

Fourier Transforms of Generalised Functions

Transforms of the Dirac Delta Function

Reminder

$$\text{FT}(\delta(t)) = \int_{-\infty}^{\infty} \delta(t) e^{-i\omega t} dt = 1,$$

$$\text{FT}(\delta(t - a)) = \int_{-\infty}^{\infty} \delta(t - a) e^{-i\omega t} dt = e^{-i\omega a}.$$

Proof

Fourier Transform of 1

$$\text{FT}(1) = 2\pi\delta(\omega)$$

Proof

Transforms of $\sin(t)$ and $\cos(t)$

$$\text{FT}(\cos(at)) = \pi(\delta(\omega - a) + \delta(\omega + a))$$

and

$$\text{FT}(\sin(at)) = -i\pi(\delta(\omega - a) - \delta(\omega + a)).$$

Proof

Powers of t

$$\text{FT}(t) = 2\pi i \delta'(\omega),$$

$$\text{FT}(t^n) = 2\pi i^n \delta^{(n)}(\omega),$$

Fourier Transform of $\text{sgn}(t)$

$$\text{sgn}(t) = \begin{cases} 1 & t > 0 \\ -1 & t < 0 \end{cases}$$

$$\text{FT}(\text{sgn}(t)) = -\frac{2i}{\omega}$$

Fourier Transform of the Heaviside Step Function

$$FT(u(t)) = \pi \left(\delta(\omega) - \frac{i}{\pi\omega} \right).$$

Proof