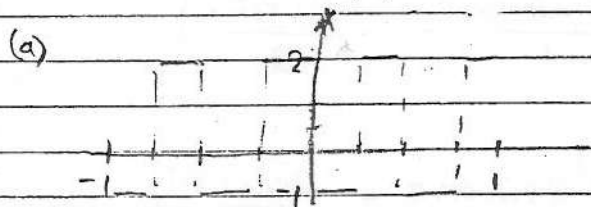


Fourier Series Tutorial 1- Solutions

1. $f(x) = \begin{cases} +2 & -0.25 \leq x < 0 \\ -1 & 0 \leq x < 0.25 \end{cases} \quad T = 0.5$

$f(x+1) = f(x)$



(b)

$$a_0 = \frac{2}{T} \int_{-T/2}^{T/2} f(x) dx = \frac{2}{0.5} \int_{-0.25}^{0.25} f(x) dx = 4 \int_{-1/4}^0 2 dx + 4 \int_0^{1/4} -1 dx$$

$$= 4 \left[2x \right]_{-1/4}^0 + 4 \left[-x \right]_0^{1/4}$$

$$= 4 \left(0 - \left(-\frac{1}{2} \right) \right) + 4 \left(-\frac{1}{4} - 0 \right) = 4 \left(\frac{1}{2} - \frac{1}{4} \right) = 1.$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \cos \frac{2n\pi x}{T} dx = \frac{2}{1/2} \int_{-1/4}^{1/4} f(x) \cos \frac{2n\pi x}{1/2} dx$$

$$= 4 \int_{-1/4}^0 2 \cos 4n\pi x dx + 4 \int_0^{1/4} -1 \cos 4n\pi x dx$$

$$= 8 \cdot \frac{1}{4n\pi} \sin 4n\pi x \Big|_{-1/4}^0 - 4 \cdot \frac{1}{4n\pi} \sin 4n\pi x \Big|_0^{1/4}$$

$$= 0$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \sin \frac{2n\pi x}{T} dx = \frac{2}{1/2} \int_{-1/4}^{1/4} f(x) \sin \frac{2n\pi x}{1/2} dx$$

$$= 4 \int_{-1/4}^0 2 \sin 4n\pi x dx + 4 \int_0^{1/4} -1 \sin 4n\pi x dx$$

$$= 8 \cdot \frac{1}{4n\pi} [-\cos 4n\pi x]_{-1/4}^0 - 4 \cdot \frac{1}{4n\pi} [-\cos 4n\pi x]_0^{1/4}$$

$$= \frac{-2}{n\pi} [1 - \cos(n\pi)] + \frac{1}{n\pi} [\cos(n\pi) - 1]$$

$$= \frac{-2}{n\pi} [1 - (-1)^n] - \frac{1}{n\pi} [1 - (-1)^n]$$

$$= \frac{-3}{n\pi} [1 - (-1)^n]$$

$$\therefore f(x) \sim \frac{1}{2} - \frac{3}{\pi} \sum_{n=1}^{\infty} \frac{1-(-1)^n}{n} \sin 4n\pi x.$$

1st 3 terms: $\frac{1}{2} - \frac{3}{\pi} \left(\frac{2 \sin 4\pi x}{1} + \frac{2 \sin 12\pi x}{3} \right)$

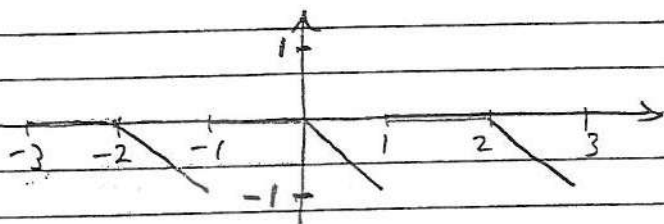
$$x=0.1 \Rightarrow f \sim 0.5 - \frac{3}{\pi} \left(2 \sin 0.4\pi + \frac{2 \sin 1.2\pi}{3} \right)$$

$$= 0.5 - 0.9549 (1.9021 - 0.3919)$$

$$= 0.5 - 1.4421$$

$$= -0.942 \text{ to 3dp.}$$

2 (a)



$$f(x) = \begin{cases} 0 & -1 \leq x \leq 0 \\ -x & 0 \leq x \leq 1 \end{cases}$$

$$(b) d_0 = \frac{2}{T} \int_{-T/2}^{T/2} f(x) dx = \frac{2}{2} \left[\int_{-1}^0 0 dx + \int_0^1 -x dx \right] = -\frac{1}{2} x^2 \Big|_0^1 = -\frac{1}{2}$$

$$a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \cos \frac{2n\pi x}{T} dx = \frac{2}{2} \left[\int_{-1}^0 0 dx + \int_0^1 -x \cos \frac{2n\pi x}{2} dx \right]$$

$$= \int_0^1 -x \cos n\pi x dx$$

$$= -x \cdot \frac{1}{n\pi} \sin n\pi x \Big|_0^1 - \int_0^1 -1 \cdot \frac{1}{n\pi} \sin n\pi x dx$$

$$= -\frac{1}{n\pi} [\sin n\pi - 0] + \frac{1}{n^2\pi^2} \cos n\pi x \Big|_0^1$$

$$= 0 - \frac{1}{n^2\pi^2} [\cos n\pi - \cos(0)] = \frac{(-1)^n - 1}{n^2\pi^2}$$

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(x) \sin \frac{2n\pi x}{T} dx = \frac{2}{2} \left[\int_{-1}^0 0 dx + \int_0^1 -x \sin \frac{2n\pi x}{2} dx \right]$$

$$= \int_0^1 -x \sin n\pi x dx$$

$$= -x \cdot \frac{1}{n\pi} \cos n\pi x \Big|_0^1 - \int_0^1 -1 \cdot \frac{1}{n\pi} \cos n\pi x dx$$

$$= \frac{1}{n\pi} [\cos n\pi - 0] - \frac{1}{n^2\pi^2} \sin n\pi x \Big|_0^1$$

$$= \frac{(-1)^n}{n\pi} - \frac{1}{n^2\pi^2} [\sin n\pi - \sin(0)] = \frac{(-1)^n}{n\pi}$$

$$\therefore f(x) = -\frac{1}{4} + \sum_{n=1}^{\infty} \left[\frac{1 - (-1)^n}{n^2\pi^2} \cos n\pi x + \frac{(-1)^n}{n\pi} \sin n\pi x \right]$$

(c) 1st 3 terms are: $f(x) = -\frac{1}{4} + \frac{2}{\pi^2} \cos \pi x - \frac{1}{\pi} \sin \pi x$

(d) Converges to -0.5 at $x=1$

