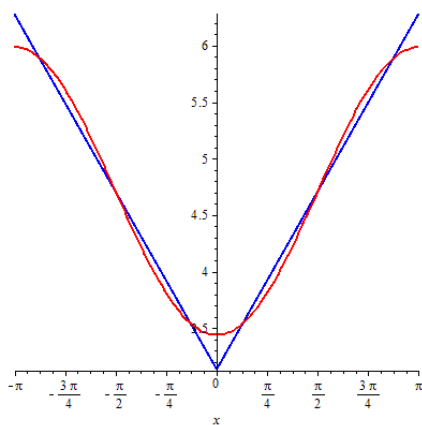
- First function

$$f := x \rightarrow (x + \pi) (u(x) - u(x - \pi)) + (\pi - x) (u(x + \pi) - u(x))$$

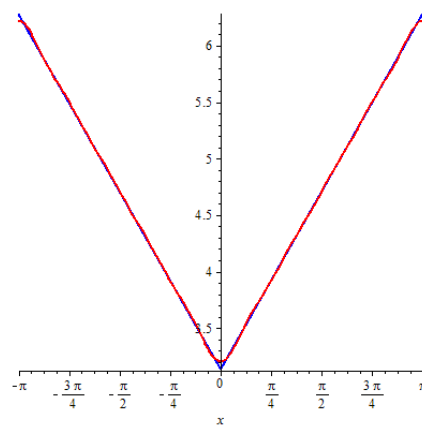


Fourier Series :

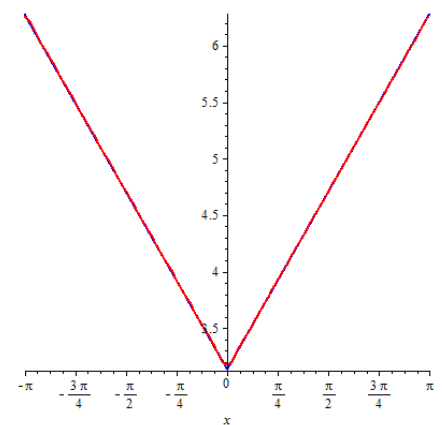
$$\frac{3}{2} \pi + \sum_{n=1}^N \frac{2(-1 + \cos(n\pi)) + 2\sin(n\pi)n\pi}{\pi n^2} \cos(nx)$$



N = 2



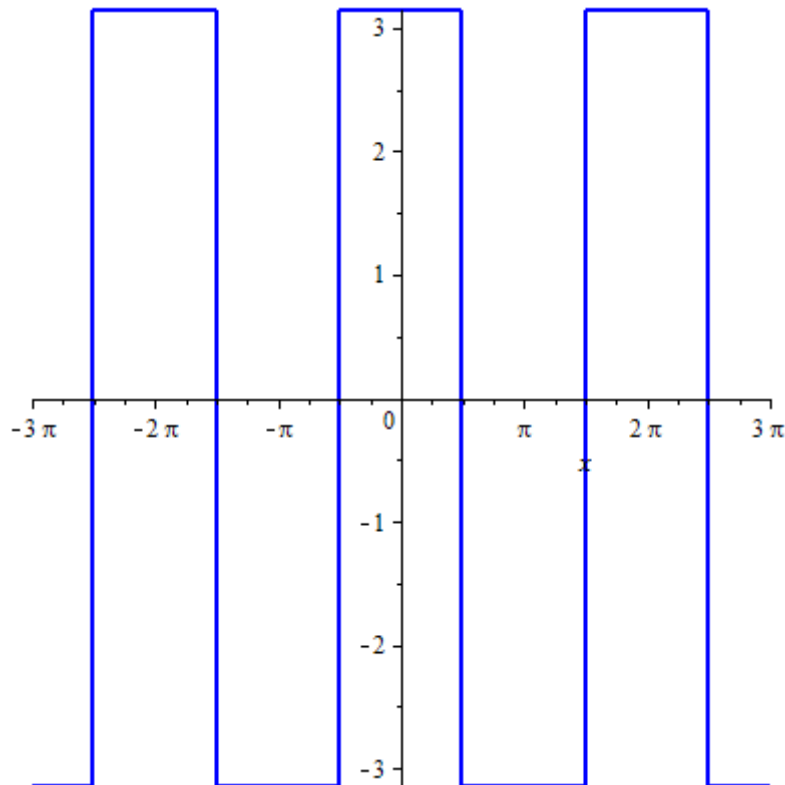
N = 10



N = 20

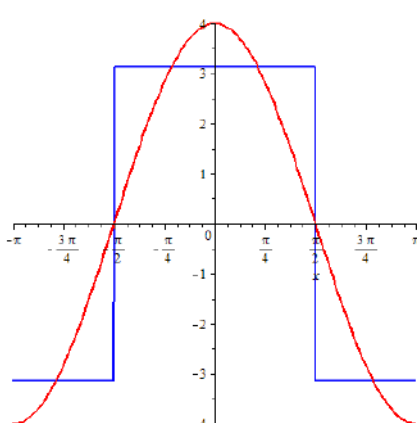
- **Second function**

$$f := x \mapsto -\pi \left(u(x + \pi) - u\left(x + \frac{1}{2} \pi\right) \right) + \pi \left(u\left(x + \frac{1}{2} \pi\right) - u\left(x - \frac{1}{2} \pi\right) \right) - \pi \left(u\left(x - \frac{1}{2} \pi\right) - u(x - \pi) \right)$$

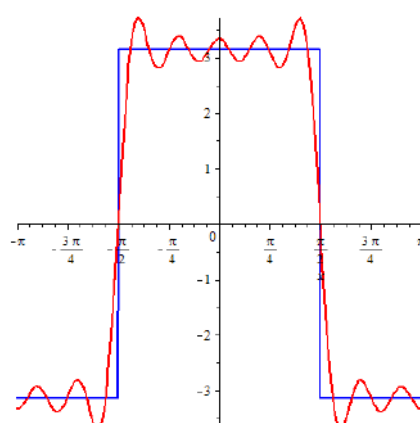


Fourier Series :

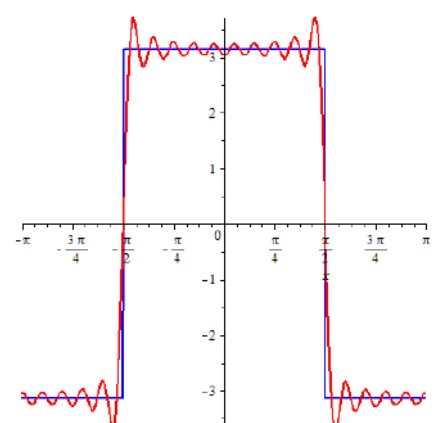
$$\sum_{n=1}^N \frac{2 \left(-\pi \sin(n\pi) + 2\pi \sin\left(\frac{1}{2} n\pi\right) \right) \cos(nx)}{\pi n}$$



N = 2



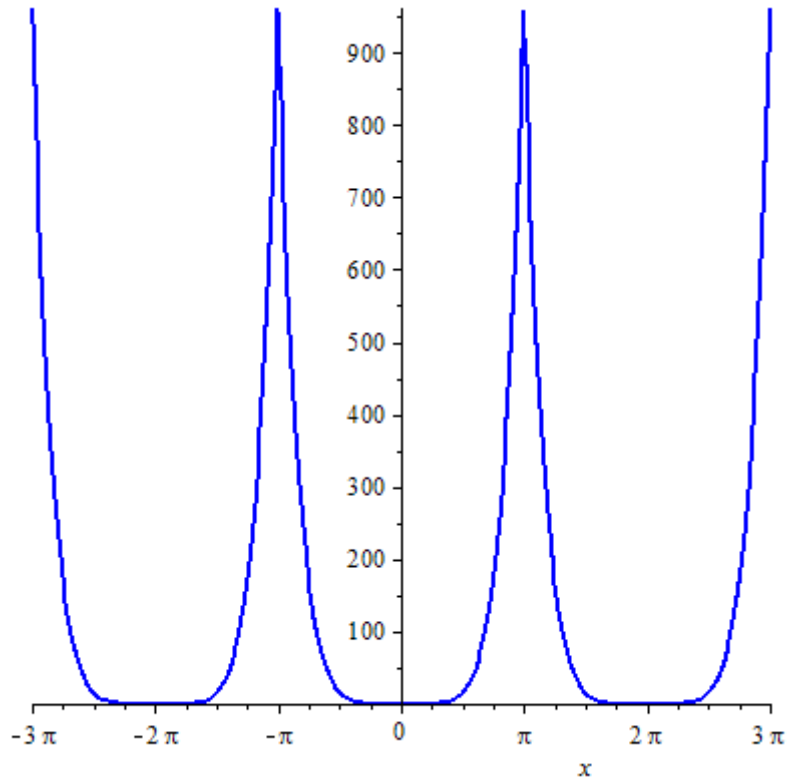
N = 10



N = 20

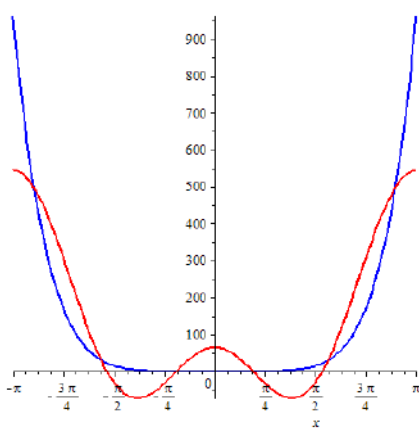
- Third function

$$f := x \rightarrow x^6 (u(x + \pi) - u(x - \pi))$$

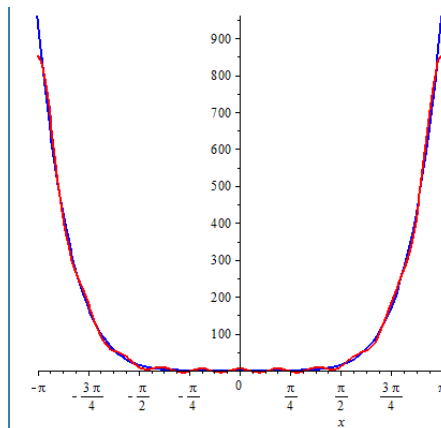


Fourier Series :

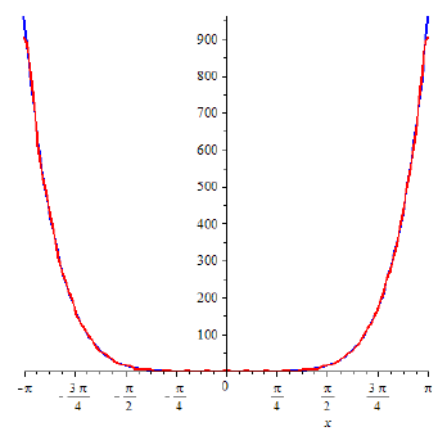
$$\frac{1}{7} \pi^6 + \sum_{n=1}^N \frac{1}{\pi n^7} (2(n^6 \pi^6 \sin(n\pi) + 6n^5 \pi^5 \cos(n\pi) - 30n^4 \pi^4 \sin(n\pi) - 120n^3 \pi^3 \cos(n\pi) + 360n^2 \pi^2 \sin(n\pi) - 720n \pi \cos(n\pi)) \cos(nx))$$



N = 2



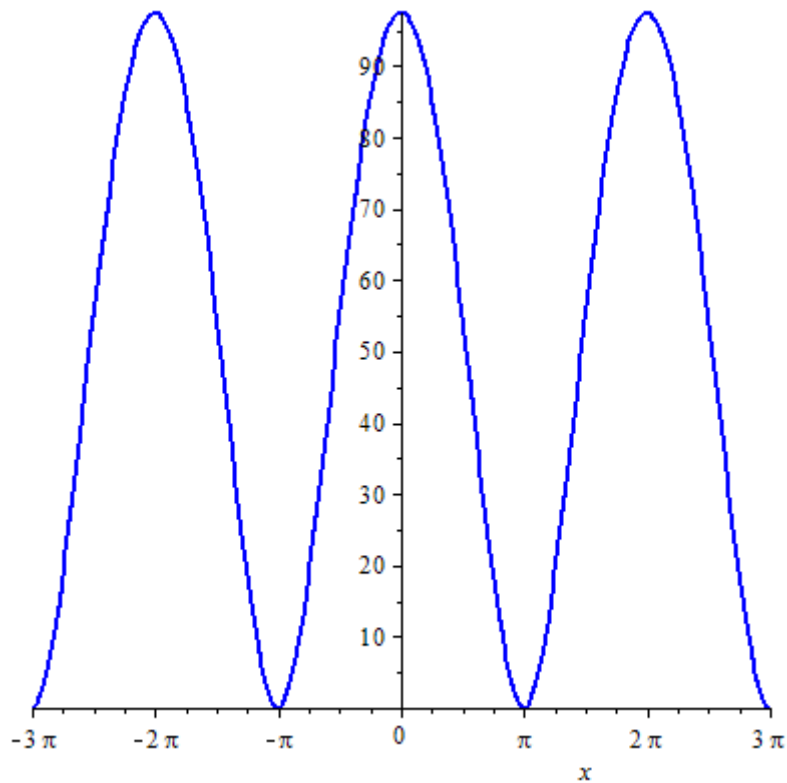
N = 10



N = 20

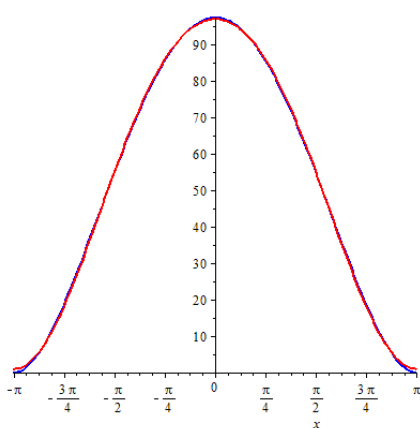
- **Fourth function**

$$f := x \rightarrow (x^2 - \pi^2)^2 (u(x + \pi) - u(x - \pi))$$

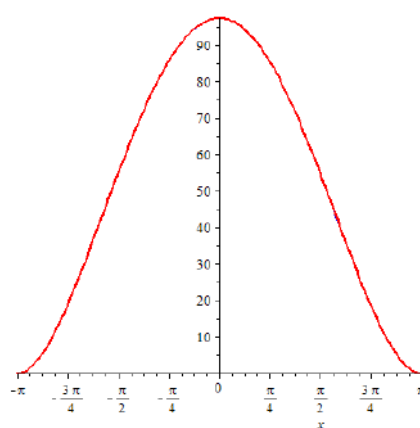


Fourier Series :

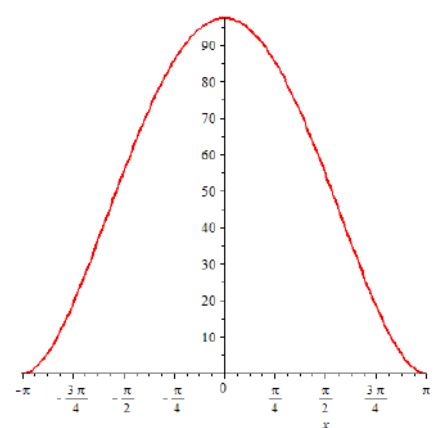
$$\frac{8}{15} \pi^4 + \sum_{n=1}^N \frac{1}{\pi n^5} (2(-8n^2 \pi^2 \sin(n\pi) + 24 \sin(n\pi) - 24n\pi \cos(n\pi)) \cos(nx))$$



N = 2



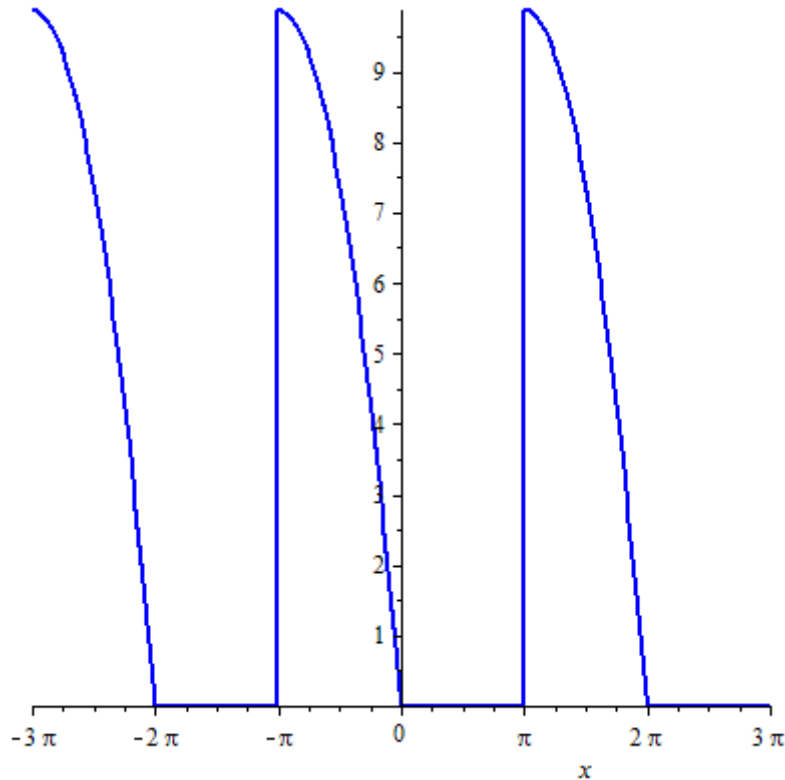
N = 10



N = 20

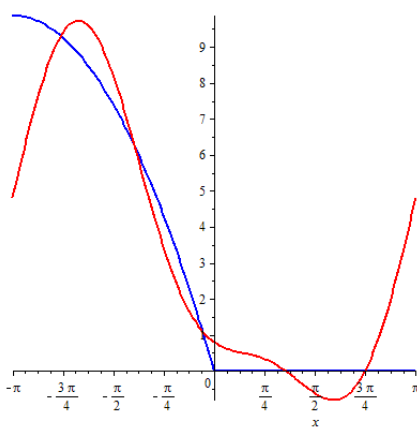
- **Fifth function**

$$f := x \mapsto (-x^2 - 2\pi x) (u(x + \pi) - u(x)) + 0 (u(x) - u(x - \pi))$$

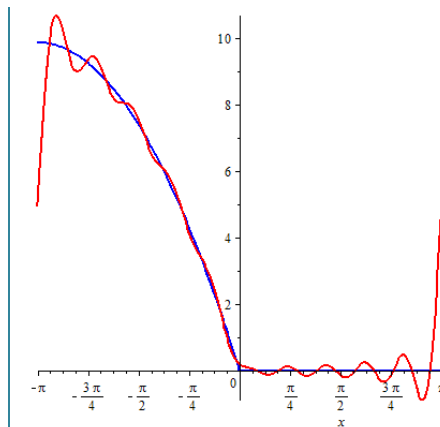


Fourier Series :

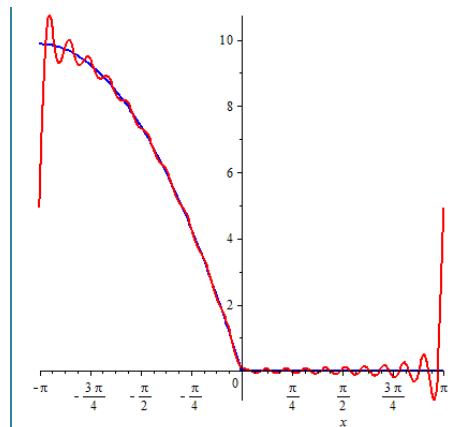
$$\frac{1}{3} \pi^2 + \sum_{n=1}^N \left(\frac{(n^2 \pi^2 \sin(n\pi) + 2 \sin(n\pi) - 2 n \pi) \cos(nx)}{\pi n^3} + \frac{(\cos(n\pi) n^2 \pi^2 + 2 \cos(n\pi) - 2) \sin(nx)}{\pi n^3} \right)$$



N = 2



N = 10



N = 20

Question 2. A

With the previous results, we can find different situations and parameters about series and convergence.

When there are jumps at the end of the fundamental interval, N must be high if we want that the series approaches the function. We can see it on the last example, f_5 has jumps at the ends of the fundamental interval and we have good results only when N is high. We have the same phenomenon when jumps are in the fundamental interval, in f_2 for example. We also see that when we have jumps, there are some irregularities at the ends of the interval, for example in f_5 when $N = 20$.

When there are corners in the fundamental interval, for example in f_1 , N must be high too, like in the previous case. But, even when it's high, there are some problems at the corners, it's the Gibbs phenomenon. We have the same problem when corners are at the ends of the fundamental interval.

In other cases, N doesn't have to be high to have a series which converge, so we don't have any problem like before.

Question 2. B

If the function looks like a sin or a cos function, N doesn't have to be big for the function to converge. When there are corners or discontinuities, N must be high for the function to converge quickly.