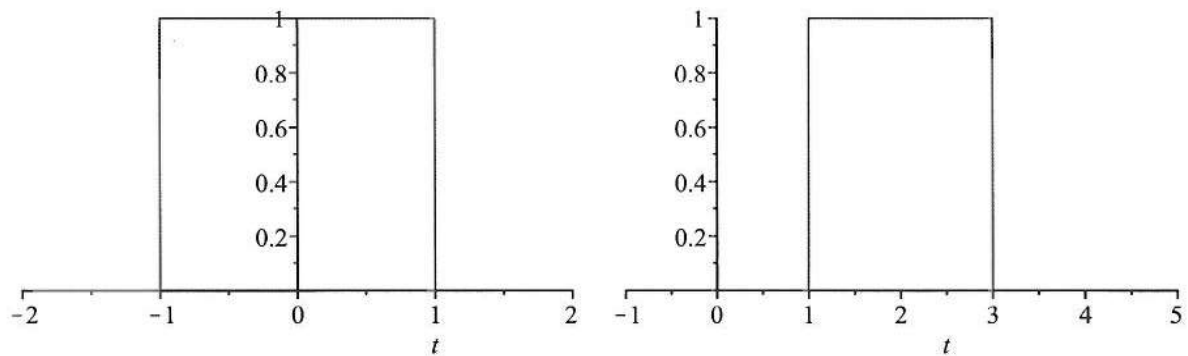


Properties of Fourier Transforms

We assume that $F(\omega)$, the Fourier Transform of $f(t)$ is known.

1. The Delay Property

Consider the function $f(t) = u(t + 1) - u(t - 1)$ and the function $f(t - 2)$. Graphs of these two functions are shown below:



We can clearly see from the graphs that $f(t - 2)$ is $f(t)$ shifted or **delayed** by 2 units. In general $f(t - b)$ is a delay of b units. The Fourier Transforms of $f(t - b)$ in terms of $F(\omega)$ is:

$$\text{FT } (f(t - b)) = e^{-i\omega b} F(\omega).$$

Proof

Example 1

Use tables to find the Fourier Transform of $e^{-|t|}$ and hence find the Fourier Transform of $e^{-|t-4|}$.

2. Modulation

$$\text{FT } (e^{ibt} f(t)) = F(\omega - b).$$

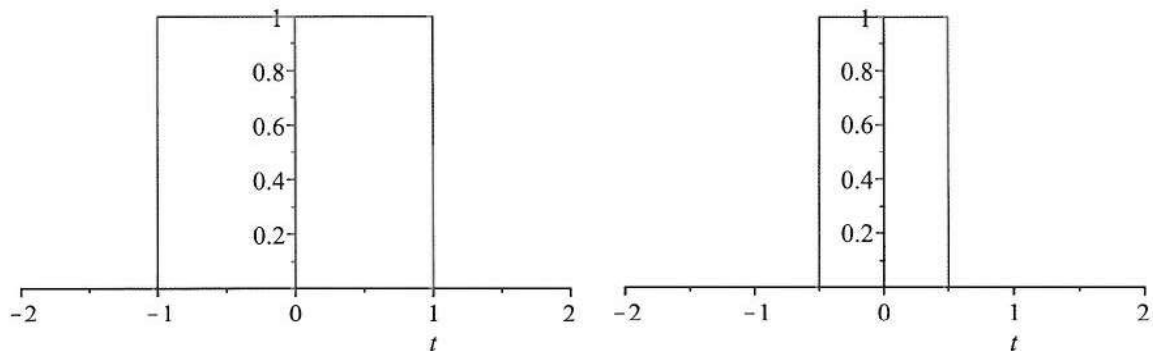
Proof - an exercise for you to do

Example 2

Use tables to find the Fourier Transform of $u(t + a) - u(t - a)$ and hence find the Fourier Transform of $e^{2it}(u(t + a) - u(t - a))$.

3. Scaling Property

Again consider the function $f(t) = u(t+1) - u(t-1)$ and the function $f(2t)$. Graphs of these two functions are shown below:



We can clearly see from the graphs that $f(2t)$ is half the width of $f(t)$. In general $f(at)$ is $\frac{1}{a}$ -th the width of $f(t)$. Notice that the height is **not** changed. The relationship between the Fourier Transforms of $f(t)$ and $f(at)$ is:

$$\text{FT } (f(at)) = \frac{1}{a} F\left(\frac{\omega}{a}\right).$$

Proof

Example 3

Use tables to find the Fourier Transform of $e^{-|t|}$ and hence find the Fourier Transform of $e^{-4|t|}$.

Note

It is possible to take combinations of these properties.

Example 4

If $f(t) = u(t + 1) - u(t - 1)$ determine $F(\omega)$ and hence find the Fourier Transform of $f(2t - 3)$.

4. Time Differentiation Property

$$\text{FT} \left(\frac{df}{dt} \right) = i\omega F(\omega).$$

More generally:

$$\text{FT} \left(\frac{d^n f}{dt^n} \right) = (i\omega)^n F(\omega).$$

Proof - an exercise for you to do

Example 5

Use tables to find the Fourier Transform of $f(t) = e^{-t^2}$. Hence find the Fourier Transform of te^{-t^2} .

5. Multiplication by t

$$\text{FT } (tf(t)) = i \frac{dF}{d\omega}.$$

Proof

Example 6

Use tables to find the Fourier Transform of $f(t) = e^{-t}u(t)$. Hence find the Fourier Transform of $te^{-t}u(t)$.

Example 7

Use tables to find the Fourier Transform of $f(t) = u(t+1) - u(t-1)$. Hence find the Fourier Transform of $t(u(t+1) - u(t-1))$.

6. Symmetry Property

$$\text{FT } (F(t)) = 2\pi f(-\omega).$$

Proof - an exercise for you to do

Example 8

Use tables to find the Fourier Transform of $f(t) = e^{-2|t|}$. Hence find the Fourier Transform of $\frac{4}{4+t^2}$.

Example 9

Use tables to find the Fourier Transform of $f(t) = u(t+1) - u(t-1)$. Hence find the Fourier Transform of $\text{sinc}(t)$.

Inverse Fourier Transforms

These properties can also be used in reverse

Example 10

Find the inverse Fourier Transforms of:

$$(a) \frac{5}{3+4iw} \quad (b) e^{-|\omega-3|} \quad (c) \frac{e^{4iw}}{9+\omega^2} \quad (d) \frac{6\omega}{9+\omega^2}$$