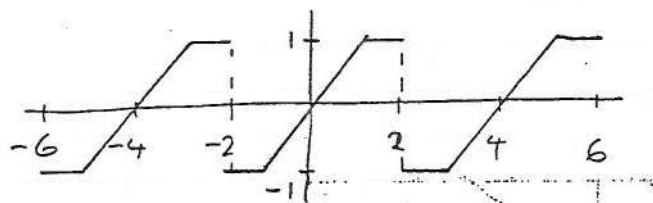


Fourier Series Tutorial. 2 Solutions

1. (a)



$T=4.$

(b) f is an odd function so $a_n = 0.$

$$b_n = \frac{4}{T} \int_0^{T/2} f(x) \sin \frac{2n\pi x}{T} dx$$

$$= \int_0^2 f(x) \sin \frac{n\pi x}{2} dx$$

$$= \int_0^1 x \cdot \sin \frac{n\pi x}{2} dx + \int_1^2 \sin \frac{n\pi x}{2} dx$$

$$= x \cdot \frac{-2}{n\pi} \cos \frac{n\pi x}{2} \Big|_0^1 - \int_0^1 \frac{-2}{n\pi} \cos \frac{n\pi x}{2} dx + \frac{-2}{n\pi} \cos \frac{n\pi x}{2} \Big|_1^2$$

$$= \frac{-2}{n\pi} \left(\cos \frac{n\pi}{2} - 0 \right) + \left(\frac{2}{n\pi} \right)^2 \sin \frac{n\pi x}{2} \Big|_0^1 - \frac{2}{n\pi} \left(\cos n\pi - \cos \frac{n\pi}{2} \right)$$

$$= \frac{4}{n^2\pi^2} \sin \frac{n\pi}{2} - \frac{2(-1)^n}{n\pi}$$

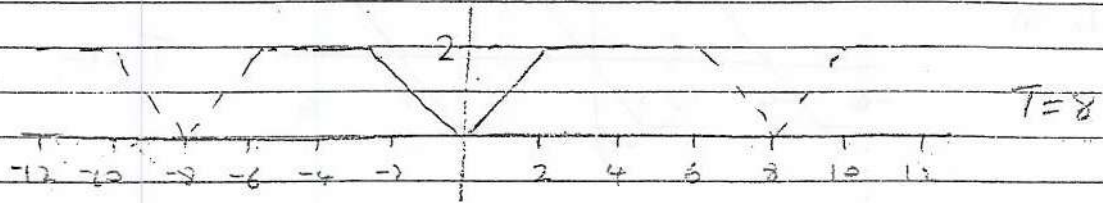
$$\therefore \text{Series is } \sum_{n=1}^{\infty} \left\{ \frac{4}{n^2\pi^2} \sin \frac{n\pi}{2} - \frac{2(-1)^n}{n\pi} \right\} \sin \frac{n\pi x}{2}$$

$$(c) \left(\frac{4}{\pi^2} + \frac{2}{\pi} \right) \sin \frac{\pi x}{2} + \left(0 - \frac{2}{2\pi} \right) \sin \pi x + \left(\frac{-4}{9\pi^2} + \frac{2}{3\pi} \right) \sin \frac{3\pi x}{2}$$

(d) zero.

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(a)

(b) Even $f_n \Rightarrow b_n = 0$

$$a_0 = \frac{4}{T} \int_0^{T/2} f(x) dx = \frac{4}{8} \left[\int_0^2 x dx + \int_2^4 2 dx \right]$$

$$= \frac{1}{2} \left[\frac{1}{2} x^2 \Big|_0^2 + 2x \Big|_2^4 \right] = \frac{1}{2} [2 + (8 - 4)] = 3.$$

$$a_n = \frac{4}{T} \int_0^{T/2} f(x) \cos \frac{n\pi x}{T} dx = \frac{4}{8} \left[\int_0^2 x \cos \frac{n\pi x}{4} dx + \int_2^4 2 \cos \frac{n\pi x}{4} dx \right]$$

$$= \frac{1}{2} \left[x \cdot \frac{4}{n\pi} \sin \frac{n\pi x}{4} \Big|_0^2 - \int_0^2 \frac{4}{n\pi} \sin \frac{n\pi x}{4} dx + 2 \cdot \frac{4}{n\pi} \sin \frac{n\pi x}{4} \Big|_2^4 \right]$$

$$= \frac{1}{2} \left[\left(\frac{8}{n\pi} \sin \frac{n\pi}{2} - 0 \right) + \left(\frac{4}{n\pi} \right)^2 \cos \frac{n\pi x}{4} \Big|_0^2 + \frac{8}{n\pi} \left(0 - \sin \frac{n\pi x}{2} \right) \right]$$

$$= \frac{8}{n^2 \pi^2} (\cos \frac{n\pi}{2} - 1).$$

$$\therefore \text{series is } \frac{3}{2} + \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{\cos \frac{n\pi}{2} - 1}{n^2} \cos \frac{n\pi x}{4}$$

$$(c) \quad \frac{3}{2} + \frac{8}{\pi^2} \left(\frac{0-1}{1^2} \cos \frac{\pi x}{4} + \frac{-1-1}{2^2} \cos \frac{2\pi x}{4} + \frac{0-1}{3^2} \cos \frac{3\pi x}{4} \right)$$